Use of the “Radiosity Importance Concept” in the Error Estimation in Radiative Heat Transfer Design of Spacecraft

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***Abstract*—The use of importance functions started in neutron transport simulations soon after World War II, it was used to accelerate ray tracing and later for global illumination, to optimize finite element methods (radiosity) and various Monte Carlo methods (path tracing, random walk radiosity, stochastic relaxation radiosity, ray bundles, and photon particle tracing for photon maps.**

In this paper, the “Radiosity Importance Concept”, an adjoint of the radiosity equation in the case of isothermal, diffuse surfaces is used to estimate the errors in radiative heat transfer in Spacecraft. The formulation of the radiosity equation is recalled as a function of the nature of the boundary conditions (either known temperature or fixed radiative heat flux). The importance is then defined as the dual quantity to radiosity. We explain how these equations can be used after the resolution of a radiative heat transfer situation, as a post processing step, to establish the accuracy of each individual radiative link between the active faces of a tri-dimensional surface geometrical model. Examples are provided.

I. Introduction

In order to design the thermal control subsystem of a spacecraft, the thermal engineer often uses dedicated software. As the radiative component can be predominant, software is often based on stochastic ray tracing to compute the energy exchanges between the surfaces which compose the tri-dimensional model of the spacecraft, as well as the heat fluxes from the heat sources to the spacecraft and the evacuation of heat to the deep space. A drawback of current software for space thermal engineering is the fact that the accuracy of the thermal results (temperatures and radiative heat fluxes) cannot be insured.

The purpose of this paper is to present an innovative method in order to bridge the gap.

II. Radiosity equation

Following Modest [1], the radiative balance of a tri-dimensional geometrical model can be expressed by the following set of equations, based on the radiosities J_i of the surfaces:

$$\frac{q_i}{\epsilon_i} = E_{b,i} - \sum_{j=1}^{N} F_{i-j} J_j - H_{0,j} \tag{1}$$

The radiative heat flux q_i, the self-emitted power $E_{b,i}$ and the radiosity J_i of a patch P_i are linked by the following relation:

$$\frac{q_i}{\epsilon_i} = \frac{1}{1-\epsilon_i}\left(E_{b,i} - J_i\right) \tag{2}$$

A. Boundary condition

For each patch P_i, two expressions of the radiosity equation can be obtained, in function of the nature of the boundary condition. Let us assume that the temperatures of the n first patches are known while the corresponding heat fluxes are unknown. On the other hand, the radiative heat fluxes of the $N-n$ other patches are fixed and their temperatures have been computed/ For each patch, the radiosity equation is given by the following relation, in function of the boundary condition:

$$\begin{cases} J_i = \epsilon_i E_{b,i} + \rho_i H_{0,i} + \rho_i \sum_{j=1}^{N} F_{i-j} J_j & \forall i \in [1,n] \\ J_i = q_i + H_{0,i} + \sum_{j=1}^{N} F_{i-j} J_j & \forall i \in [n+1,N] \end{cases} \tag{3}$$

B. Radiosity equation

The set of radiosity equations can be rewritten as follows:

$$\mathcal{T} J = S \tag{4}$$

The thermal source vector S is based on the boundary conditions and the external irradiation; this vector is assumed to be exact.

$$S = \begin{pmatrix} \epsilon_1 E_{b,1} + \rho_1 H_{0,1} \\ \vdots \\ \epsilon_n E_{b,n} + \rho_n H_{0,n} \\ q_{n+1} + H_{0,n+1} \\ \vdots \\ q_N + H_{0,N} \end{pmatrix} \tag{5}$$

$\mathcal{T}$ refers to the transport operator. It is a matrix of real numbers, which implies that $\mathcal{T}^* = \mathcal{T}^T$.

$$\mathcal{T} = \begin{pmatrix} 1-\rho_1 F_{1-1} & \dots & -\rho_1 F_{1-n} & \dots & -\rho_1 F_{1-N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ -\rho_n F_{n-1} & \dots & 1-\rho_n F_{n-n} & \dots & -\rho_n F_{n-N} \\ -F_{n+1-1} & \dots & -F_{n+1-n} & \dots & -F_{n+1-N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ -F_{N-1} & \dots & -F_{N-n} & \dots & 1-F_{N-N} \end{pmatrix} \quad (6)$$

To simplify the notation of the operator, an alternative set of thermo-optical properties ρ' is defined, such that the patches with fixed heat fluxes are associated with a diffuse reflectance ρ' equal to 100%. If $\mathcal{I}|_N$ denotes the identity matrix of rank N and F corresponds to the view factor matrix, the transport operator can be rewritten as:

$$\mathcal{T} = \mathcal{I}|_N - \rho' F \quad (7)$$

III. Importance

We define an abstract quantity, that we call importance and which is governed by the adjoint set of equations based on the transport operator $\mathcal{T}$ [2]:

$$\mathcal{T}^T I = R \quad (8)$$

where R is the reception vector, corresponding to the initial distribution of importance I. Importance is the quantity dual to radiosity; it is propagated throughout the geometrical model like radiosity but in the opposite direction. If radiosity J_i denotes the energy diffusely emitted by a surface i, due to a distribution of thermal sources S, importance I_i corresponds to the energy impact of surface i onto the whole model, represented by the distribution of receptors R.

Considering the previous definition of importance, it appears naturally that the reception term R_i of a patch P_i is equal to its area A_i. This relation has been rigorously demonstrated in [3].

$$R_i = A_i \quad \forall i \in [1, N] \quad (9)$$

The surface importance W_i of patch P_i is defined as the importance per unit area:

$$W_i = \frac{I_i}{A_i} \quad (10)$$

Given the definition of the reception vector R, the surface importance is always greater than or equal to unity. While the importance strongly depends on the discretization of the geometry, the surface importance does not. Literally speaking, the surface importance of a patch corresponds to the impact of this patch on the thermal balance. This can be shown thanks to the radiative energy.

IV. Radiative energy

The radiative energy $v(J)$ is defined as the scalar product of the radiosity vector J by the reception vector R [4]. After some mathematical operations, it can be shown that it is equal to the scalar product of the importance vector I by the source vector S, illustrating the duality of radiosity and importance.

$$v(J) = R^T J = I^T S \quad (11)$$

In radiative heat transfer, the reception vector is equivalent to the area vector, yielding:

$$v(J) = \sum_{i=1}^{N} A_i J_i \quad (12)$$

Each term $A_i J_i$ is the diffuse energy emitted by patch P_i; it is the sum of the self-emission and the diffuse reflection of all incident radiations. $v(J)$ corresponds to the energy which is diffusely emitted by all the surfaces of the geometrical model.

V. Radiative error

Based on the radiative energy $v(J)$ and the notion of importance, a measure of the energy error can be derived. We assume that the main source of error in the radiative computation is linked to the view factors. If the view factors are computed by stochastic ray tracing, an error measure, based on statistics, can be derived for each single view factor. This error measure is only geometrical; it is not representative of the energy error. Here,w e present a way to derive such an error measure. First, a global measure of the error affecting the total radiative energy is established. Then, a local measure, affecting each individual patch, is given.

A. Global error

Let $\widetilde{\mathcal{T}}$ be the approximated transport operator:

$$\widetilde{\mathcal{T}} = \mathcal{T} + \Delta\mathcal{T} \quad (13)$$

The diffuse reflectances can be assumed to be exact. The error is only due to the view factors. Each term $\mathcal{T}_{i-j}$ of the transport operator is affected by an error $\rho'_i \Delta F_{i-j}$. The thermal radiosities, computed by inverting equation (3), are also approximated:

$$\widetilde{J} = J + \Delta J \quad (14)$$

As the thermal source S is assumed to be exact, equation (4) yields the following relation:

$$\widetilde{T}\widetilde{J} = S \quad (15)$$

By combining equations (13) and (15), we obtain the following expression, where the operator is the exact transport operator and where the source term is perturbed by a quantity $\Delta\mathcal{T}\widetilde{J}$:

$$\mathcal{T}\widetilde{J} = S - \Delta\mathcal{T}\widetilde{J} \quad (16)$$

The energy error is defined thanks to the radiative energy $v(J)$:

$$v(\Delta J) = R^T \left(J - \widetilde{J}\right) = I^T \Delta\mathcal{T}\widetilde{J} \quad (17)$$

This quantity is the error introduced in the radiative energy $v(J)$ by the approximations of the transport operator and the radiosities. The importance cannot be exactly computed; it is associated with an error due to the approximation of the transport operator. $\widetilde{I}^T \Delta \mathcal{T} \widetilde{J}$ is used as the approximation of $I^T \Delta \mathcal{T} \widetilde{J}$. Equation (17) is a double sum on all the surfaces which compose the geometrical model. A particular term $\widetilde{I}_i^T \Delta \mathcal{T}_{i-j} \widetilde{J}_j$ corresponds to the error energy from surface j to surface i. This expression allows us to establish the error induced by the approximation of the transport term $\mathcal{T}_{i-j}$.

B. Local error

On the basis of the definition of the global radiative error (equation (17)), we can derive the expression of the error which is associated with each radiosity J_i. The global error can be rewritten as follows:

$$v(\Delta J) = \sum_{i=1}^{N} A_i \underbrace{W_i \sum_{j=1}^{N} \Delta \mathcal{T}_{i-j} \widetilde{J}_j}_{\Delta J_i} \tag{18}$$

It appears that the radiosity error ΔJ_i is given by the following expression:

$$\Delta J_i = \frac{I_i}{A_i} \rho_i' \sum_{j=1}^{N} \Delta F_{i-j} \widetilde{J}_j \tag{19}$$

$$= W_i \rho_i' \sum_{j=1}^{N} \Delta F_{i-j} \widetilde{J}_j \tag{20}$$

This equation states that the radiosity error of a patch P_i is proportional to its surface importance I_i, *i.e.* its influence on the energy balance of the tri-dimensional model. The influence of the thermo-optical properties depends on the nature of the boundary condition. If the radiative heat flux q_i of the surface is fixed, its reflectance does not appear in equation (20); if its temperature T_i is given, the radiosity error is proportional to the diffuse reflectance ρ_i. Finally, the radiosity error of a patch i is due to the diffuse reflection of the radiosities received from the other surfaces, which is not correctly estimated because of the errors on the view factors.

The radiosity error can be linked to an error in terms of temperature or radiative flux, in function of the boundary of the surface. An error ΔJ_i on a surface with fixed radiative heat flux corresponds to an error in terms of temperature given by:

$$\Delta T_i = \frac{\Delta J_i}{4 \sigma T_i^3} \tag{21}$$

where σ is the Stefan-Boltzmann constant, equal to $5.669\,10^{-8} W/m^2/K^4$. On the other hand, an error ΔJ_i on a surface with fixed temperature is equivalent to an error in heat flux equal to:

$$\Delta q_i = \frac{\epsilon_i}{1 - \epsilon_i} \Delta J_i \tag{22}$$

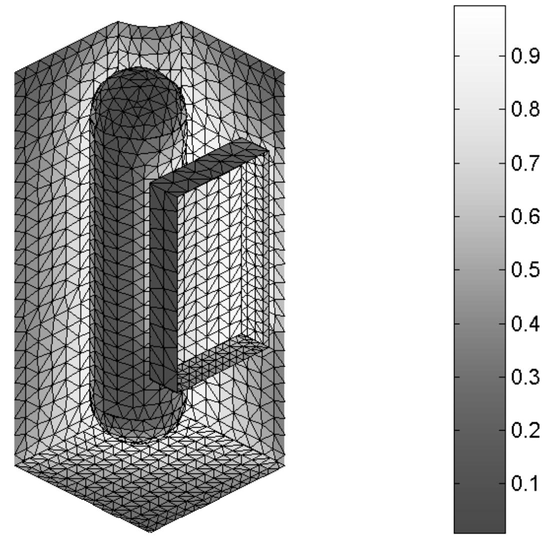

Fig. 1. View factors distribution.

VI. APPLICATION

In order to illustrate the use of adjoint equations, the model presented in Figure 1 has been considered. It represents a quarter of the inside of a spacecraft; it is composed of 12 geometrical primitives modeling a tank, an electronic box, a cylinder and two panels. The geometry has been meshed into finite elements (about 1 800 triangles of the first degree).

A. Boundary conditions

The temperature of the basis is fixed at $293K$ while the tank is maintained at $303K$. The other components are assumed to be adiabatic. An additional node represents the cold space and is set at $0K$. The basis of the cylinder, the reinforcements and the electronic box are characterized by a coating `coating1` while the tank presents a coating `coating2`.

Coating	Emittance	Diffuse reflectance
`coating1`	0.95	0.05
`coating2`	0.5	0.5

TABLE I
THERMO-OPTICAL PROPERTIES.

B. Computation of the view factors

In this document, we assumed that the computation of the view factors was accompanied by an error measure associated with each single view factor. This error measure is easily accessible with stochastic ray tracing methods [5]. In this study, we used a particular stochastic ray tracing method, which is called stratified hemisphere and which is characterized by an improved convergence w.r.t. classical ray tracing approaches. The distribution of the view factors is given in Figure 1.

The distribution of the associated geometrical error, associated with the view factors. This error distribution is linked to the particular method used for the computation of the view factors (in our case, the stratified hemisphere method).

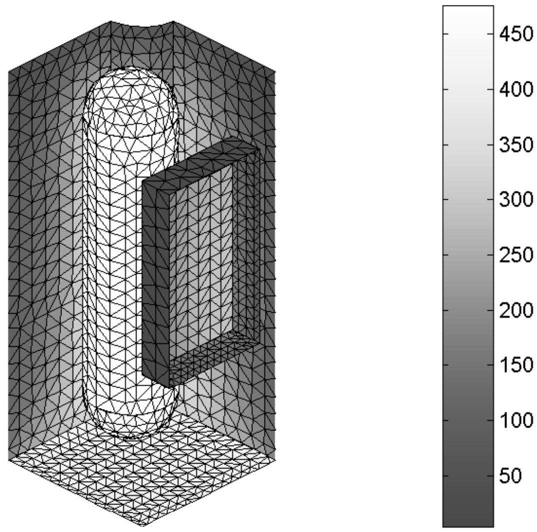

Fig. 2. Thermal radiosity distribution.

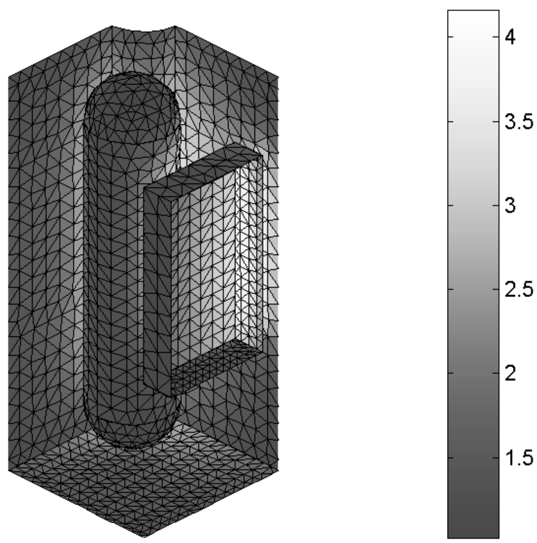

Fig. 3. Surface importance distribution.

C. Thermal radiosities

Based on the boundary conditions, equation (4) is solved. The distribution of thermal radiosities J_i is detailed in Figure 2.

D. Importance

The last quantity to compute is the importance. The initial importance vector is defined by the area of the elements. In function of the view factors and the surface thermo-optical properties, we obtain the distribution of importance given in Figure 3 (the displayed quantity is in fact the importance per unit area). This quantity is always larger than (or equal to) unity.

E. Radiosity error

On the basis of the stratified hemisphere method, we are able to compute the absolute error affecting the view factors.

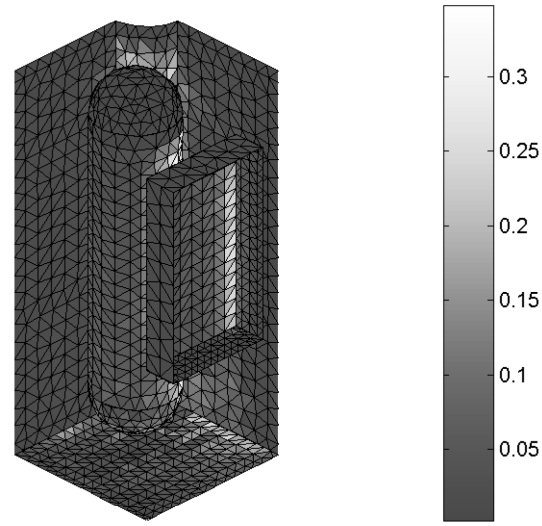

Fig. 4. Radiosity error distribution.

This error is combined with the computed thermal radiosities and the surface importance in equation (20). Figure 4 presents the evolution of the radiosity error. For each patch P_i, we can compute the error associated with the unknown quantity (temperature or heat flux) based on this distribution and the nature of the boundary condition. Giving access to the accuracy of the thermal computation, the use of importance and adjoint equations is a clear improvement for radiative heat transfer computation.

VII. Conclusion

In this paper, we have established the basis of an innovative formulation using adjoint equations for radiosity and importance, which can be used in a post-process step to estimate the error characterizing each radiative link in a 3D geometrical model. This method, which yields guarantees on the accuracy of the thermal results, is a clear improvement for radiative heat transfer computation.

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