EXPERIENCE WITH THE MULTIPLE CENTRALITY CORRECTIONS INTERIOR POINT ALGORITHM FOR OPTIMAL POWER FLOW

Florin Capitanescu Mevludin Glavic Louis Wehenkel
Department of Electrical Engineering and Computer Science, University of Liège, Belgium

ABSTRACT

This paper analyzes the ability of the *Multiple Centrality Corrections* (MCC) interior point algorithm to solve various classical *optimal power flow* (OPF) variants, namely: the minimization of generation cost, the minimization of active power losses, the maximization of power system loadability and the minimization of the amount of load curtailment. The performances of the MCC approach are assessed with respect to the predictor-corrector algorithm, which is widely recognized as the best interior point method based optimizer to date. Illustrative examples on three test systems up to 300 buses are provided.

1 INTRODUCTION

Since the early 60's [1] the Optimal Power Flow (OPF) problem has become progressively an indispensable tool in power systems planning, operational planning and real-time operation, and that whatever the electricity market environment: liberalized or not [2].

The OPF is stated in its general form as a nonlinear, non-convex, large-scale, static optimization problem with both continuous and discrete variables. It aims at optimizing some objective by acting on available control means while satisfying network power flow equations, physical and operational constraints.

The Interior Point Method (IPM) [3] is a very appealing approach to the OPF problem due to its speed of convergence and ease of handling inequality constraints by logarithmic barrier functions [14, 8, 9]. Another advantage of this method is that a strictly feasible initial point is not required. On the other hand, the main drawback of the IPM stems from the fact that slack variables and their corresponding dual variables must remain positive at every iteration, which may drastically shorten the Newton step length(s). This is especially the case of the pure Primal-Dual (PD) algorithm. Two classes of methods mainly aiming to overcome this flaw can be distinguished: *higher-order IPMs* (e.g. the predictor-corrector [4], multiple predictor-corrector [6], multiple centrality corrections [5]) and *non-interior point methods* (e.g. the unlimited point algorithm [11], the complementarity method [12]).

The Multiple Centrality Corrections (MCC) algorithm belongs to the class of higher-order interior-point methods. The latter rely on the observation that the factorization of the Hessian matrix is, by far, the most expensive computational task of an interior-point algorithm iteration. Indeed, in most power system applications, the Hessian factorization takes much more time than the backward/forward solution of the already factorized linear system [14, 9]. The aim of these methods is hence to draw maximum of profit from the factorized matrix, with little additional computational effort. Practically, they solve one or more extra linear system(s), based on the same factorized matrix, expecting thereby to yield an improved search direction. Obviously, higher-order methods are of interest as long as they are able to reduce the computational time with respect to the pure primal-dual algorithm.

The MCC algorithm was initially proposed by Gondzio for linear programming [5]. Then, it was successfully applied also to solve nonlinear programming problems such as: the minimization of active power losses [13, 10], the maximization of power system loadability and the minimization of the amount of load shedding [10], respectively. The MCC algorithm is motivated by the Gondzio's observation that the convergence of an interior-point algorithm is worsened by a large discrepancy between complementarity products at an iteration. The MCC algorithm is based on the belief that the closer the point to be optimized to the central path, the larger step length(s) can be taken afterwards. The aim of the MCC approach is twofold: (i) to increase the step length in both primal and dual spaces at the current iteration and (ii) to improve the centrality of the next iteration.

In this paper we extensively explore the ability of the MCC algorithm to solve various OPF variants while comparing its performance to the Mehrotra's Predictor-Corrector (PC) algorithm, which is widely recognized as the best IPM-based optimizer.

The paper is organized as follows. Section 2 introduces very briefly the OPF problem. Section 3 offers an overview of the MCC algorithm. Section 4 provides numerical results while conclusions are drawn in section 5.

2 OPTIMAL POWER FLOW STATEMENT

An OPF can be compactly formulated as a general nonlinear programming problem:

$$\min f(\mathbf{x}) \tag{1}$$

$$\text{subject to :} \quad \mathbf{g}(\mathbf{x}) = \mathbf{0} \tag{2}$$

$$\underline{\mathbf{h}} \leq \mathbf{h}(\mathbf{x}) \leq \overline{\mathbf{h}} \tag{3}$$

where $\mathbf{x}$ is an n-dimensional vector that encompasses both control and state variables, f is a scalar function that

represents the optimization goal, $\mathbf{g}$ is an m-dimensional vector of functions that involve mainly bus active and reactive power balance equations, $\mathbf{h}$ is a p-dimensional vector that comprises functional (e.g. branch current) and simple variables (e.g. voltage magnitude, active and reactive generator powers, variable ratio of transformer, shunt reactance), ($\underline{\mathbf{h}}$) and ($\overline{\mathbf{h}}$) are lower and upper bound vectors corresponding to operational and physical limits of the power system.

3 MULTIPLE CENTRALITY CORRECTIONS ALGORITHM

3.1 Obtaining the Optimality Conditions

Let us consider the general nonlinear programming problem (1-3). One first add slack variables to inequality constraints, transforming them into equality constraints:

$$\begin{aligned} & \min f(\mathbf{x}) \\ \text{subject to :} \quad & \mathbf{g}(\mathbf{x}) = \mathbf{0} \\ & \mathbf{h}(\mathbf{x}) - \underline{\mathbf{h}} - \underline{\mathbf{s}} = \mathbf{0} \\ & -\mathbf{h}(\mathbf{x}) + \overline{\mathbf{h}} - \overline{\mathbf{s}} = \mathbf{0} \\ & \underline{\mathbf{s}}, \overline{\mathbf{s}} \geq \mathbf{0} \end{aligned}$$

where the vectors $\mathbf{x}$, $\underline{\mathbf{s}} = [\underline{s}_1, \ldots, \underline{s}_p]^T$ and $\overline{\mathbf{s}} = [\overline{s}_1, \ldots, \overline{s}_p]^T$ are called *primal variables*.
Slack variables $\underline{s}_i$ and $\overline{s}_i$ ($i = 1, \ldots, p$) are added to the objective function as logarithmic barrier terms, resulting the following equality constrained optimization problem:

$$\begin{aligned} & \min f(\mathbf{x}) - \mu \sum_{i=1}^{p} (\ln \underline{s}_i + \ln \overline{s}_i) \\ \text{subject to :} \quad & \mathbf{g}(\mathbf{x}) = \mathbf{0} \\ & \mathbf{h}(\mathbf{x}) - \underline{\mathbf{h}} - \underline{\mathbf{s}} = \mathbf{0} \\ & -\mathbf{h}(\mathbf{x}) + \overline{\mathbf{h}} - \overline{\mathbf{s}} = \mathbf{0} \end{aligned}$$

where μ is a positive scalar called *barrier parameter* which is gradually decreased to zero as iteration progresses.
The Lagrangian of the above equality constrained optimization problem is:

$$\begin{aligned} \mathcal{L}_\mu(\mathbf{y}) = \quad & f(\mathbf{x}) - \mu \sum_{i=1}^{p} (\ln \underline{s}_i + \ln \overline{s}_i) - \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x}) \\ & - \underline{\boldsymbol{\pi}}^T (\mathbf{h}(\mathbf{x}) - \underline{\mathbf{h}} - \underline{\mathbf{s}}) - \overline{\boldsymbol{\pi}}^T (-\mathbf{h}(\mathbf{x}) + \overline{\mathbf{h}} - \overline{\mathbf{s}}) \end{aligned}$$

where the vectors of Lagrange multipliers $\boldsymbol{\lambda}$, $\underline{\boldsymbol{\pi}}$ and $\overline{\boldsymbol{\pi}}$ are called *dual variables* and $\mathbf{y} = [\underline{\mathbf{s}} \ \overline{\mathbf{s}} \ \underline{\boldsymbol{\pi}} \ \overline{\boldsymbol{\pi}} \ \boldsymbol{\lambda} \ \mathbf{x}]^T$.
The *perturbed* Karush-Kuhn-Tucker (KKT) first order necessary optimality conditions of the resulting problem are [3]:

$$\begin{bmatrix} -\mu\mathbf{e} + \underline{\mathbf{S}}\,\underline{\boldsymbol{\pi}} \\ -\mu\mathbf{e} + \overline{\mathbf{S}}\,\overline{\boldsymbol{\pi}} \\ -\mathbf{h}(\mathbf{x}) + \underline{\mathbf{h}} + \underline{\mathbf{s}} \\ \mathbf{h}(\mathbf{x}) - \overline{\mathbf{h}} + \overline{\mathbf{s}} \\ -\mathbf{g}(\mathbf{x}) \\ \nabla f(\mathbf{x}) - \mathbf{J_g}^T \boldsymbol{\lambda} - \mathbf{J_h}^T (\underline{\boldsymbol{\pi}} - \overline{\boldsymbol{\pi}}) \end{bmatrix} = \mathbf{0} \qquad (4)$$

where $\underline{\mathbf{S}}$, $\overline{\mathbf{S}}$ are diagonal matrices of slack variables, $\mathbf{e} = [1, ..., 1]^T$, $\nabla f(\mathbf{x})$ is the gradient of f, $\mathbf{J_g}$ is the Jacobian of $\mathbf{g}(\mathbf{x})$ and $\mathbf{J_h}$ is the Jacobian of $\mathbf{h}(\mathbf{x})$.

3.2 Solving the KKT Optimality Conditions

The perturbed KKT optimality conditions (4) can be solved by the Newton method. Let us remark that at the heart of IPM is the theorem [3], which proves that as μ tends to zero, the solution $\mathbf{x}(\mu)$ approaches $\mathbf{x}^\star$, the solution of the problem (1-3). The goal is therefore not to solve completely this nonlinear system for a given value of μ, but to solve it approximately and then diminishing the value of μ iteratively until convergence is reached.
MCC algorithm consists of a *predictor* and a *corrector* step as in the Mehrotra's predictor-corrector procedure. The predictor step is responsible for the optimization, i.e. reducing the primal and dual infeasibilities and the complementarity gap. The corrector step is responsible for improving centrality as well as for keeping the current iterate away from the feasibility boundary.

3.2.1 The Predictor Step

The predictor step objective is to dynamically estimate the barrier parameter μ. To this purpose one solves the system (4) for the *affine-scaling* direction, obtained by neglecting μ in its right-hand side, that is:

$$\mathbf{H} \begin{bmatrix} \Delta\underline{\mathbf{s}}_{af} \\ \Delta\overline{\mathbf{s}}_{af} \\ \Delta\underline{\boldsymbol{\pi}}_{af} \\ \Delta\overline{\boldsymbol{\pi}}_{af} \\ \Delta\boldsymbol{\lambda}_{af} \\ \Delta\mathbf{x}_{af} \end{bmatrix} = \begin{bmatrix} -\underline{\mathbf{S}}\,\underline{\boldsymbol{\pi}} \\ -\overline{\mathbf{S}}\,\overline{\boldsymbol{\pi}} \\ \mathbf{h}(\mathbf{x}) - \underline{\mathbf{h}} - \underline{\mathbf{s}} \\ -\mathbf{h}(\mathbf{x}) + \overline{\mathbf{h}} - \overline{\mathbf{s}} \\ \mathbf{g}(\mathbf{x}) \\ -\nabla f(\mathbf{x}) + \mathbf{J_g}^T \boldsymbol{\lambda} + \mathbf{J_h}^T (\underline{\boldsymbol{\pi}} - \overline{\boldsymbol{\pi}}) \end{bmatrix}$$

where $\mathbf{H}$ is the second derivatives Hessian matrix.
Next the affine complementarity gap ρ_{af} is computed:

$$\begin{aligned} \rho_{af} \quad = & (\underline{\mathbf{s}}_{af} + \alpha_{af} \Delta\underline{\mathbf{s}}_{af})^T (\underline{\boldsymbol{\pi}}_{af} + \alpha_{af} \Delta\underline{\boldsymbol{\pi}}_{af}) + \\ & (\overline{\mathbf{s}}_{af} + \alpha_{af} \Delta\overline{\mathbf{s}}_{af})^T (\overline{\boldsymbol{\pi}}_{af} + \alpha_{af} \Delta\overline{\boldsymbol{\pi}}_{af}) \end{aligned}$$

where $\alpha_{af} \in (0, 1]$ is the step length which would be taken along the affine scaling direction if the latter was used.
Finally, the barrier parameter for the next iteration is estimated from:

$$\mu_{af} = \min \left\{ \left(\frac{\rho_{af}}{\rho} \right)^2, 0.2 \right\} \frac{\rho_{af}}{2p}$$

where $\rho = \underline{\mathbf{s}}^T \underline{\boldsymbol{\pi}} + \overline{\mathbf{s}}^T \overline{\boldsymbol{\pi}}$ denotes the complementarity gap at the current iterate.
The goal of this adaptive scheme is to significantly reduce the barrier parameter when a large decrease in complementarity gap from affine direction is obtained ($\rho_{af} \ll \rho$) and to slightly reduce it otherwise.

3.2.2 The Corrector Step

The aim of this step is to compute a corrector direction $\Delta\mathbf{y}_{co}$ such that: (i) a larger step size can be taken for the composite direction $\Delta\mathbf{y} = \Delta\mathbf{y}_{af} + \Delta\mathbf{y}_{co}$ and (ii) all complementarity products are driven in a vicinity of the *central path*.

We describe in the sequel the procedure to accomplish these objectives. Let us assume that we propose to increase the step length α_{af} to:

$$\tilde{\alpha} = \min(\alpha_{af} + \delta_{\alpha}, 1)$$

where δ_{α} is the desired improvement of the step length. The empirical observation that the two goals: improving centrality and increasing the step size might be contradictory, especially for large values of δ_{α}, constrains to use small values for δ_{α} (very often $\delta_{\alpha} \in [0.1, 0.2]$) [5].

Let $\mathbf{y}$ be the solution at the current iteration. Obviously, whenever $\alpha_{af} < 1$ at the *trial* point

$$\tilde{\mathbf{y}} = \mathbf{y} + \tilde{\alpha}\Delta\mathbf{y}_{af}$$

some slack variables and/or their corresponding dual variables violate strictly positivity conditions $(\underline{\mathbf{s}}, \overline{\mathbf{s}}, \underline{\boldsymbol{\pi}}, \overline{\boldsymbol{\pi}}) > \mathbf{0}$. The corrector term $\Delta\mathbf{y}_{co}$ has thus to offset for these negative terms as well as to drive the trial point in the neighbourhood of the central path. To this end a *target* close to the central path must be defined. Because the most natural such target, the analytic center $\mu_{af}\mathbf{e}$ is usually unreachable, for practical purposes, we require instead that all complementarity products belong to the interval $[\beta_{min}\mu_{af}, \beta_{max}\mu_{af}]$. Typical values for β_{min} and β_{max} are: $\beta_{min} = 0.1$ and $\beta_{max} = 10$.

Next, one computes the complementarity products at the trial point: $\tilde{\mathbf{v}} = \tilde{\underline{\mathbf{S}}}\,\tilde{\underline{\boldsymbol{\pi}}}$ and $\tilde{\mathbf{z}} = \tilde{\overline{\mathbf{S}}}\,\tilde{\overline{\boldsymbol{\pi}}}$. Then one identifies components of $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{z}}$ that do not belong to the interval $[\beta_{min}\mu_{af}, \beta_{max}\mu_{af}]$, called *outlier* complementarity products. The corrector step effort focuses in correcting the outliers only in order to improve the centrality of the next iterate. Vectors of complementarity products $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{z}}$ are projected to a hypercube $H = [\beta_{min}\mu_{af}, \beta_{max}\mu_{af}]^{2p}$ to define the target:

$$v_i^t = \begin{cases} \beta_{min}\mu_{af}, & \text{if } \tilde{v_i} < \beta_{min}\mu_{af} \\ \beta_{max}\mu_{af}, & \text{if } \tilde{v_i} > \beta_{max}\mu_{af} \\ \tilde{v_i}, & \text{otherwise} \end{cases}$$

$$z_i^t = \begin{cases} \beta_{min}\mu_{af}, & \text{if } \tilde{z_i} < \beta_{min}\mu_{af} \\ \beta_{max}\mu_{af}, & \text{if } \tilde{z_i} > \beta_{max}\mu_{af} \\ \tilde{z_i}, & \text{otherwise} \end{cases}$$

Finally, the corrector direction $\Delta\mathbf{y}_{co}$ is obtained as the solution of the following linear system:

$$\mathbf{H}\begin{bmatrix} \Delta\underline{\mathbf{s}}_{co} \\ \Delta\overline{\mathbf{s}}_{co} \\ \Delta\underline{\boldsymbol{\pi}}_{co} \\ \Delta\overline{\boldsymbol{\pi}}_{co} \\ \Delta\boldsymbol{\lambda}_{co} \\ \Delta\mathbf{x}_{co} \end{bmatrix} = \begin{bmatrix} \mathbf{v}^t - \tilde{\mathbf{v}} \\ \mathbf{z}^t - \tilde{\mathbf{z}} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

where the nonzero components of the right-hand-side correspond to the outlier complementarity products only. The new search direction can now be obtained:

$$\Delta\mathbf{y} = \Delta\mathbf{y}_{af} + \Delta\mathbf{y}_{co}$$

Then, the actual step length α is computed so as to preserve non-negativity conditions and variables are updated:

$$\mathbf{y} \leftarrow \mathbf{y} + \alpha\Delta\mathbf{y}$$

The corrector step can be applied a desired number of times. In such a case, the current corrector becomes the predictor for a new corrector, that is

$$\Delta\mathbf{y}_{af} \leftarrow \Delta\mathbf{y}_{af} + \Delta\mathbf{y}_{co} \quad \text{and} \quad \alpha_{af} \leftarrow \alpha$$

3.3 Some Implementation Issues of MCC Algorithm

For the sake of facilitating the presentation, we have considered a common step length α for updating both primal and dual variables. Our experience showed, however, that the MCC algorithm behaves better when applying separate step lengths in primal and dual spaces.

As regards the choice of the desired improvement of the step size δ_{α}, two solutions are proposed: either the use of a constant value $\delta_{\alpha} = 0.1$ [5, 13], or the use of an adaptive value according to the formula $\delta_{\alpha} = (1 - \alpha_{af})/K$ while imposing additionally that δ_{α} not be smaller than 0.1 or greater than 0.2 [13, 10]. In order to reduce the number of centrality corrections computed at an iteration while aiming to obtain the largest increase in the step size we suggest to set $\delta_{\alpha} = 0.2$. We have found that this setting offers better convergence performances in terms of CPU time and iterations number than the above mentioned proposals. We have not encountered any convergence trouble caused by this "optimistic" setting.

In our implementation we compute a new centrality correction only if it improves considerably the step lengths. To this end, we allow the computation of a new centrality correction only if the gain in step length is superior to $\varepsilon_{\alpha} = 0.03$. In contrast, according to other authors, even as small value as $\varepsilon_{\alpha} = 0.01$ may be accepted [5, 13].

The most challenging problem of the MCC algorithm is the choice of the maximal number of corrections allowed K, as the objective is not only to reduce the number of iterations comparatively to the PC algorithm but also to save some CPU time. To this respect some heuristics are proposed [5, 13]. Their idea is to solve first a set of problems by progressively increasing the maximal number of corrections allowed K (going from 1 to 10, for instance). Results are then gathered together and let nbiter_{MCC}^{K} and nbiter_{PC} be the number of iterations needed to converge for the MCC and PC algorithm, respectively. When dealing with a new problem, K is established automatically as follows. One counts at the first iteration the elapsed time of the Hessian factorization (f) and of the backward/forward solution of the linear system (s), respec-

tively. After computing the ratio $r_{f/s} = f/s$, K is chosen such that [5]:

$$\frac{\text{nbiter}^K_{MCC}}{\text{nbiter}_{PC}} \left[1 + \frac{K}{(r_{f/s}+2)}\right] < 1$$

Note, finally, that the algorithms performance in terms of the CPU time may be somewhat dependent to the solver used for the solution of the linear system of equations.

4 NUMERICAL RESULTS

In this section we present numerical results of four OPF applications while comparing both MCC and PC algorithms performance. The OPF has been coded in C++ and runs under Cygwin or Linux environments. It has been tested on three test systems, namely a 60 bus system which is a modified variant of the Nordic32 system, and the popular IEEE118 and IEEE300 systems. A summary of these test systems is given in Table 1, where nn, ng, nc, nb, nl, nt, no, and ns are the number of: buses, generators, loads, branches, lines, all transformers, variable ratio transformers and shunts.

Table 1: Test systems summary

system	nn	ng	nc	nb	nl	nt	no	ns
Nordic32	60	23	22	81	57	31	4	12
IEEE118	118	54	91	186	175	11	9	14
IEEE300	300	69	198	411	282	129	50	14

All tests have been performed on a PC Pentium IV of 1.7-GHz and 512-Mb RAM. The MCC algorithm runs with the following parameters: $\beta_{min} = 0.1$, $\beta_{max} = 10$, $\delta_\alpha = 0.2$ and $\varepsilon_\alpha = 0.03$. For the sake of illustration of how the MCC algorithm works we set, before of each OPF run, the maximum number allowed of corrections allowed K. The latter is let to vary from $K = 1$ to $K = 8$, by a step of 1.

The convergence tolerances are set to 10^{-4} for the primal feasibility and the scaled dual feasibility, and 10^{-6} for the scaled complementarity gap and the scaled objective function variation from an iteration to the next [7].

4.1 Minimizing Overall Cost of Generation

We first focus on minimizing, by means of a full OPF, the overall cost of generators active output. The control variables used are: the active and reactive power of generators, controllable transformers ratio and shunt reactances. The equality constraints are the bus active and reactive power flow equations. The inequality constraints include bounds on all above mentioned control variables as well as limits on voltage magnitudes and branch currents.

Table 2 displays the number of iterations to convergence as well as the CPU times [1] (in seconds) for the MCC algorithm with different values of maximal number of corrections allowed. For comparison purposes this table also contains the information relative to the PC algorithm (marked in bold), which is considered the benchmark, and the PD one.

Table 2: Number of iterations to convergence and CPU times

interior point	Nordic32		IEEE118		IEEE300	
algorithm	iters	time	iters	time	iters	time
MCC ($K = 1$)	20	0.92	16	1.13	20	3.10
MCC ($K = 2$)	16	0.79	14	1.09	16	2.82
MCC ($K = 3$)	14	0.77	13	1.06	14	2.50
MCC ($K = 4$)	12	0.69	11	0.97	13	2.35
MCC ($K = 5$)	12	0.71	11	0.99	12	2.23
MCC ($K = 6$)	12	0.72	10	0.90	12	2.28
MCC ($K = 7$)	12	0.74	10	0.92	11	2.16
MCC ($K = 8$)	12	0.75	10	0.94	11	2.20
PC	**13**	**0.61**	**11**	**0.80**	**15**	**2.33**
PD	21	0.82	18	1.10	23	3.02

Table 3 shows the number and the type of binding constraints at optimum, where P_g, Q_g, I, V, r and b reffer to constraints relative to generator active power, generator reactive power, branch current, bus voltage magnitude, controllable transformer ratio and shunt reactance, respectively.

Table 3: Number and type of active constraints

	active constraints						
system	P_g	Q_g	I	V	r	b	total
Nordic32	15	1	5	26	0	4	51
IEEE118	3	8	3	25	0	5	44
IEEE300	3	26	4	59	3	2	97

Expectedly, for the MCC algorithm, the higher the value of K, the less the number of iterations. Recall that the computation of a new centrality correction at an iteration is allowed only if it leads to a significant improvement of step lengths. Consequently, a distinction should be made between the maximal number of corrections allowed K and the actual number of corrections at an iteration, which is less or equal to K.

Note that the MCC algorithm leads to a reduction of number of iterations with respect to the PC algorithm for the Nordic32 system (when $K \geq 4$), for the IEEE118 system (when $K \geq 6$) and for IEEE300 system (when $K \geq 3$). On the other hand, except of the IEEE300 system (for $5 \leq K \leq 8$), this reduction of iterations count does not translate in a CPU time saving.

An interesting comparison can be made between the PC and the MCC algorithm run with $K = 1$. In such case the computational effort per iteration of both algorithms is almost the same. Observe that, for all systems, when using a single corrector only the PC algorithm clearly outperforms the MCC one. For this OPF variant the use of the MCC algorithm is of interest for higher values of K.

One can also remark that, for all the test systems used, the MCC algorithm with $K \geq 2$ converges faster and takes a less number of iterations than the pure PD algorithm. This observation holds true also for most of the OPF variants studied hereafter.

[1] CPU time concerns the optimization process only except of processing data and print of results

4.2 Minimizing Active Power Losses

We now deal with the minimization of transmission active power losses, counted as the sum of active losses over all branches of the system. The control variables considered are: slack generator active power, generators reactive power, controllable transformers ratio and shunt reactance. The equality constraints involve the buses power balance. The inequality constraints concern bounds on: slack generator active power, generators reactive power, voltage magnitudes, transformer with controllable ratio and shunt reactance. Voltage magnitudes are allowed to vary between 0.95 pu and 1.05 pu in all buses.
Table 4 provides the number of iterations to convergence and the CPU times for the MCC, PC and PD algorithms, while Table 5 shows the number and the type of binding constraints at optimum.

Table 4: Number of iterations to convergence and CPU times

interior-point	Nordic32		IEEE118		IEEE300	
algorithm	iters	time	iters	time	iters	time
MCC ($K=1$)	9	0.29	11	0.61	18	2.14
MCC ($K=2$)	7	0.25	10	0.59	13	1.66
MCC ($K=3$)	7	0.25	9	0.58	11	1.52
MCC ($K=4$)	6	0.24	8	0.54	11	1.55
MCC ($K=5$)	6	0.24	7	0.52	10	1.48
MCC ($K=6$)	6	0.25	7	0.53	10	1.52
MCC ($K=7$)	6	0.25	7	0.54	10	1.52
MCC ($K=8$)	6	0.25	7	0.55	10	1.52
PC	**8**	**0.26**	**10**	**0.58**	**12**	**1.44**
PD	11	0.31	13	0.62	16	1.63

Table 5: Number and type of active constraints

	active constraints				
system	V	Q_g	r	b	total
Nordic32	21	0	0	4	25
IEEE118	15	12	0	4	31
IEEE300	60	23	1	4	88

The active losses at optimum are: 151.74 MW for the Nordic32 system, 116.52 MW for the IEEE118 system and 386.6 MW for the IEEE300 system, respectively, that is, of 7.90%, 12.31%, and 5.37% less than in the base case for the Nordic32, IEEE118 and IEEE300 system, respectively.
The MCC algorithms compares favorably with the PC one, for the Nordic32 system (when $K \geq 2$) and for the IEEE118 system (when $K \geq 3$), in terms of both iterations count and CPU time. On the other hand, for the IEEE300 system, despite of the smaller number of iterations needed to converge for the MCC algorithm (for $3 \leq K \leq 8$), the PC algorithm is slightly faster for all values of K.
Note that, the number of iterations and CPU times do not change for the Nordic32 system (when $6 \leq K \leq 8$) and for IEEE300 system (when $6 \leq K \leq 8$) despite of the increase of K. This is due to fact that the computed step length improvement is lower than $\delta_\alpha = 0.03$ which forbids the computation of new centrality corrections.

4.3 Maximizing Power System Loadability

We now focus on the determination of the maximum loadability of a power system. We assume that all loads are increased proportionally with their base case consumptions, covered by the slack generator only. We consider the following control variables: generators reactive power, variable ratio of controllable transformers, shunts reactance and slack generator active output. The equality constraints concern buses active and reactive balance. The inequality constraints include limits on: generator reactive power, transformer variable ratio, shunt reactance and bus voltage magnitudes only. The latter are allowed to vary between 0.95 pu and 1.05 pu.
Table 6 gives the number of iterations to convergence and CPU times for the MCC, PC and PD algorithms, while Table 7 shows the number and the type of binding constraints at optimum.

Table 6: Number of iterations to convergence and CPU times

interior-point	Nordic32		IEEE118		IEEE300	
algorithm	iters	time	iters	time	iters	time
MCC ($K=1$)	13	0.42	div		23	3.25
MCC ($K=2$)	10	0.35	div		20	2.96
MCC ($K=3$)	10	0.36	div		17	2.71
MCC ($K=4$)	9	0.33	div		15	2.52
MCC ($K=5$)	9	0.34	div		15	2.59
MCC ($K=6$)	9	0.35	div		14	2.46
MCC ($K=7$)	9	0.36	div		14	2.48
MCC ($K=8$)	8	0.33	div		14	2.49
PC	**11**	**0.37**	**15**	**0.83**	**19**	**2.60**
PD	19	0.48	21	0.99	47	4.65

Table 7: Number and type of active constraints

	active constraints				
system	V	Q_g	r	b	total
Nordic32	17	5	0	6	28
IEEE118	27	21	0	10	58
IEEE300	57	29	4	5	95

The maximum loadability margin is: 537.8 MW for the Nordic32 system, 1119.1 MW for the IEEE118 system and 445.9 MW for the IEEE300 system.
The MCC algorithm behaves better than the PC one for the Nordic32 system (when $K \geq 2$) and for IEEE300 system (when $K \geq 4$) while the opposite happens for the other values of K. Note that, the MCC algorithm fails inexplicably to optimize the IEEE118 system. We mention, however, that such convergence problems have not been experienced for other systems and/or OPF variants and, therefore, this should not be taken as conclusive.

4.4 Minimizing the Amount of Load Shedding

We finally tackle the problem of minimizing the amount of load shedding for an infeasible power system situation such that an equilibrium point is restored. We assume that at each load bus the load shedding is done under constant power factor and not exceed 10%. The slack generator compensates for the active power imbalance only.

The control variables used are: load consumptions, controllable ratio of transformers, shunts reactance and slack generator active output. The equality constraints involve the buses power balance. The inequality constraints include limits on: the amount of bus load shedding (ϕ), controllable transformer ratio and shunt reactance.
Table 8 displays the number of iterations to convergence and CPU times for the MCC, PC and PD algorithms, while Table 9 shows the number and the type of binding constraints at optimum.

Table 8: Number of iterations to convergence and CPU times

interior-point	Nordic32		IEEE118		IEEE300	
algorithm	iters	time	iters	time	iters	time
MCC ($K=1$)	12	0.26	8	0.44	14	1.25
MCC ($K=2$)	11	0.23	8	0.45	12	1.14
MCC ($K=3$)	10	0.21	7	0.42	11	1.12
MCC ($K=4$)	10	0.22	7	0.43	11	1.13
MCC ($K=5$)	10	0.23	6	0.39	10	1.08
MCC ($K=6$)	10	0.24	6	0.39	9	1.02
MCC ($K=7$)	10	0.25	6	0.39	9	1.04
MCC ($K=8$)	10	0.26	6	0.39	9	1.04
PC	**11**	**0.23**	**10**	**0.55**	**13**	**1.14**
PD	25	0.44	47	1.98	20	1.47

Table 9: Number and type of active constraints

	active constraints			
system	ϕ	r	b	total
Nordic32	20	0	12	32
IEEE118	91	0	0	91
IEEE300	177	14	5	196

The overall load curtailed is 349.3 MW for the Nordic32 system, 1078.8 MW for IEEE300 system while no load is shed for the IEEE118 system. The curtailment effort is shared by 9 and 88 loads for the Nordic32 system and IEEE300 system, respectively.
One can observe that for the IEEE118 and the IEEE300 systems the MCC algorithm clearly outperforms the PC one for most values of K, while for the Nordic32 system their performances are very close.

5 CONCLUSION

This paper has presented and extensively tested the MCC interior point algorithm on various OPF variants. The results obtained suggest that the MCC algorithm is a highly viable alternative to the successful PC algorithm. The performances of the MCC algorithm emphasize once more the importance of the centrality theory for the IPM. Future work aims to find a heuristic scheme to automatically chose the maximal number of corrections allowed K for a given problem, in as much as the best value of K shifts from an OPF variant to another and from a test system to another. Nevertheless, choosing any value of $K \in [2, 6]$, generally provides very close performances comparatively with the PC algorithm.
A natural extension of this work is the development of an hybrid algorithm, in which the corrector step combines both MCC and PC features, in order to take advantage of their respective qualities, as reported in [10].

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