# Report of Committee V.1 12th Int. Ship and Offshore Structures Congress 1994 St.John's, Newfoundland, Canada

# APPLIED DESIGN - STRENGTH LIMIT STATES FORMULATIONS

Concern for the applicability of formulations related to the ultimate, buckling and the fatigue strength of structural components of ships, offshore platforms, and other marine structures. Emphasis shall be given to comparisons with available experimental data compiled in terms of probabilistic measures to evaluate the accuracy of these formulations for use in reliability-based design procedures.

CHAIRMAN:

Prof. T. Moan

MEMBERS: Dr. P. A. Frieze

Prof. C. Ganapathy

Dr. T. Hori

Prof. A. E. Mansour Mr. G. Parmentier Dr. P. H. Rigo Dr. V. Zanic

## CONTENTS

1.	INTRODUCTION
	LIMIT STATES AND UNCERTAINTY MODELLING Limit state formats Model uncertainty Model, parameter and total uncertainty Characteristic and design values of resistance
3.1 3.2 3.2.1 3.2.2 3.3 3.3.1 3.3.2 3.4.1 3.4.2 3.5 3.6.1 3.6.2 3.6.3 3.6.4 3.7.1 3.7.2 3.7.2	Stresses General Plate failure modes Overall failure of narrow girder flanges and stiffeners Uncertainty measures Girders under predominantly in-plane shear stresses General Effect of direct and bending stresses - interaction between shear and bending Stiffener requirements
	Effect of web cut-outs
3.7.5	Uncertainty measures
4. 4.1 4.2 4.2.1 4.2.2 4.2.3	Failure modes and formulations
4.2.4 4.2.5 4.2.6 4.3 4.3.1 4.3.2	Axial compression and bending Axial compression, bending and hoop compression Uncertainty measures Local buckling of unstiffened and ring stiffened shells Failure modes Axial compression and bending
4.3.3 4.3.4	Hoop compression

4.5.3	Frames in ring stiffened cylinders Orthogonally stiffened shells Failure modes and formulations Axial loading Radial loading Combined axial and radial loading Uncertainty measures
5.1 5.2 5.3 5.4	ULTIMATE STRENGTH OF TUBULAR JOINTS General Formulations for simple joints Model uncertainty for simple joints Other joints
6.3.2	FATIGUE LIMIT STATES FOR WELDED JOINTS General SN curves and stress concentration Model uncertainties General Stress concentration factors Miner Palmgren hypothesis
<b></b>	CONCLUCTONS

#### 7. CONCEOUS

# ACKNOWLEDGEMENT

REFERENCES

#### 1. INTRODUCTION

Marine structures are designed to fulfill serviceability (SLS) and ultimate limit state (ULS) criteria. The latter category in general comprises criteria for structural integrity, and may be further classified in terms of /1/:

- ultimate limit state of components
- ultimate limit state of systems
- fatigue limit state of components

The ultimate limit may be characterized by loss of equilibrium, fracture or excessive deformation. The fatigue limit state is defined by a crack size that represents an ultimate limit /1, 2/.

Marine metal structures are made of beams, stiffened panels, shells, and joints. Ultimate limit states may be expressed by parametric formula, numerical calculation procedures (e.g., nonlinear finite elements), or by testing. Existing component ultimate limit states are generally given by parametric formulae in terms of stresses, forces or moments; and exceptionally by strains, which have been justified by test results. Fatigue criteria in design codes are typically expressed by SN curves. Numerical approaches usually need to be used in connection with systems capacity. Design by testing (alone) may be used in special cases of innovative design.

There are significant differences between formulae used in different codes for the same failure mode. Hence, there is a need to compare and harmonize strength expressions, to identify the "best" one. Efforts to harmonize requirements for ships are made through IACS; and for offshore structures through ISO. There is a need to harmonize ship and offshore strength requirements, especially for steel-plated components.

The purpose of this report is to review ultimate and fatigue limit states for steel components in marine structures. The focus is on formulations of beam-columns, stiffened plates and shells and planar and tubular joints given in terms of parametric equations. Approaches specified as a numerical procedure will be briefly commented upon, in cases where they seem to offer significant improvement compared to parametric formulae.

The mathematical expressions for the limit states will be used as a basis for:

- failure function for reliability analysis. In this case the parameters in the limit state are considered to be random variables.
- design equations. In this case the parameters in the limit state according to the partial factor method are given by their characteristic values, each multiplied by a partial safety factor.

However, for a given phenomenon, the same limit s formulation may not necessarily be applied for the de equation and the failure function; but a more optimal calibration is achieved by choosing the same limit state.

A particular issue of concern in connection with buo marine structures is the definition of ultimate limit state view of conditions where large deformations/strains cause I failure/fracture and, hence, possible leakage. "W tightness" may in principle be considered as a separate I state, which may imply a lower limit on deformations dictated by the pure load carrying capacity.

Generally, several formulations are applied for diffe limit states. Significant efforts would be required to comexisting formulations to establish the "best" approach. committee has attempted to collect available information selected failure modes that could be considered as a mocontribution to future efforts of improving the correspondimit states.

Clearly, existing codes for marine and civil enginee structures should form the basis for the assessment of l states. Recent systematic experimental or numerical analy as reviewed by other ISSC committees, are useful as a basis code modification. Due to space limitation, only selected references can be quoted by this committee.

Ship rules have traditionally represented prescrip criteria for scantlings, which reflect strength and fatigue other considerations. So-called direct calculation methods ships are usually based on explicit loads and streformulae. However, these formulations are often based on fyielding and elastic buckling with a simple correction plasticity, i.e., not truly ultimate limit states. For lace space, such methods will only briefly be touched upon her On the other hand, explicit fatigue criteria that have received introduced for ships, are considered.

The current availability of accurate numerical metallows determination of the strength on a case basis as well to accomplish systematic studies of complex structures. particular advantage of numerical methods is that the effect each factor can be studied separately. However, numer methods to trace nonlinear behavior up to collapse susceptible to various sources of uncertainty, including herrors of different kinds. It is, therefore, necessar standardize the relevant numerical analysis procedures. It even be necessary to "certify users" and validate resulting the such analyses can be used by designers at large

Up to now, ultimate strength formulations in codes been established primarily on the basis of simple structure mechanics models, with appropriate "knock down" factormulations are achievable when failure mode follow simple mechanism (buckling, yield hinge mechanism, etc.).

determine resistances.

achieve such simple failure modes, e.g., for complex stiffened, thin walled plates and shells, the stiffeners are considered to meet certain assumptions, which commonly are conservative.

This review refers to "strength of materials" type design formulations supported by experiments or systematic numerical studies. The main source of information for the limit states of concern herein are codes issued by the industry, governmental regulatory bodies and classification societies. Review of recent developments of analytical and numerical techniques as well as experimental methods as such, is outside the scope of this report. The aim is rather to address some possible directions of further code developments. To come up with a new code, is certainly beyond the scope of this ISSC committee.

Chapter 2 deals with various formats for limit states and a consistent assessment of the inherent uncertainties.

Chapters 3-6 present comparisons between limit states used for the different types of components.

A summary and recommendations for future work are given in Chapter 7.

#### 2 LIMIT STATES AND UNCERTAINTY MODELLING

#### 2.1 Limit state formats

Limit states should /6,7/:

- preferably be based on structural mechanics theoret formulations (not pure regression to data)
- explicitly contain the parameters of influence
- have as small (random) model uncertainty as possible
- be as simple as possible

Limit states should, as far as possible, be based strength of materials, elastic buckling formulae, plastic l loads, etc., appropriately modified to account for resi welding stresses and geometrical imperfections, and should be obtained by a pure regression analysis of test results. would also be advantageous that a formulation converges elastic buckling formulae and yield stress for slender and stocky components, respectively. Besides nominal dimensions of the components, the stress-strain cu geometrical imperfections and residual stresses are parame of influence. Only the yield stress, and not the stress-st curve, is commonly referred to in resistance formulati Maximum geometrical deviations are usually assumed strength limit states; and toler developing the requirements are imposed in design to ensure that they complied with during fabrication. The residual stress pat depends upon the fabrication, and is generally not explic accounted for.

Limit states are subject to a systematic and rauncertainty. As discussed below, the systematic component accounted for directly, while the random component implipartial safety (material) factor.

Simplicity in the design formulation implies a red chance of gross errors.

## 2.2 Model uncertainty

For a structural component subject to one load (i.e., plane compression, or lateral pressure),  $s_i$  at a time, predicted resistance,  $r_{\rm pred.,i}(\underline{z})$  is a function of a set, variables, which may include yield strength, plate thick geometrical imperfection, etc. Usually these parameter vary over the component, and each type of parameter represented by its value at a few locations, assuming that variation follows some pattern over the structure. Typics out of plane imperfections may be described by Four components /3/. The yield stress is different in different controlled in the components of the present of the present of the structure. Typics out of plane imperfections may be described by Four components /3/. The yield stress is different in different in the present of the present of

assessed using the same definition of yield stress as use

design analyses. The resistance model is uncertain. By characterizing this model uncertainty by  $\Delta_i$ , the actual resistance,  $r_{\text{true},i}$  corresponding to a load  $s_i$ , may be written as /4/:

$$r_{\text{true,i}} = \Delta_{i} r_{\text{pred,i}} (\underline{z})$$
 (1)

The model uncertainty,  $\Delta_i$  is assessed by comparing predictions with results obtained by experiments or advanced numerical analyses, most often by assuming that the error in experimental or numerical calculations is negligible. The predictions are made by using a representative value of the parameters in the set,  $\underline{z}_j$  corresponding to the specimen no. j. The corresponding sample of  $\Delta_i$  is:

$$\Delta_{i}^{(j)} = r_{(test)}^{(j)} / r_{(pred)}^{(j)} \left(\underline{z}_{j}\right)$$
 (2)

The statistics of  $\Delta_i$  may then be calculated based on a sample  $\left\{\Delta_i^{(j)}\right\}.$  Usually limited information is available, and  $\Delta_i$  is simply modelled by a normal or lognormal distribution.

In some cases, it may be interesting to determine the model uncertainty  $\Delta_i$  as a function of some (geometrical or other) parameters. Hence, the uncertainty for beam-columns may firstly be determined for each type of member cross-section. Secondly, for a given type of component under compressive stresses, it may be convenient to determine  $\Delta_i$ , as a function of slenderness, because the uncertainty will depend upon whether it is buckling or plastic collapse that leads the component to reach its ultimate limit state. Such information may be useful in the first place to assess whether a trend in the bias may warrant a modified strength formulation, and secondly whether the partial safety factor should depend upon this parameter (e.g., slenderness).

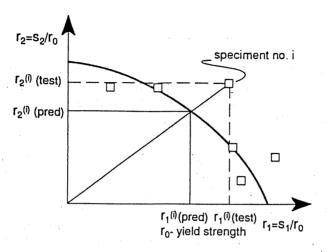
It follows from the above assumptions, that before applying experimental or numerical data to assess the uncertainty, they should be critically reviewed to eliminate data influenced by "gross" errors, or experimental data which involve incompletely defined tests. This is a major task in an uncertainty assessment.

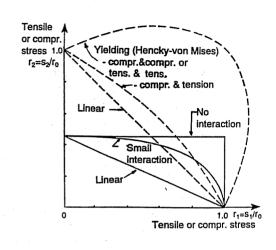
Model uncertainties for the ultimate strength of components which are subject to two or more load components also need to be described. In general, the interaction equation may be written as:

$$g\left(\left\{\frac{s_i}{r_i}\right\}, \left\{c_k\right\}\right) = 0 \tag{3}$$

where  $s_i$  and  $r_i$  are load effects and resistances corresponding to load component no. (i),  $\{c_k\}$  is a set of coefficients used

in the model. The interaction for the case of two loads illustrated in Fig. 1.





- (a) Experimental vs. predicted values
- (b) Typical interaction equati

Fig. 1 Strength interaction for two action effects

The model uncertainty of the interaction equation sho be expressed so that it represents the model uncertainty each basic load case as a special case when only one load, is acting. This means that each  $r_i$  should be modified with which is determined for each  $r_i$  as explained above. addition, uncertainties,  $\Delta_k$  need to be associated with other parameters,  $\{c_k\}$  in the interaction formula, Eq. e.g.:

$$g(\{s_i/\Delta_i r_i\}, \{\Delta_k c_k\}) = 0$$

If, for instance, the basic interaction equation consider of terms like  $(s_i/r_i)^{\alpha_i}$ , terms of the type:

$$\Delta_k c_k (s_i/r_i)^{\alpha_i} (s_j/r_j)^{\alpha_j}$$

may be introduced to account for uncertainties of interaction. A convenient way to characterize the uncertainties the case of two loads (Fig. 1a) is by:

$$\Delta_{12}^{(j)} = r_{1(\text{test})}^{(j)} / r_{1(\text{pred})}^{(j)} = r_{2(\text{test})}^{(j)} / r_{2(\text{pred})}^{(j)}$$

where  $r_{i(\text{test})}^{(j)}$  is the experimental resistance corresponding the load i. Then, it needs to be demonstrated by a cert choice of  $\Delta_k$ , that Eq. (4) implies the same uncertainty expressed by  $\Delta_{12}$  /7/. If the discrepancy is unacceptable, improved interaction equation should be introduced.

If, however, interaction between, e.g., two loads s1

limited to the range 0 to  $\overline{s}_2/r_2$ , the interaction equation may be expressed in terms of  $s_1/r_1$ , and the model uncertainty,  $\Delta_1$  may be calculated by:

$$\Delta_1 = r_{1(\text{test})}^{(j)} / r_{1(\text{pred})}^{(j)}$$
 (5b)

In this case (all) the uncertainty  $\Delta_1$  should be assigned to the variable  $r_1$ , also for the case when  $s_2 > 0$ . Clearly, this approach will not be particularly accurate when  $s_2$  varies from 0 to  $r_2$ , which implies a variation between a pure  $s_1$  and a pure  $s_2$  load condition.

An analogous interaction problem exists for fatigue, both in relation to crack initiation and crack growth, under multiaxial stress conditions. While the equivalent (plastic) strain would be a representative measure for crack initiation, tensile stresses normal to crack and shear stresses are the relevant stresses for crack growth. However, the normal stress usually dominates and the shear contribution is rarely considered for marine structures.

#### 2.3 Model, parameter and total uncertainty

The resistance is influenced, for instance, by overall scantlings (a,b,...), plate thickness (t), initial imperfection ( $\delta$ ), modulus of elasticity (E), yield stress ( $\sigma$ <sub>o</sub>) and residual stress ( $\sigma$ <sub>r</sub>). Representative values of the parameters a, t, and  $\sigma$ <sub>o</sub> are commonly explicitly accounted for in ultimate resistance formulae. Values of initial imperfections and residual stresses are often only implicitly assumed when establishing the resistance formula. Imperfections ( $\delta$ ) are, however, controlled by tolerance requirements. Residual stresses,  $\sigma$ <sub>r</sub> are treated explicitly, e.g., in connection with stiffened cylinders.

When, for instance, initial imperfections are not explicitly considered in  $r_{\text{pred}}(\underline{z})$ , and the variation of imperfections in real structural components is not the same as in the test specimens used to estimate  $\Delta_i$ , the model uncertainty  $\Delta_i$  needs to be modified to be considered representative for real components, /7/.

If the sample of specimens used to determine  $\Delta_i$ , is small, the statistical uncertainty should be combined with the model uncertainty using Bayesian methods /5/.

However, in some cases, the experimental/numerical data basis is applied in the first place to fit the theoretical model to the data. In that case, the model uncertainty  $\Delta_i$  determined by the above procedure, obviously does not reflect the true model uncertainty, and  $\Delta_i$  needs to be (subjectively) adjusted.

Finally, the relevance of the material, geometry and boundary as well as load conditions used in the experimental/numerical investigation to determine the model

necessary to modify  $\Delta_i$  for a particular application of prediction method,  $r_{\text{pred}}.$ 

#### 2.4 Characteristic and design values of resistance

Uncertainties should be handled by probabilistic method If a simple design criterion in terms of a single resistant  $R_i$  and load effect,  $S_i$  is considered, and, e.g., a lognor probability distribution is assumed for  $R_i$  and  $S_i$ , the design values of  $R_i$  and  $S_i$  can be separated, and the design value  $R_i$  becomes /1/:

$$r_{id} \equiv \mu_{r_i} \exp \left[-\alpha_{r_i} \cdot \beta_T \cdot V_{r_i}\right]$$

where  $\mu_{r_i}$  and  $V_{r_i}$  are the mean and COV of  $R_i$ , respectively. and  $\beta_T$  are the so-called sensitivity factors and the tarreliability index, respectively.  $\alpha_{r_i}$  depends upon the relative magnitude of the uncertainties in  $R_i$  and  $S_i$ , and, variety between 0.2 and 1.0.  $\alpha_{r_i}$  will typically be smaller for slemmembers in bottom supported platforms subjected to wave lotten for panel and shell components in large volume markstructures, or structures subjected predominantly hydrostatic pressure, because of the relative magnitude of land resistance uncertainties. In the design value method which yields a simplified code calibration,  $\alpha_{r_i}$  is taken to 0.8.

The characteristic value for a lognormal variable,  $R_i$  be defined by means of  $\mu_{\rm r_i}$  and  $V_{\rm r_i}$  as:

$$r_{ik} \cong \mu_{r_i} \exp \left[-k_{r_i} V_{r_i}\right]$$
 for  $V_{r_i} \leq 0.2$ 

where  $k_{r_i}$  is a constant which is selected so that  $r_{ik}$  is defactording to a certain fractile value, and is typically of order 0.5 to 1.5.

The partial resistance factor,  $\gamma_{Ri}$  may be defined by:

$$\gamma_{Ri} = \frac{r_{ik}}{r_{id}} \cong exp[(\alpha_{r_i}\beta_T - k_{r_i})V_{r_i}]$$

Hence, with  $k_{r_i}$  = 1.5,  $\alpha_{r_i}$  = 0.5 and a target safety 1 corresponding to  $\beta_T$  = 4.2,  $\gamma_{Ri}$  ~ 1 + 0.6V  $_{r_i}$  .

Eq. (8) applies if the design resistance  $r_{id}$  is based Eq. (1), e.g., corrected due to  $\Delta_i$ . If the design is based  $r_{pred.}$ , the safety factor to achieve a target reliability would be  $\gamma_{Ri}/\mu_{\Delta}$ , where  $\mu_{\Delta}$  is the mean  $\Delta_i$ . This shows important influence of the bias in the resistance.

#### 3. ULTIMATE LIMIT STATES FOR STEEL-PLATED COMPONENTS

#### 3.1 Introduction

Basically two main types of steel-plated components are applied in marine structures (Fig. 2):

- stiffened panels subjected to predominantly in-plane axial stresses, but with some in-plane transverse and shear stresses and possibly lateral pressure
- stiffened girders subjected primarily to in-plane shear and longitudinal axial and bending stresses

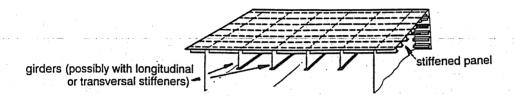


Fig. 2 Stiffened panels/girders

The collapse behavior of stiffened panels and girders received considerable attention in the 1970's and 1980's, especially in connection with several box girder bridge failures, culminating in the codes such as BS 5400 /9/ and EC3/ECCS /13,14/. Much of this work has been focused on modern limit state methods.

While the civil engineering developments of ultimate limit states for stiffened panels for bridges had some influence on marine practice, the significant efforts on stiffened girders in bridges have so far had less effect on the design practice for similar components in marine practice.

The disadvantage with EC3/EECS, BS5400 and other codes primarily developed for civil engineering structures is that they do not cover geometries and load conditions (e.g., inplane and lateral pressure) typical of marine structures. rules of classification societies are based on elastic/plastic buckling/first yield criteria, but since the formulations are implicit in scantling requirements, they are not pursued Especially, the involvement of some classification societies in structures for offshore oil exploitation, which initially was heavily influenced by civil engineering practice, has made them introduce limit state formulations. Det norske Veritas has introduced a comprehensive guidance /11/, while Bureau Veritas /10/, Germanischer Lloyd (GL) /15/ and other societies follow. GL, for instance, base their recent (offshore) codes on Eurocode 3 (ECCS) and DIN 18800. Besides classification societies, governmental regulatory bodies such as HSE /16/ and NPD /17/ in the UK and Norway, respectively; and API (in the USA) issue codes for offshore structures.

Recent state-of-the-art reports on steel-plated structu/18-21/ refer primarily to civil engineering components, are partly relevant to marine structures.

However, the research goes on in this area, as reported ISSC Committee III.1, generally by using experimental numerical methods. In this report, the focus will be on us data obtained by systematic numerical or experimental study to validate strength codes based on parametric formulae. publications deal with the uncertainties in the strength. this reason, this committee launched a benchmark study; /22/ and the brief summary in Section 3.5. The benchmark st was also done to assess the robustness of the formulations calculations, with respect to human errors.

Stiffened panels may exhibit the following failure mod depending upon the geometry and material property:

- local buckling or collapse of plate between stiffeners
- local buckling or collapse of stiffeners. The basic desphilosophy is to avoid such failure modes by slenders requirements
- overall buckling of stiffened panel between frames bulkheads

Limit states for the first failure mode are discussed Section 3.2. Overall collapse of stiffened panels under an and possible lateral loads (bending moments) is treated Sections 3.3 and 3.4, while the benchmark test for such partis described in Section 3.5. The overall collapse of stiffe girders subjected to in-plane bending and shear is discussed Sections 3.6 and 3.7.

#### 3.2 Plate elements

#### 3.2.1 Formulations

The basic component is a plate element subjected to plane or lateral loads.

In modern limit state approaches the design principle to assume that the plate element reaches its collapse 1 before overall stiffener failure. Estimates of the ultiplimit capacity of plate elements are therefore crucial.

Significant efforts have been devoted to improving original Von Karman estimate of the ultimate strength axially loaded plate. Various in-plane and flexural boun conditions, and imperfections and residual stresses need to considered. Recent reviews or the various formulations been made in /18,23-29/. Some of the discrepancies bet different specifications may be explained by diffe assumptions regarding initial imperfection, boundary condi

and residual stress levels, which are all factors of influe

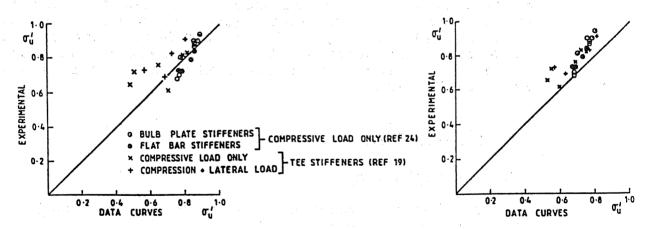
In particular, it is noted that the different codes specify different level of acceptable imperfection tolerance!

For instance, the tolerance requirement for initial imperfection of plating varies, and the imperfection amplitude to plate width is, e.g., w<sub>o</sub>/b < 0.01 (API Bul. 2V /8/, DnV /11/, 1987), w<sub>o</sub>/b  $\leq$  0.002 (ECCS /13/). According to BS 5400 /9/, it varies with a/b and  $\sigma_{\rm o}$ , but is approximately w<sub>o</sub>/b  $\leq$  0.01. The allowable initial deflection of stiffeners is in the range 1/750 of the span (BS 5400) to 1/500 (ECCS). Considering moderate imperfections (w/t = 0.1 $\beta^2$ , where  $\beta$  is normalized plate slenderness) and compressive residual stresses (0.15 $\sigma_{\rm o}$ ), the following ultimate to yield strength,  $\sigma_{\rm u}/\sigma_{\rm o}$  was proposed in /23,28/:

$$\sigma_{11}/\sigma_{0} = 0.23 + 1.16/\beta - 0.48/\beta^{2} + 0.09/\beta^{3}$$
 (9)

which slightly deviates from previously proposed formulae.

An assessment of the sensitivity of  $\sigma_u$  to imperfections is given in /26/. Experimental results are compared with numerical results using moderate imperfections ( $w_o/t=0.1\beta^2$ ) in real plates as well as numerical results using actual imperfections of the specimens; see Fig. 3. The effect of residual stresses and imperfections was also studied in /29,30/.



(a) Average imperfection

(b) Actual imperfection

Fig. 3 Comparison of numerical and experimental results /20/.

Numerical predictions using average or actual imperfection.

An interaction formula for biaxial compression was proposed in /24,27/, which converges to the von Mises-Hencky and linear interaction for stocky and very slender plates, respectively. Also, an interaction curve for biaxial compression and shear is given in /24,27/. Based on ultimate limit state formulation, it is concluded for the shear strength in /24/ that the DnV curves are rational, but generally

conservative and the LR rules are too simplistic and leavarying levels if the same safety factor is applied.

A special phenomenon is observed for slender pelements (e.g.,  $\beta \gtrsim 3$ ) supported by strong stiffeners subjected to shear loading, in that a tension field developed and, hence, leads to significant capacity be linear shear buckling (see Section 3.7).

Concerning lateral pressure combined with bia compression, interaction curves are provided in /25/particular imperfections and a range of aspect ratisfied slenderness and pressures. These curves are translated parametric formulations to give the variation of the strewith the lateral pressure. This model predicts that mode pressure has a much greater effect on compressive resist than suggested by current design rules.

#### 3.2.2 Uncertainty measures

The uncertainty of predicted ultimate strength (effectivity) of plating can be assessed based on the redisplayed in Fig. 3 /26/. It is seen that the smallest defined according to Eq. (2) (of 1.05) is achieved predictions are made on the basis of average imperfect. The scatter then corresponds to a COV of 13%. If the assimperfection is used, the bias surprisingly increases to but the COV is reduced to 7%.

#### 3.3 Stiffened panels subject to axial compression

## 3.3.1 Formulations

The strength of stiffened panels under axial compre may be treated as

- a series of disconnected beams
- an orthotropic plate
- a discretely stiffened plate

For most (wide) stiffened panels, the "column" appropriate considering flexural collapse of a single stiffener with adjacent plate, is adopted. Local buckling of the stiff is avoided by assuming that their geometry fulfills ce local slenderness constraints. Generally, two diff "column" approaches are applied. The first approach is on reduced slenderness concept with a strength expresse means of the Euler buckling load, modified for elasto-pl behavior by the Johnson-Ostenfeld parabola (API Bul. 2V, ship rules). The second method is the Perry approach (e.g. BS 5400, DnV, ECCS). Recently, various methods used

predicting the ultimate strength of stiffened panels (

axial compression) have been assessed and new methods proposed /31,32/.

The beam model is sometimes established by taking the plate flange to have a given effective width, while in others, the relative effective width is to taken to be the ultimate strength  $\sigma_{\rm u}/\sigma_{\rm o}$ , according to, e.g., Eq. (9) for axial compression. For tension flanges, the relative width is 1.0. The basic Perry equation for a column subjected to an axial compressive force P and a bending moment, M at a cross section with an effective area  $A_{\rm eff}$  and an effective moment of inertia,  $I_{\rm eff}$  is:

$$\frac{P}{A_{eff}} + \omega \frac{M}{I_{eff}} y = \frac{P_o}{A_{eff}}$$
 (10)

where y is the distance from the neutral axis to the outer fiber,  $\omega$  is  $\pm 1$  and Po the yield load. For an imperfect column under axial compression, the maximum bending moment is approximately taken as:

$$M = \Delta_o \cdot P/(1 - P/P_E)$$
 (11a)

where the  $\Delta_{\rm o}$  is the initial imperfection and  $P_{\rm E}$  the Euler load.

In evaluating the collapse stress using the Perry equation, it is necessary to examine two cases as the overall column initial geometrical imperfection  $(\Delta_{\rm o})$  may occur in a direction towards the plate or towards the stiffener tip. For each of these two cases, the check point (i.e., the value of y) may be different, depending on whether the stress check for compressive yield is applied to the plating (PC) or to the stiffener (SC).

To account for transverse or shear stresses in the plating, an efficient yield stress may be applied; see, e.g. DnV-CNB /11/.

In Eq. (10),  $\omega=1$  when checking against compressive failure either in the plate panel (PC) or at the stiffener tip (SC) while, on the other hand,  $\omega=-1$  when checking against tensile failure at the stiffener tip (ST).

Eqs. (10-11a) may be readily solved and  $\sigma_u = P_u/A_{eff}$  expressed in terms of  $\Delta_o$ ,  $\omega$ ,  $\sigma_o$  and  $\sigma_E = P_E/A_{eff}$ . Different codes specify different values of the "effective" imperfection parameter  $\Delta_o$ . It is noted that the ECCS (column) approach is a modified "Perry approach" by referring to the stress at the cross-section centroid rather than the stiffener tip stress. DnV introduces an imperfection for the collapse mode associated with the column deflection towards the stiffener tip. However, it is pointed out in /30,31/ that in the analysis of continuous panels, the line of action of the load is allowed to follow this shift in neutral axis, and the correction is unnecessary.

The column approach does not normally cater for tripping of stiffeners. At present, it is usual to specify slenderness

criteria to prevent this mode from occurring prior to over collapse. Some relaxation on the slenderness limitation possible if the stiffener is not fully stressed or if stress distribution across the depth has a high ber component. In most cases, these criteria are based on buckling response of a single stiffener outstand, modeled plate panel under uniform axial compression with three estimply supported and one edge free, but this model caproperly take into account the interaction between the panel the stiffener /34,35/. Plate buckling, which preceeds ultimate limit state, may initiate deformation and premastiffener failure /35/. While this problem can be analyzed numerical methods, appropriate "strength of materials design code formulations", are still lacking.

The strength of narrow stiffened flanges with only a longitudinal stiffeners can be underestimated by the comproach, since transverse action may be significant a ECCS provides a strength formulation for orthotropic stiffened panels.

#### 3.3.2 Uncertainty measures

The uncertainty of existing code formulations (DnV, 185 5400, and API) and a method proposed by the research at Imperial College has been assessed by using the result 23 tests for axially compressed panels /31,32/. imperfections and residual stresses in the prediction makes taken as fixed, not varied according to those in specimens. The results are summarized in Table 1. The results are not included because this method does not see have been correctly used in /31/. It is seen that the bias COV are almost the same (1.10 and 15%) for all methods, exthat the proposed IC-method yields less scatter. This makes used in the theoretical predictions by the IC-method closer look at the results /33/ reveals that the uncert depends upon plate and stiffener slenderness:

- for stocky plates (normalized plate slenderness,  $\beta$  = 0.9
  - the ECCS orthotropic approach seems over-conservative the normalized stiffener slenderness,  $\lambda <$  0.8
  - the ECCS column approach could over-estimate the str for a very slender column  $(\lambda < 0.2)$
- for slender plates  $(\beta > 2.9)$  all codes are conservative

When applying the model uncertainty in Table 3.1 should be recognized that it somewhat accounts for variable of e(x,y) and  $\sigma_r(x,y)$ . The uncertainty measures, hence, to be appropriately modified to reflect the real variable and the tolerance level set.

Table 1 Comparison of Model Uncertainty Based on 23 Tests of Axially Compressed Panels /31/

Code	Minimum	Maximum	Mean	COV
	Value	Value	Value	(%)
BS5400 (1982) DnV, Buckl. Notes 30.6 (1987) ECCS (Column Appr.) (1990) ECCS (Orth. Appr.) (1990) Method proposed by Imp. College	0.89	1.46	1.10	13
	0.94	1.52	1.10	16
	0.86	1.44	1.08	14
	0.94	1.68	1.14	15
	0.94	1.34	1.10	08

The model uncertainty of the proposed IC method was also estimated by comparing predictions with this method with accurate numerical results for 63 panels. The resulting bias and COV of 1.05 and 5% are, as expected, less than 1.10 and 8%, previously obtained for the IC method. This is partly because imperfections etc. are more well defined in the "numerical test" panels. However, since the latter uncertainty measures do not contain any effect of parameter uncertainties, the bias and random uncertainty must be appropriately modified when applied to actual panels.

It seems that several prediction methods yield a reasonable accuracy for panels under axial compression. Moreover, the uncertainty varies with the slenderness of the plate and panel, and it could be further reduced by expressing it as a function of slenderness.

# 3.4 Stiffened panels subject to axial compression, bending or lateral pressure

#### 3.4.1 Formulations

Some codes specify ultimate strength for panels under combined axial load (P) and bending, due to end moments or lateral load by ultimate strength interaction equations, such as Eq. (17). The DnV approach for combined axial lateral loading, q, in multispan stiffened plates is essentially such an approach using an equivalent moment equal to qba²/16 (where a and b are plate length and width, respectively) and an assumed effective length of 60% of the span. This involves carrying out a stress check for each possible failure mode (that is PC, SC and ST; see Section 3.3.1) at the most critical section within the span under consideration. Considering both the normal and bending stresses in a simply supported strut under bending, the Perry equation (10), should be applied with /31,32/:

$$M = (M_{eq.} + P\Delta_o)/(1 - P/P_{cr})$$
 (11b)

$$M \cong M_{q} + (P(\Delta_{o} + \Delta_{q}))/(1 - P/P_{cr})$$
 (11c)

for the case of end bending moments and uniform lat pressure, q, respectively.  $M_{\rm eq}$  is the equivalent bending moment for the case of variable moment.  $\Delta_{\rm o}$  is the initial column deflection, and  $M_{\rm q}$  and  $\Delta_{\rm q}$  are the bending moment deflection at the midspan due to lateral load, respective When calculating the effective width for the plate element cases with lateral pressure, the effect of lateral pressure between the pressure of the considered.

All the proposed equations (Eqs. 10-11) are related simply supported single-span model /31,32/, and they can used to study axially compressed stiffened panels between of frames. However, a simply-supported single-span model is appropriate for the design of multi-bay longitudinal stiffened panels subjected to axial and lateral loads, du continuity effects and interaction in between adjacent shaving different initial imperfections.

The Perry model for the case with axial and lateral I may be extended to clamped ends. While Eqs. (10,11a) are realistic for multispan panels when axial loads predoming the clamped end case is expected to be a better approximate when lateral loads on multispan panels predominate.

The IC method /31-33/ deals with multi-span by introduced an additional deflection and an additional central span module to the lateral pressure acting on a fixed ended stated, each span (column) with its additional deflection moment is calculated as an axially loaded simply-suppospan.

Interaction equations such as Eq. (17), are conservative than methods based on a stress check at critisections (Eqs. 9-11), but are still more convenient implementation in design rules since various forms and leading can be catered for.

The Perry equations, however, are essentially a stacked at one point, and their use in complex load cases be involved. Reduced uncertainty usually may require a complicated checking procedure. Hence, this refinement must balanced against the consequences of potential errors made using the method.

Interaction curves of stiffened plates under combined plane compression and shearing stresses, was also investign /37/. Buckling, ultimate and fully plastic strength states are considered for flat plates and unidirection stiffened plates. Minimum stiffness ratio of stiffened plate for buckling  $(\gamma^{\rm B}_{\rm min})$  and for ultimate strength  $(\gamma^{\rm U}_{\rm min})$  determined. For  $\gamma$  <  $\gamma^{\rm B}_{\rm min}$ , the buckling interaction curve valid, for  $\gamma^{\rm B}_{\rm min}$  <  $\gamma$  <  $\gamma^{\rm U}_{\rm min}$  the ultimate strength curve is and, if the stiffened plate is stiff enough, a fully plastrength interaction curve is proposed. Ultimate strength determination is based on stress coefficients that may

analytically evaluated using parametric formulae.

#### 3.4.2 Uncertainty measures

There is very limited information about the uncertainties in the strength models for panels with axial and lateral loads. Comparisons have been made in /31/ between four experimental tests of the axial capacity for given lateral load levels and values predicted by the DnV and IC-methods. The ratio of test/predicted values ranged between 1.1 to 1.35 for the DnV approach and between 1.05 to 1.25 for the Perry formula (10, 11c). Interaction Eq. (17) yields even larger (conservative) bias factors.

#### 3.5 Benchmark test of stiffened panels

Several classification societies and other organizations that have engaged in developing strength formulations stiffened panels, were asked to determine the ultimate capacity of 10 multi-span stiffened panels which were selected by this The focus was on the ultimate strength of ISSC committee. stiffened panels located between transverse (multi-span) frames, failure modes and effective width of flange plating. Within the 10 panels, there are 8 longitudinally compressed panels and 2 subjected to longitudinal compression and lateral pressure. There are 6 Smith's panels /39/ (same geometries and actual imperfections) and 4 typical panels in the VLCC "Energy Concentration" ship that broke its back during discharge of oil in 1980 /40/.

The contibutors to the benchmark study were first asked to analyze the 10 panels. Then all participants were informed about the outcome of all analyses and, if they wished, allowed to rerun their analysis /22/. The resulting model uncertainties are shown in Table 2. The names of contributors and codes/methods used, are made anonymous. The contributors are referred by one capital letter (A, B, C, etc.) and the codes by the symbols (M1, M2, etc.). The codes are in alphabetic order, BS5400 /9/, Bureau Veritas /10/, DnV /11,12/, ECCS column and orthotropic approaches /13/, Imperial College method /31/, Lloyds Register /41/ and Ueda-Yao's method /30/. Some contributors used codes as well as finite elements and other numerical methods. The latter types of results are not included in this summary.

It should be noted that the initial imperfections of the specimens have a large variation, and especially that the initial deflection of the stiffener for case 3 is two times larger than the tolerance. Initial plate deflections exceed the particularly restrictive tolerance requirement of the ECCS (column) code for all cases. Moreover, some methods had to be used outside their specified range of validity in this study. It was found that the effective width used in the various methods, differed by 0-15%, except for the method M6, which gave up to 45% lower value than other methods.

Some codes have been used by several contributors. It is

Table 2 Ultimate Strength to squash load for stiffened panels predicted by various organizations using different methods, obtained by the participants after being informed about the outcome of all first analyses.

								T	-			 	
				COV	0.126	0.098	0.145	0.237	0.106	0.063	0.039	0.043	0.106
				Mean	9/9'0	0.800	0.719	0.501	0.818	0.798	0.888	0.662	0.818
Œ	M9	Single	beam column	Pu/Po	0.650	0.870	0.870	0.467	0.907	0.689	0.881	<b>. (1</b>	0.907
E	M8	Single	beam column	Pu/Po	0.751	0.543	0.508	0.378	0.731	0.874	0.923	0.670	0.731
D	M7	2°	edge Stiff.	Pu/Po	0.672	0.803	0.771	0.455	0.877	0.858	0.915	0.645	0.877
D	9W	With	edge Stiff.	Pu/Po	0.708	0.812	0.775	0.495	0.892	0.870	0.924	0.684	0.892
သ	9W	Single	beam column	Pu/Po	0.930	0.930	0.940	0.920	0.700	0.850	0.880	0.930	0.770
၁	MS	Single	beam column	Pu/Po	0.580	0.790	0.770	0.480	0.580	0.750	0.840	0.540	0.580
8	MS	ž	edge Stiff.	Pu/Po	0.564	0.833	0.784	0.461	0.824	0.802	0.893	0.529	0.824 0.774
В	M4	%	edge Stiff.	Pu/Po	989.0	0.797	0.560	0.542	0.858	0.804	0.892	0.512	0.858
æ	M3	8 Z	edge Stiff.	Pu/Po	0.659	0.811	0.556	0.482	0.839	0.752	0.887	1 1	0.839
B	M2	%	edge Stiff.	Pu/Po	0.572	0.764	0.652	0.411	0.771	0.735	0.813		0.771
В	M1	%	edge Stiff.	Pu/Po	0.672	0.803	0.570	0.493	0.880	0.791	0.923		0.880
V	M2	ž	edge Stiff.	Pu/Po	0.631	0.812	0.544	0.432	0.810	0.794	0.843	0.613	0.810
V	M2	With	edge Stiff.		0.680	0.832	0.703	0.484	0.845	0.819	0.863	0.662	0.845
Ą	M1	%	edge Stiff.	Pu/Po	0.672	0.800	0.585	0.495	0.871	0.783	0.917	0.664	0.871
A	M1	With	edge Stiff.	Pu/Po	0.708	0.805	0.593	0.527	0.899	0.798	0.928	0.702	0.889
CONTRIBUTORS	METHODS/CODES		Boundary conditions	Pu/Po Experim. C.SMITH	0.76	0.83	0.61 0.82 (1)	0.72				0.73	1 1
CONT	METHO		Boundar	Panel Reference No	Smith 1.a	Smith 2.b	Smith 3.b Smith 4.a	Smith 5	Energ. Conc.	Bottom Shell Energ. Conc.	Upper Deck Energ. Conc. Side Shell	a Smith 1.b without lateral pressure with lateral pressure P=103.4kN/m2	O Energ. Conc., Sottom Shell without lateral pressure with lateral pressure P=200kN/m2

) C. Smith panel 4.a has longitudinals interspersed with two tee-bars of 40 inches that are not considered by the contributors.

different contributors are different. Hence, this benchmark test shows two kinds of uncertainties: model (and parameter) uncertainties associated with the method for strength prediction, and uncertainties/"errors" associated with transforming the physical problem into the mathematical model handled by the method (i.e., modeling the geometry, boundary conditions, etc.), possible errors in the respective computer codes, use of the computer code, etc.

The benchmark study is described in more detail in /22/.

# 3.6 Stiffened girders subject to in-plane axial and bending stresses

#### 3.6.1 General

Girders may be part of monocoque steel-plated structures as shown in Fig. 2. The flanges may be wide and stiffened; or narrow, unstiffened plates. The ultimate capacity of flanges with uniform in-plane loads is reviewed in Sections 3.2-3.3. However, it is noted that the stress in wide box girder flanges will be non-uniform due to shear-lag. Except for cases with very slender stiffeners, the effect of shear-lag on ultimate strength, however, can be ignored, as demonstrated, e.g., in the review /43/ and the recent study /36/.

The focus here is on the stiffened web subjected to inplane axial bending or shear forces, and the interaction between the web and flanges. The international research effort in response to the box-girder bridge failures in the early 70's has led to the publication of numerous ultimate shear strength analytical models (based on the tension field approach), as well as some numerical models, which attempt to provide the most efficient and effective solution to the design of structures. Ultimate strength formulations for plate and box girders of civil engineering structures have recently been reviewed in /18,43,44/ and quantitatively assessed in /45,46/. As codes for marine structures have not been based on truly ultimate strength formulations, the review provided in Sections 3.6-3.7, is partly done as a tutorial presentation.

Currently, two approaches are applied in estimating the contribution to the ultimate capacity from the web subjected to in-plane direct and bending stresses:

- for webs with no stiffeners or transverse stiffeners connected to wide flanges (box girders) the contribution from the web is ignored (and the web capacity is utilized to carry shear forces); see, e.g., BS 5400 (Pratt truss approach)
- for webs with transverse or especially longitudinal stiffeners connected to narrow webs, the web is accounted for by a complete analysis of the load shedding from webs to flanges by means of an effective width approach.

Overall collapse is, by the current design pract governed by failure of longitudinal load carrying stiffe and flange failure. By assuming that the plate buckles be the stiffeners and flange, the effect of the plate car accounted for by the effective width concept.

#### 3.6.2 Plate failure modes

For unstiffened webs, the "efficiency"  $(\eta)$  of the compression zone, may be calculated by /18/:

$$\eta = (1 - 0.05(3 + \psi)/\lambda_p)/\lambda_p$$

This expression is a simple, generalized form of Eq. (9) plates under uniform axial stress. The efficiency depend the normalized web slenderness  $(\lambda_p)$  and the ratio  $(\psi)$  of axial stress in the upper and lower flange. Alternatex expressions are also provided in /18/. The effective cresction method may also be employed for the case longitudinally stiffened webs with the difference that each the forming subpanels must be considered separately /18/. addition, Cooper's formula /47/ is commonly used for do symmetric I sections (e.g., North America) as well as in Cardiff approach (paragraph 3.7.1) /48,49/.

The information about the effect of perforations/cutin girder webs on the capacity under direct stresses bending, is limited, but the effect is also, especially girders with longitudinal stiffeners, relatively small.

Local failure of narrow compression flange(s) stiffeners are prevented by requiring that slenderness critare fulfilled. For instance, the width to thickness of girder flange should satisfy the following criterion:

$$c/t_f \leq 18.5 \beta \sqrt{235/\sigma_{of}}$$

where  $\sigma_{of}$  (in N/mm²) is the yield stress of the flange mater. The semi-empirical Winter formula requires  $\beta \cong 0.72$  and value has been adopted in Eurocode 3 and in the Recommendations /18/. AISC /50/ specifies a value of  $\beta \equiv 0.76$ . When the while BS5950 /51/ requires a value of  $\beta \equiv 0.76$ . When the to thickness ratio does not comply with the limiting value "effective" compression flange should be defined /44/. Sin local buckling criteria apply for stiffeners aligned in direction of loading.

To avoid buckling of the flanges into the web, the depth, d to thickness, tw should satisfy /18/:

$$d/t_{w} \leq 0.55\sqrt{A_{w}/A_{f}}(E/\sigma_{of})$$

where A<sub>f</sub> is the cross-sectional area of the larger fla

of-plane "breathing" which can cause fatigue cracks under repeated loading conditions.)

#### 3.6.3 Overall failure of narrow girder flanges and stiffeners

For girders with narrow flanges, the collapse of the flange out of the girder plane needs to be considered.

For plate girders with high web slenderness ratios, the pure torsion contribution is neglected and the critical torsional buckling stress is expressed only by the elastic warping resistance,  $\sigma_{\text{LT,Cr}}$ . There is no general agreement on the formula to be used for the ultimate lateral torsional buckling stress. In Europe the following has been recommended /53/:

$$\sigma_{LT,u} = \sigma_o \left(1 + \lambda_{LT}^{2n}\right)^{-1/n} \tag{14}$$

with n = 2.5 and  $\lambda_{\rm LT} = \sqrt{\sigma_{\rm o}/\sigma_{\rm LT,cr}}$ . Eurocode 3 indicates a value of n = 1.5 and suggests another approach based on a column buckling type curve. Formulae similar to the above are suggested in /19/.

Transverse stiffeners have a limited effect on the capacity under longitudinal direct stresses. Such stiffeners are more important in the case of predominantly shear loads and are, hence, discussed in Section 3.7.

Longitudinal stiffeners must be able to support the load shedding due to plate buckling in the web subpanels, up to the ultimate limit state, and must maintain nodal lines in a buckled web subject to shear. Hence, they must be appropriately sized. Experiments and numerical simulations indicate /44/ that the relative flexural rigidity ( $\gamma_L = 10.92I_L/dt_w^3$ ) of the longitudinal stiffeners need to be increased beyond the optimum relative stiffness ( $\gamma_L^*$ ), i.e.,  $\gamma_L \geq m_L \gamma_L^*$  where  $m_L$  is a semi-empirical amplification factor. The required rigidity depends, e.g., on the location of the longitudinal stiffeners, the aspect ratio of the longitudinally stiffened web panel, the web slenderness, the relative cross-sectional area of the stiffener and the web, the relative torsional rigidity of the stiffener (the effect of which is usually disregarded) and the type of loading /54/. In calculating the second moment of area about the neutral axis parallel to the web plate, an effective width of plate given by  $b_e = t_w \sqrt{E/\sigma_{ow}}$  /18,44/ should be assumed.

Longitudinal stiffeners are load-carrying and a direct strength check may be more reasonble (BS 5400), e.g., considering a strut approach. However, in general, for girders subjected to both shear and bending, a stiffness or strength approach is recommended /44/.

#### 3.6.4 Uncertainty measures

The uncertainty for direct and bending in-plane load expected to be as for the stiffened panels treated in Sect 3.2-3.3. The uncertainty associated with the BS 5400 appr for bending loading, was estimated to correspond to a bias COV of 0.988 and 4%, respectively /55/. Similarly, based sample of 18 test specimens, the bias and COV for the Comodel was estimated to be 0.99 and 4-5%, respectively /56/.

#### 3.7 Girders under predominantly shear stresses

#### 3.7.1 General

Several theories to calculate the ultimate capacity girders under shear, have been reviewed in detail in /44-The general conclusion /46/ is that the Cardiff model /48 represents the state-of-art. This view is also supporte the results of uncertainty analysis referred in Section 3.7

The Cardiff model has been implemented, e.g., in the of 19,14,51,52/, however, with slight modifications. BS5400 BS5950 do not allow the use of tension field theory longitudinally stiffened girders. EC3 does not of longitudinally stiffened girders as yet.

The ultimate strength of girders in shear consists of sum of three terms /48,49/ as shown in Fig. 4:

- (1) The elastic/inelastic critical buckling load  $V_1$  of the plate (beam action).
- (2) The load  $V_2$  carried by the post-critical tension (tension field action). The tension field contribution will be reduced by the presence of web perforate especially if they are located at the geometrical centrof the web plate.
- (3) The load  $V_3$  carried by the flanges at the instant plastic hinge formation in the flanges (Vierendeel mechanism).

From vertical equilibrium, the ultimate strength in shear plate girder is determined by (Fig. 4):

$$V_{ult} = V_1 + V_2 + V_3 = V_{cr} + 0.5\sigma_t^{\circ} t_w \sin 2\theta (d - b \tan \theta) + \sigma_t^{\circ} t_w (c_c + c_t)$$

where  $\sigma_t^\circ$  is the tension field stress to cause yielding of web material in the tension band (based on the von Mises equation); d and two are the depth and thickness of the web is the distance between transverse stiffeners;  $\theta$  is the intension of the tension band to the horizontal; and  $(c_c, c_t)$  the anchorage lengths of the tension band on the compression

and tension flanges, respectively.  $c_c$  and  $c_t$  identify the positions of the plastic hinges. The latter are dependent on the flange rigidities which are themselves influenced by the presence of any direct and bending in-plane stresses.

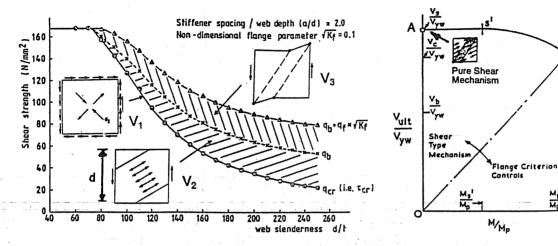


Fig. 4 Contributions to the ultimate shear strength of a plate girder /48/.

Fig. 5 Interaction curve between shear and bending effects /45/.

Bending

C Failure

The Cardiff model has been validated for plate girders with narrow, fairly thick flanges (see Section 3.7.5). The third term  $(V_3)$  may not be fully developed in box girders with wide, thin flanges, by using the "conventional" effective width of the flanges. Hence, limited use of the plastic frame mechanism action is recommended for box girders in, e.g., BS5400; see /43/. If  $V_3$  is neglected, the Cardiff model effectively reduces to the Basler model /44,48/.

Transverse stiffeners affect all terms in the expression, and are crucial to the shear capacity. Longitudinal stiffners are only accounted for by their influence on the critical buckling stress of the web, and hence the term  $V_1$ . Although the Cardiff model does not represent the most accurate approach /18/, it is recommended in /18,57/ because of its simplicity.

# 3.7.2 Effect of direct and bending in-plane stresses - interaction between shear and bending

Any direct and bending in-plane stresses affect the membrane stress tension field  $\sigma_{\rm t}^{\circ}$  and the plastic moment capacity  $M_{\rm pf}$  of the flanges, in addition to the critical buckling stress of the web plate.

The stress resultant of the diagonal tension band gives rise to a horizontal component in the flanges. The Cardiff model does not account for the effect of this horizontal component on the bending capacity of the flanges, while /46,58/propose to do so. However, the Cardiff model makes allowance of axial stresses in the flanges caused by in-plane girder

Interaction between shear and bending and direct ax stresses is accounted for by the Cardiff model using a diag as shown in Fig. 5. Parts AC and CD are based on the sh capacity (Section 3.7.1) and bending (direct stress) capac (Section 3.6), respectively. A transition in failure mo occurs at point B, where the applied bending moment is assu to be solely carried by the flanges. Between points S' (wh  $M_s \sim 0.5 \ Mp$ ) and B; and B and C, the curve may be represen by parabolas, or, without much loss of accuracy, strailines. Points D and E refer approximately to first yield plastic moment, respectively.

#### 3.7.3 Stiffener requirements

The shear capacity can be achieved only if the stiffen fulfill certain stiffness and strength requirements.

Transverse stiffeners (including an effective width of web) should fulfill stiffness requirements analogous to the mentioned for longitudinal stiffeners in Section 3.6. Howev transverse stiffeners carry compression forces equal to difference in predicted ultimate strength  $(V_u)$  of the gir and the critical buckling strength of the web panel in she acting in the plane of the web. The eccentricity to the sti ener's neutral axis of the loads introduces an addition bending moment in the stiffener. In addition, the outstraightness of possible longitudinal stiffeners (in the c pression zone only) causes a destabilizing moment in transverse stiffener. The intensity of each transverse for is recommended to be taken as 1% of the compression load in longitudinal stiffener (without any web effective width) /4 The strength of the transverse stiffeners is then verified the beam-column approach, similar to those outlined in Sect 3.3, using a buckling length of 70% of the stiffener de /18,44/. Some transverse stiffeners may have to carry p loads from other major components, e.g., in frame corners. additional stiffeners requirement for transverse stiffeners briefly reviewed in /44/.

Longitudinal stiffeners should fulfill certain stiffer and strength criteria, considering the destabilizing effect shear stress.

#### 3.7.4 Effect of web cut-outs

Significant information only exists for shear loading wery low axial bending stresses and for vertically stiffed plate girders containing one cut-out per panel. Accur formulations for special cases with (reinforced) cut-outs presented, e.g., in /59,60/.

The ultimate capacity can be assessed approximately linear interpolation between the unperforated ultimate strength ( $V_u$ ) and the Vierendeel (frame mechanism) load ( $V_v$ ) /6

$$V_{ult} = V_{v} + (V_{u} - V_{v})(1 - D/d)$$
 (16)

where D/d is the ratio of the opening diameter to girder depth. As mentioned before, this approach has been validated for plate girders where the relatively heavy flanges provide bending capacity in the frame action.

#### 3.7.5 Uncertainty measures

Uncertainty measures for plate girders under shear loading are given in Table 3.

Table 3 Model uncertainty for stiffened girders (without cutouts) subject to shear loading

	Cardiff	EC 3 <sup>2</sup>	BS 5400 <sup>1</sup>	EC 3 <sup>2</sup>
Ref.	/46/	/46/	/62-63/	/62-63/
Sample	164/67	164/67	102	109
Bias	0.98/1.00	$1.08/1.09^3$	1.14/1.6	1.10
COV (%)	6/7	16/13 <sup>3</sup>	16/36	14.5

<sup>1)</sup> BS 5400 for longitudinally stiffened panels is not based on ultimate limit state and tension field approach

limit state and tension field approach.

2) EC 3 do not provide criteria for longitudinally stiffened girders.

3) Pofore to provide criteria for longitudinally stiffened girders.

Refers to evaluation of model developed in Germany for longitudinally stiffened girders, for inclusion in EC 3.

While the uncertainties for the girder models presented in Table 3 are determined for "all available" data, a closer examination of the results shows that the uncertainty depends, for instance, upon the web slenderness bw/tw. The COV for bw/twless than 250 is about twice that for larger slendernesses (up to 400) /46/. By comparing the model uncertainties with and without accounting for the effect of the horizontal component of the tension field force in the flanges, the effect of these different formulations is found to be neglibible. Also, it is observed that the slightly different versions of the Cardiff model implemented for transversely stiffened girders in EC3 and British standards do not yield very different model uncertainties.

The BS 5400 model for longitudinally stiffened girders is based on empirical considerations /63'/ conducted before the accuracy of the Cardiff model for such girders had been proven.

The simplified approach, Eq. (16) mentioned above for accounting for cut-outs implies a bias and COV equal to 1.60 and 0.35, respectively. The more refined formulations for circular and rectangular cut-outs with/without reinforcement typically yield bias' in the range 0.83 to 1.07, and COV's between 6 and 17% /56,59-61/.

#### 4. ULTIMATE STRENGTH OF CYLINDRICAL SHELLS

#### 4.1 Introduction

Cylinders are frequently used in offshore platfor Significant research on the ultimate strength of cylindric columns and beam-columns was accomplished for civil engineer structures in the 1960-70's, see, e.g., /20/, and for logical failure modes of more thin-walled cylinders in aerosp structures in the 1960's; see, e.g., /64/. However, the modes of structures and the radial pressure loads acting offshore structures were not considered until the late 197 and 1980's, partial reliance being placed on long-term researched to submarine shells /65/.

Cylinders in offshore platforms may suffer both local overall failure. The relatively long, unstiffened thick-wal tubulars used in bottom supported platforms exhibit over failure; however, for deep water platforms, external press affects the strength. Thin-walled, stiffened tubulars floating platforms primarily suffer local failure modes, whare particularly sensitive to imperfections.

#### 4.2 Overall column and beam-column collapse

#### 4.2.1 Failure modes and formulations

With the relatively stocky cross-section of members bottom supported platforms, overall collapse is of prim concern. For the design for local buckling, cross-sections normally classified into four groups as follows:

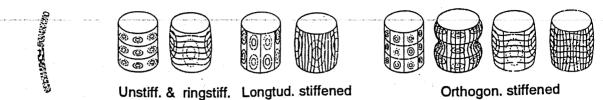
- plastic full plastic action can develop and maintained through significant rotations
- compact plastic hinge capacity can be achieved but limited rotations only
- semi-compact yield can develop at the extreme fibers
- slender local buckling occurs limiting flexu capacity.

The slenderness limits applicable to each group can vedepending on the structural action although this subtlety frequently ignored.

#### 4.2.2 <u>Column capacity</u>

The capacity of compression members is determined by interaction between buckling and yielding, and depermantly upon yield strength, geometrical imperfections, well as residual stresses (and, hence, the fabrication method as represented by column curves in Fig. 7 /74/. The effect residual stresses vanishes at high slenderness,  $\lambda$ , since

total (including residual) stress is less than the proportional limit. For really high slendernesses, both effects vanish and the capacity approaches the ideal elastic buckling strength.



a) Column mode

b) Local modes

Fig. 6. Buckling forms of cylindrical shells, see e.g. /11/.

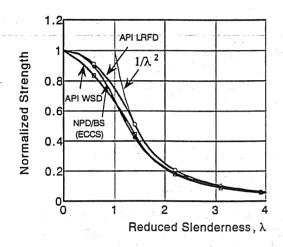


Fig. 7 Column curves

Fig. 8 Local buckling capacity under hoop compression.

Typically, differences in cross-section shape and plate thickness will influence the residual stress (and the ratio of I and A), and multiple column curves are applied. For tubular beam columns in offshore structures, there is no distinction between hot- and cold-formed tubulars /75/. Some codes (e.g., API LRFD, and some class society rules) are based on the SSRC Curve 1, while others (BS5950, DnV, NPD) apply the ECCS curve. The SSRC curve is similar to a Johnson-Ostenfeld correction to This curve follows the Euler the elastic buckling stress. It may be interpreted as a correction for curve for  $\lambda > \sqrt{2}$ . residual stresses (tangent stiffness formulation). curve more explicitly accounts for both residual stresses and In principle, a reduced yield stress geometrical imperfection. should be used when applying the European column curve to fabricated columns. However, since offshore tubulars constructed by cold rolling (which enhances the strength in the first place) and welding, this reduction is dispensed with. noted that API WSD is based on the AISC WSD, considered to be somewhat conservative for tubulars in offshore structures. This is because it is based on WF-shapes that possess high residual stresses compared with, e.g., cold-formed

#### 4.2.3 Bending

If D/t satisfies the requirement to fully plastic cresctions, the plastic moment is applicable (API codes, BS59 For larger D/t the appropriate reduction due to local buck is applied; see Section 4.3. NPD/DnV refer to the first y for the whole D/t range.

#### 4.2.4 Axial compression and bending

Interaction between axial compression  $(\sigma_a)$  and ber  $(\sigma_b)$  is established in some codes (API, DnV tubular memby:

$$\frac{\sigma_{a}}{\sigma_{au}} + \frac{C_{M} \cdot \sigma_{b}}{\sigma_{bu} \left(1 - \frac{\sigma_{a}}{\sigma_{E}}\right)} = 1.0$$

where  $\sigma_{au}$  is determined by the appropriate column curve,  $\sigma$  the ultimate stress under pure bending and  $\sigma_E$  is the buckling stress.  $C_M$  is a factor to account for the variation of (first order) bending stress along the member.

In addition to the overall "stability" limit state, (17) there are local yielding and buckling criteria. For a columns experiencing plastic strains before collapse, interaction between local and global behavior is complex /

The ECCS interaction equation is based on a Perry approach. The axial capacity,  $\sigma_{au}$  is represented by two to  $\sigma_a + e^*\sigma_a/(1-\sigma_a/\sigma_E) = \sigma_{au}$ , such that it corresponds to the concurve by suitably choosing a fictitious eccentricity exaccount for the reduction in strength due to imperfection residual stresses. The interaction equation for a compression and bending is then [ECCS; e.g., NPD/DnV]:

$$\frac{\sigma_a}{\sigma_0} + \frac{e^*\sigma_a + C_M \sigma_b}{(1 - \sigma_a / \sigma_E) \sigma_0} = 1.0$$

If the plastic bending capacity is utilized in bending, (18) should be modified by increasing e\* and  $\sigma_0$  in the seterm by the ratio of the plastic to elastic section mode of the plastic to elastic section mode of the conservative value of  $C_M$  is used in many conservative when the first order moments cause the beam-contonbend in an S-shape.

#### 4.2.5 Axial compression, bending and hoop stress

Current codes do not account for the combined effective external pressure, axial compression and bending loads on columns, or they represent this interaction poorly, e.g. API 2U. Such effects are important for deep-water platform

Loh /76/ modified the (elasto-plastic part of the) RP2A column curve and the moment capacity to account for external pressure.

#### 4.2.6 Uncertainty measures

Uncertainty in overall column curves is given in Table 4.

Table 4 Uncertainty in the ultimate capacity of columns under compression

Formulation	Data	No. of Bias		COV (%)
API RP2A	/77/	56	1.14	7
API LRFD	/77/	56	1.02	8
ECCS (DnV/NPD)	/74/	280	see 1)	5-16 <sup>1)</sup>

<sup>&</sup>lt;sup>1)</sup>The bias may be represented by B = 1.0 + 0.1  $\lambda$  (for  $\lambda \le 2$ ) and B = 1.0 (for 2 <  $\lambda$  < 3) and the COV also varies depending upon the reduced slenderness,  $\lambda$ .

It is seen that the LRFD approach corresponds better to the mean of experimental results than the RP2A (WSD) approach. The random uncertainty (COV) for the relevant formulations is seen to be relatively small.

Ref. /78/ provides an estimate of the model uncertainty for tubular beam-columns which have so small cross-section slenderness that local buckling effects have an effect. By using Eq. (17) with  $\sigma_{au}$  and  $\sigma_{bu}$  that account for local buckling, the linear interaction between axial and bending stresses that include second order effects, is found to have a bias of the order 1.1.

The Loh correction of the LRFD-column curve for hoop compression yields a bias of 1.07 and a COV of 6%, while the corresponding formulation for bending capacity implies a bias and COV of 1.24 and 7%, respectively. The latter uncertainty is reflected in a bias of 1.23 and a COV of 14% for beam-columns subjected to axial compression and bending under hoop compression.

Code formulations for ultimate bending capacity and axialbending under hydrostatic pressure, should be improved.

A crucial factor in applying the column curves is the effect of boundary conditions, as taken into account by the K-factor. Often this is done for offshore structures by using a constant value for braces and legs of jackets. This implies a significant conservative bias and some variability /79/.

# 4.3 Local buckling of unstiffened and ring stiffened shell

#### 4.3.1 Failure modes

Frequently, codes for ring stiffened shells spestiffness requirements for the stiffeners to prevent stiff collapse until the shell has failed. Hence, interframe fails generally of concern. Nevertheless, explicit ultistrength criteria are given for densely stiffened cylind see Section 4.4. Generally, the API codes LRFD, WSD and 2U differ although it should be observed that the former codes are applicable for D/t < 300 while Bul. 2U is also v for more slender shells.

#### 4.3.2 Axial compression or bending

Axial compression stresses cannot reach yield if D/t for  $\sigma_a$  = 240 MPa, and the bending capacity will be less the plastic moment if D/t > 30-50 for  $\sigma_0$  = 360-240 MPa, w the capacity under external pressure is essentially a l failure mode.

The three API formulations /WSD,LRFD,2U/ provided local buckling under axial load is based on the class elastic buckling formula in the elastic range. In the inelarange the strength is expressed as a function of D/t, whice fitted as a lower bound to experimental data. The NPD/DnV code uses a Merchant-Rankine approach /80/ to represent interaction between elastic buckling and yielding. Cho Frieze /81/ propose to improve the correlation with test by using a knock-down factor,  $\rho$  on the elastic buckstrength:

$$1/\sigma_{\rm u}^2 = 1/\sigma_0^2 + 1/(\sigma_{\rm E}')^2 = 1/\sigma_{\rm u}^2 = 1/\sigma_0^2 + 1/(\rho\sigma_{\rm E})^2 = (1 + \lambda^4)/\sigma_0^2$$

where  $\sigma_u$ ,  $\sigma_0$  and  $\sigma_E$  are the ultimate, yield and elastic buck strengths, respectively.  $\lambda$  is the normalized slenderness by  $\lambda^2 = \sigma_0/\sigma_E^2$ .

#### 4.3.3 <u>Hoop compression</u>

The hoop capacity is described by the elastic buckformulae for D/t > 60 for mild steel; the API-WSD and formulations are a lower bound and LRFD a mean fit to test in the elasto-plastic regime, described by several expression the NPD approach is again based on the MR approach and below the API predictions, because the  $\sigma_{\rm E}$  used is conservate A modified MR method can be used to replace the capacity single expression over the whole D/t range /81/. Fig. 8 so  $\sigma_{\rm u}/\sigma_{\rm 0}$  vs.1/ $\lambda^2$  using  $\sigma_{\rm E}'$  and  $\sigma_{\rm u}$  given by API and Eq. respectively. For D/t between 10 and 40 (a relevant range pipelines and tethers), a Timoshenko formulation, we

essentially is a stress check of an imperfect ring, is four

yield a good estimate, e.g., better than consideration of a plastic mechanism /82/.

#### 4.3.4 Axial loading and hoop compression

API codes specify different interaction equations for all combinations of axial tension/compression and bending and hoop compression. For instance, the interaction between axial and hoop compression is described by a quadratic relationship in 2U, while WSD and LRFD take the smallest values implied by the interaction equation for elastic buckling and the elastoplastic buckling capacities for individual loads.

NPD/DnV is based on a generalization of the Merchant-Rankine formula for interaction between elastic buckling and plastic yielding /80/. The advantage of this approach is that it allows also tensile loads to be easily handled and that it converges into linear interaction and the Von Mises-Hencky interaction for large and small D/t, respectively.

A further development of this method for combined axial and hoop compression was made in /81/ by calibrating knock-down factors on the elastic buckling stresses against test data.

For shells under axial compression and radial pressure, the effects of the various parameters controlling strength such as L/R, R/t, L/t, and E/ $\sigma_0$  were examined with a view to identifying those important for inclusion in knockdown factors. The effects were found to differ markedly between the two actions, but bearing in mind a preference to use the same parameters within all of the knockdown factors in order not to complicate the equations, the parameter chosen was  $(E/\sigma_0)\sqrt{Z}$ , where the Batdorf slenderness parameter  $Z = (L^2/Rt)\sqrt{1-v^2}$ ; R = D/2 is the mean shell radius.

The determined equations in the presence of axial stress  $\sigma_x$  and hoop stress  $\sigma_\theta$  were:

$$\left(\frac{\sigma_{x}'}{\rho_{x}\sigma_{xc}} + \frac{\sigma_{\theta}'}{\rho_{\theta}\sigma_{\theta c}}\right)^{2} + \left(\frac{\sqrt{\left(\sigma_{x}^{2} - \sigma_{x}\sigma_{\theta} + \sigma_{\theta}^{2}\right)}}{\sigma_{0}}\right)^{2} = 1.0$$
 (20)

where  $\sigma_x'=\sigma_x$  for  $\sigma_x<0$  and = 0 for  $\sigma_x\geq0$  ,  $\sigma_\theta'=\sigma_\theta$  for  $\sigma_\theta<0$  and = 0 for  $\sigma_\theta\geq0$  .

Clearly, this approach can be extended to other combinations of loads, but different knock-down factors will then have to be applied for a given load in different combinations to achieve a best fit.

#### 4.3.5 <u>Uncertainty measures</u>

The model uncertainty for current API codes (especially the LRFD approach) have been estimated by data reported in

/77/. Experimental results for ring stiffened cylinders compiled in /81/ and used to assess the uncertainty of modified Merchant-Rankine formulation, BS5500 and an version of the DnV formulation, where appropriate. While nominal geometrical data and yield stress are given for specimens, no information is given about initial imperfect and residual stresses.

Because a number of tests on single bay and st relieved shells also had been conducted, the effects of t particular factors were also examined in detail.

Among the API codes, 2U was found /79/ to give the fit to data for local buckling under axial compression, wi bias and COV of 1.07 and 8%, respectively. The NPD/DnV approach for this loading yields a bias and COV of 1.59 32%, respectively. However, as shown, e.g., in /81/, the muncertainty increases with slenderness, suggesting applying correction factors on the elastic buckling st would improve the accuracy. By using such a modified appraisable and COV of 1.05 and 14% was achieved /81/. different data bases applied in estimating these muncertainties should be observed.

For bending the API LRFD formulation, which converge the nominal plastic capacity for small D/t, yields a best to data within 15 < D/t < 80, with a bias and COV of 1.16 9%, respectively. Hence, there is a significant rescapacity beyond the plastic moment of an ideal plamaterial, essentially due to strain hardening. Howe another investigation /82/ for long cylinders with 10 < D 40 shows that the bias and COV are 0.87 and 7%, respectively. For this D/t range, it is demonstrated that other approaches, used for pipelines by Shell, yield a better fit.

The local hoop buckling stress in the elastic region approximately the same for the API WSD, LRFD and 2U. The equation gives significantly higher stresses in the inelaregion than the "lower bound" estimates that WSD and provide. The bias and COV of the LRFD is 1.12 and respectively, by using data for D/t in the range 30 to The NPD/DnV approach is found to yield a bias of 1.15 and of 8%. The corresponding numbers for the ECCS approach 1.21 and 11%. For more thick-walled tubulars (10  $\leq$  D/t  $\leq$  the Timoshenko formula is found to yield a good fit /82/.

Comparison of the three API formulations for a tension-hoop compression shows that the LRFD formulation yields a mallest bias with 1.2, while the COV of 16% is slightly larger than that for the 2U method, which has a bias and CO 1.3 and 14%. A closer look at the results shows that the predicts the inelastic capacity well, while the discrepancy long ("unstiffened") shells show a bias of the order 1.4 While the initial NPD/DnV-CNB formulation yields a bias and of 1.50 and 13%, a modified MR-formulation yields a bias COV of 1.16 and 9%, respectively.

The uncertainties in the three API formulations for interaction between axial and hoop compression are quite close; with, e.g., a bias and COV of 1.15 and 10% for API LRFD. The bias of 1.45 and COV of 20% in the NPD/DnV-CNB approach can be reduced to 1.03 and 8%, respectively, by introducing knockdown factors on the elastic buckling formulae.

For interaction between bending and hoop compression the API formulations, LRFD and 2U represent the extemities, with a bias and COV of 1.29 and 14%; and 1.44 and 8%, respectively.

An examination of stress relieved ring stiffened shell data demonstrated that, while residual stresses had little effect on axial compressive strength, under increasing levels of radial pressure, the shells exhibited increasing strength, up to 37%, in the case of pure radial pressure. It was assumed that stress relieving allowed substantial moments to develop at the frames in contrast to when residual stresses were present and yielding over the frames began early in the loading history thereby inhibiting the development of significant boundary moments.

#### 4.4 Frames in ring stiffened cylinders

Among the API formulations, the 2U approach seems to be a refinement of the other formulations. This is also borne out by the model uncertainties determined /77/. The model uncertainty of 2U is clearly smaller, amounting to a bias and COV of 0.96 and 6.8%. Since this sample is small, a statistical uncertainty should be added. Since the LRFD and WSD approaches are based on buckling in two modes in the circumferential direction, they become very conservative for high buckling modes. The BS 5500 approach is also considered relevant for shells subjected to radial or hydrostatic pressure, while for axial loading the NPD/DnV-CNB may be suitable. However, no uncertainty measures are available.

## 4.5 Orthogonally stiffened shells

#### 4.5.1 <u>Failure modes and formulations</u>

A wide range of failure modes can occur in orthogonally stiffened shells, involving the shell and stiffening elements in isolation and in combination. Thus the shell alone can buckle, the shell and longitudinal stringers combined can suffer interframe buckling, the stringers and frames alone can trip, and the shell, frames and stringers can suffer general instability — orthotropic buckling. The stiffened shell may itself not buckle, but it can suffer overall column buckling. As for ring stiffened shells, the frames are generally designed not to fail. Stringers can be proportioned not to fail in a local or overall lateral torsional mode.

The most relevant formulations for orthogonally stiffened shells in marine structures are ART Rul 2H /68/ DDY-CNR /11/

ECCS /73/ and RCC /83/. Approaches based on discretiffeners and orthogonal shell theory are given.

This review is to a large extent based on comprehensive study /84,85/.

## 4.5.2 Axial loading

API Bul. 2U calculates the strength of the shell plate under axial compression based on the theoretical elast buckling stress, a knock-down factor which (implicit accounts for imperfections and inelastic behavior dure collapse. The axial capacity of the stiffened panel determined by first calculating the elastic buckling stress for the plate and stiffener separately. The elastic buckling stress of the stringer is determined by assuming it to simply supported, including an effective width of the showith a reduction factor for residual welding stresses. elastic buckling of the stiffened panel is then calculated the sum of the two contributions, and the collapse strength obtained by using the Ostenfeld-Bleich tangent modapproach. The resulting stress  $\sigma_{ic}$  is used to calculate revised slenderness parameter and an effective width,  $b_{em}$ . ultimate axial capacity,  $\sigma_{u}$  is then

$$\sigma_{u} = \sigma_{ic} (A_{s} + b_{em} t) / (A_{s} + bt)$$

when the ultimate force is averaged over the total cr section area.

The RCC approach, which preceded Bul. 2U, follows the procedure, however, with a modified elastic buckling stress knock-down factor for the plating, and introducing reductations for residual stresses depending on the weldment.

The ECCS approach is similar to that of the ab mentioned codes, but uses a less refined knock-down factor shell buckling. The DnV approach (formulation C) deviates the API/RCC methods by referring only to the stiffener wit effective shell, rather than to contributions from stiffener and shell, and like to ECCS approach, by using si "knock-down" factors.

# 4.5.3 Radial loading

The formulation for radial loading is analogous to for axial loading.

API Bul. 2U predicts the ultimate capacity by superpotent buckling pressure for the (unstiffened) shell and plastic collapse of the ring stiffener including an effection flange width, with a knock down factor.

The RCC approach is based on the plastic capacity of ring stiffener, using several empirical geometrical correct

factors, and an Ostenfeld-Bleich tangent modulus to account for plasticity. However, the effective flange of the shell considered seems to be small.

The NPD/DnV-CNB and ECCS approaches for radial pressure are similar to those used for axial load, however, using the appropriate theoretical buckling coefficients.

## 4.5.4 Combined axial and radial loading

API Bul. 2U recommends the following ultimate strength interaction equation for combined axial load and radial load

$$R_{x}^{2} + CR_{x}R_{\theta} + R_{\theta}^{2} = 1 \tag{22}$$

where  $R_i$  is the utilization ratio for load no. i). The initial RCC approach used a different quadratic interaction function. However, in the recent study /84/, Eq. (22) was recommended, but with a different C than in the RCC formulation.

DnV-CNB uses a generalized Rankine-Merchant approach described above for unstiffened shells.

## 4.5.5 Uncertainty measures

Table 5 shows the model uncertainties estimated in /84/ using a selection of available test results for orthogonally stiffened cylinders. It is shown particularly that aluminium shells tested for aerospace structures failing by elastic buckling are not relevant for validating prediction methods for offshore structures. Also, a closer look at the model uncertainties, reveals that they vary depending upon which test sample is considered. For instance, the bias of the RCC formulation for axial load is 1.13 using 6 small scale specimens (IC1-6) and 0.93 when using 5 specimens B1-5. This fact may be related to differences in residual stresses and geometrical imperfections, even if imperfections were reported to be within the tolerance level.

Among the methods compared, the RCC method gave the best fit. Various possible improvements of the RCC formulation for axial capacity were investigated in /84/, but only resulted in minor reductions in the model uncertainty. The NPD/DnV-CNB formulation is not satisfactory for radial and combined loading. The large bias and COV are attributed to considering the shell unstiffened when the Batdorf width parameter  $Z_{\rm s} < 8.586$ . If the more applicable narrow panel DnV model for axial compression is used, the figures would probably improve. Even if the RCC model for radial load implies a small uncertainty, it is observed in /84/ that this is partly due to using adjustment factors to fit the limited test data available. Hence, further improvement of the basic formulation is necessary. It is also desireable to improve the interaction equation to reduce the random uncertainty (COV).

Table 5 Model uncertainties estimated by comparison with results for steel shells /84/

Load:	Axial load		Radial load		Interaction	
Test data <sup>1)</sup> :	48 specimens		11 specimens		38 specimens	
Code	Bias	COV (%)	Bias	COV (%)	Bias	COV (8
API 2U		,				
- discrete	1.02	14	1.21	15	1.13	16
- orthotropic	0.86	25	0.87	46	_	
RCC	1.02	14	0.97	10	1.09	25
DnV-CNB <sup>2)</sup> (1984)	0.97	21	1.40	39	1.68	25
ECCS (1983)	1.21	24	_	<u></u> .		

 $<sup>^{1)}\</sup>sigma_0$ : 310 to 510 MPa; R/t: 95 to 505; L/R: 0.64 to 4.8; s/t: 25 to 130.  $^{2)}$ Only DnV-CNB formulation C (orthotropic theory) is applied.

It is noted that the bias of the API Bul. 2U and formulations for axial or radial loads is practically invary with respect to the predicted strength to squash load, the for the other formulations decreases towards 1.0 increasing strength (slenderness).

The advantage of the general interaction approach used in DnV-CNB is that it converges to the von Mises-He yield criterion and linear interaction for stocky and slender structures, respectively. A further advantage is it accounts for the effect of tensile stresses. However, fact that the bias/COV varies with the slenderness sugg that the formulation may be improved by introducing addition buckling stress, the elastic factor onknockdown demonstrated for unstiffened/ringstiffened shells in Sec However, it is desireable to do this based on be information about imperfections and residual stresses, als view of the magnitude of such fabrication factors in a structures.

Finally, it is noted that formulations for trip failure of stiffeners in general is based upon a hinge bet stiffener and shell plating. DnV-CNB allows using an elementariate on the stiffener, but the fact that shell buck may initiate stiffener failure, is not yet covered.

### 5. ULTIMATE STRENGTH OF TUBULAR JOINTS

#### 5.1 General

Joints between panels, and rectangular tubulars with the same width can be designed (with diaphragms) to transfer forces by membrane actions, and the ultimate strength may be fairly easily estimated. Joints between cylindrical members have a much more complex - shell - behavior. Such joints may be classified as follows:

- unstiffened, plane joints
- unstiffened, multiplane joints
- stiffened joints
- grouted joints

Current codes /16,17,66,67,86-88/ specify parametric formula for unstiffened, plane K, T, Y, X shaped joints, corroborated by tests. Other joints, e.g., plane KT and DT and multiplane joints, need to be considered on a case basis. Other unstiffened joints are usually classified based on the geometry and loading, see, e.g., Fig. 9b. General reviews of ultimate strength of tubular joints may be found in, e.g., /86,90/.

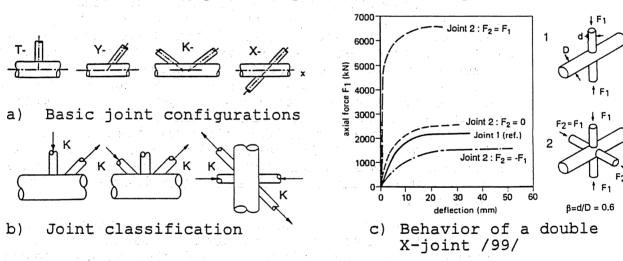


Fig. 9 Tubular joints

material and Depending upon geometrical properties, ultimate failure of a joint may occur due to buckling (due to compressive loading), excessive deformation, fracture or gross separation, shear failure of the chord-brace joint, lamellar tearing (of thick chords). For Y- and T-joints, distinguished compression and therefore, between Different definitions of crack size that implies tensile failure have been used. While API, CSA, NPD and DnV use first crack, HSE /16,91/ refers to ultimate load capacity. The former approach is conservative for static loading, but is interaction convenient avoid consideration of to fatigue and fracture. In case of tension and interaction between tension and in- or out-of-plane bending, the joint does not exhibit a distinct limit capacity in the load-deflection

behavior and the limit state should be given by max deformation, and possibly maximum strain (to avoid fracture

By merging all failure modes in one ultimate limit stit is important to ensure ductility, especially for mate with yield stress,  $\sigma_{\rm o}$  exceeding 400 Mpa. Different capacity the ratio  $\sigma_{\rm o}/\sigma_{\rm u}$  in the range 0.70 to 0.83.

Current formulations have been established by fit parametric formulae to experimental data. Recently, system studies using nonlinear finite element methods have also used to generate a data basis for fitting ultimate limit s equations. While API adopts a pragmatically determined 1 bound of test data, European codes are based more statistical analysis to determine mean and 5% fractile curv

## 5.2 Formulations for simple joints

Strength provisions in the API WSD/LRFD codes are widely used. They contain parametric formulae establishe the basis of a lower bound on experimental data selected /92/. The formulae are generally of the form /66/:

$$P_{u} = Q_{u} Q_{f} \frac{\sigma_{o} T^{2}}{\sin \theta}$$

$$M_{\rm u} = Q_{\rm u}Q_{\rm f} \frac{\sigma_{\rm o}T^2}{\sin\theta} (0.8d)$$

in which  $\sigma_o$  and T are the chord yield stress and thickness;  $Q_u$  and  $Q_f$  are nondimensional factors for brace load and of stress effects, respectively. These factors are function the ratios of the diameters of the tubulars, of diameter plate thickness etc.

The following interaction equation is recommended by A

$$1 - \cos\left[\frac{\pi}{2} \frac{P}{P_u}\right] + \sqrt{\left(\frac{M}{M_u}\right)_{IPB}^2 + \left(\frac{M}{M_u}\right)_{OPB}^2} = 1.0$$

where IPB and OPB refer to in- and out-of-plane bending.

Since the review /92/ was accomplished, new data become available and data used before, have been obsolete. New data bases have been established by /93//94/. Other code formulations have been proposed by, e.g. HSE, DnV, and NPD. In particular, improved formulations been introduced for X-joints and in-plane bending. Also, Canadian and European codes refer to a 5% (or 95%) fractionstead of a lower bound.

The deviation of the NPD formulation from, e.g., All partly caused by NPD introducing the constraint that the devalues shall be obtained from the characteristic value using the same material (safety) factors.

Instead of Eq. (24) other codes apply an interaction which is linear in the axial force and out-of-plane moment and quadratic in terms of the in-plane moment.

It is also noted that the strength formulae given in various codes have different range of validity.

In view of the harmonization of offshore codes, a new data basis /95/ was established by a critical screening of existing data, and recommendations on selection of formula were given in /96/. See also Table 6.

In particular, API LRFD /66/ applies a  $Q_u$  (in Eq. (23)) for in-plane bending, which is  $Q_u=3.4+19\beta$ , where  $\beta$  is the ratio of brace and chord diameters. Other codes (e.g., DnV, HSE, NPD) apply a formula of the type  $Q_u=k\sqrt{\gamma}\,\beta$  (where  $\gamma$  is the chord radius to thickness ratio), however, with different values of k. Recently, these formulations have been reviewed /97/ in view of a new experimental and numerical data basis for T and Y joints. While the latter form of  $Q_u$  with k=4.75, fits the mean numerical data generated by nonlinear shell analysis, it represents a lower bound on experimental data.

A closer look at the data for axially loaded T/Y joints reveals that the strength factor  $Q_u$  is determined from tests with loads in the chord, which makes  $Q_u$  smaller than it otherwise would be. However, when using Eq. (5.1) with this  $Q_u$ , also a  $Q_f$  factor is applied. To avoid that the effect of load in the chord is accounted for twice the initial  $Q_u$  should be adjusted to correspond to zero beam-bending /98/.

# 5.3 Model uncertainty for simple joints

Table 6 summarizes the uncertainty data of the formulations proposed by Yura /92/ and used in existing API guidelines. This table also shows the uncertainties implied by one slightly modified formulae, using a new data basis, which appears to be an improvement. The large scatter experienced for tensile loading is in part due to imprecise formulation of the ultimate limit state. For this reason, it may be necessary to establish two types of limit states for tensile loading, namely one referring to initial crack occurrence and one based on the ultimate load capacity.

Clearly, the uncertainties depend on how the joints are categorized in terms of geometry and load conditions. The wider variety of cases considered in one category, the larger is the scatter. It is also, therefore, necessary for instance, when all joints with in-plane bending are treated together in one category, to ensure that the data basis is representative. But for joints subject to bending there is very limited reliable experimental data for other joints than T/Y-joints, and the uncertainty measures need to be judgementally adjusted.

Table 6 Model Uncertainty of Design Formulae for Unstiff Tubular Joints

		Yura/API /92/		Study /96/1)		
Joint type	Load type	В	COV (%)	В	COV (%)	Formu
T & Y	tension	7 47	4.1	1.06	15	CS
X & DT	tension	1.41	41	1.15	30	DEn/
T & Y	compression	1 07	8	1.11	23	DEn/
X & DT	compression	1.07		1.14	12	CS
K & YT	compression	1.17	17	1.34	16	DEn/
All	in-plane-bending <sup>2)</sup>	1.23	14	0.97	21	CS
All	out-plane-bending	1.09	27	1.17	15	CS

<sup>1)</sup> All formulations except the one for X & DT joints and in-plane be are based on Yura. The model uncertainty is determined by using data basis.

Based on a new numerical data basis, the bias and COV for  $Q_u = k\beta\sqrt{\gamma}$ , = 4.75, are found /97/ to be 1.02 and 7.8%.

While some qualitative information about the uncerta of interaction equation is given in /97/ more work is neces to quantify the uncertainty and choice of interactions.

#### 5.4 Other joints

In practice, other plane joint configurations than simple T, Y, X, KT and K are applied. For instance, KT joints may have to be classified according to the predomi loading, as shown in Fig. 9b. However, the presence o unloaded member would necessarily influence the strengt some extent, and, hence, the model uncertainty when formulae for simple joints are applied.

In connection with multiplanar joints, it is necessar distinguish between situations where joints in a plane loaded and hence refer back to the cases discussed, situations with truly three-dimensional loading. The effect the 3D-character of the loading is substantial for doubl joints as illustrated in Fig. 9c. Double K-joints discussed in /89/. Guidance on multiplanar joints is limit but some correction factors for multiplanar effects are in /86,99,112/, however, with no explicit assessment uncertainties.

Code provisions for ring stiffened joints are also ralimited. Obviously, it is not practicle to establishment expressions for such joints, and one is referred using calculation procedures.

Clearly, there is a need for system experimental/numerical studies, including some benchmark to of the implied uncertainty of current methods used to determ the strength of complex tubular joints.

## 6. FATIGUE LIMIT STATES FOR WELDED JOINTS

### 6.1 General

In the design of marine structures against failure under cyclic loading, high cycle fatigue in welded joints, is of main concern. In general, limit states for fatigue and fracture could most accurately be established by fracture mechanics methods, especially when the effect of inspection is to be taken into account. However, the basis for current design practice both for platforms and ships subject to variable amplitude loading, is still primarily SN curves combined with the hypothesis of linear cumulative damage according to Miner-Palmgren (MP) /16,17,106-110/. Despite the deficiency of the MP method often observed, there is no competitive approach for use in design.

Adopting the SN-MP approach, the limit state may then be written:

$$\sum \frac{n_{i}}{N_{i}} - D = 0, \quad N_{i} = KS_{i}^{-m}$$
 (25a,b)

where  $n_i$  and  $N_i$  are the number of cycles in stress range block (i) and  $N_i$  the corresponding number of cycles to failure, K and m are material/geometrical parameters. Possibly different (K,m) sets can be applied for different regimes of S. D is the cumulative damage at failure. The stress (load effect) may either be a nominal member stress, often used for steel plated structures or a hot spot stress, e.g., used for tubular joints in offshore structures.

Fatigue failure in connection with constant amplitude loading is typically defined as visible crack, through thickness crack, or loss of strength of the specimens used. Hence, there is residual fatigue endurance both for tubular joints, and especially for steel-plated structures when the former two types of limits are reached. However, to utilize this residual life under variable amplitude loading, it is necessary to document that premature fracture does not take place due to an overload occuring in the random load history.

# 6.2 SN curves and stress concentration

The fatigue strength of welded steel joints depends largely upon their geometrical properties: the initial crack size, weld geometry and other geometrical parameters causing stress concentration. The fatigue (crack growth) strength is practically independent of, e.g., yield stress, even if ref. /115/ reports an increase in fatigue life by a factor of 1.5 to 2.0 for cruciform joints joints made of HT-steel compared to mild steel.

In the nominal stress approach, the SN curve refers to a

concentration, while the effect of weld geometry etc. is t into account in the SN curve. In principle, an infinite nu SN curves then need to be considered to represent variety of welded joints encountered in practice. However actual fatigue design codes, nine curves (BS 5400 /102/ fifteen curves (ECCS /103/, IIW /104/) have been found to appropriate. BS 5400 has been widely used for offshore of (e.g., BV /107/, HSE/DEn /16,10/, NPD /17/, DnV-CNF /10 together with an appropriate SN curve for tubular joints. recently, BS 5400 has been adopted as a basis for ship r (e.g., ABS /109/, DnV-S /110/). GL /15,111/ uses IIW / (ECCS) supplemented by a classification of many details typ for ship structures. API codes are based on ANSI/AWS of /112/, however, with their own version of SN curves for tub In the recent revision of US bridge code /113/ joints. curves were adjusted to conform with the EECS/IIW curves /1 In the studies /115-116/, an alternative fatigue j classification for ship details was proposed.

Tubular joints are treated by the hot spot stress con using one (e.g., DEn/HSE, NPD, ...) or two curves (e.g., which account for the local weld geometry. The stresses ( effects) applied are of nominal member stresses times st concentration factor which takes into the overall s behavior of the tubular joint. The T-curve used for tub joints by HSE/DEn and NPD/DnV-CNF, is practically identica the D-curve for steel-plated structures. This curve applied butt welds. API refers to one SN curve for tubular joints a smooth weld profile and one with no postweld treatm Clearly, in the hot spot stress approach, the SCF and SN of are inexorably linked. The determination of SCF is define DEn/HSE, but no explicit procedure is given by API; howe different definitions of hot spot stress is one reason for The hot spot stress approach difference in SN curves. recently also introduced for steel-plated joints in Two basic SN curves - one for a sm structures (DnV-S). welded joint and one for base material, are applied. The curves are taken to be in-between the BS 5400 C and D cur due to gross geome concentration factors eccentricities as well as weld geometry is applied on effects.

While the SN curves in the nominal stress approach directly based on experimental data, the hot spot stapproach refers to a few SN curves and relies on a calculate measured SCF. Hence, it is crucial to calibrate the hot approach using experimental data for fatigue endurance.

Ref. /117/ describes an interesting correlation of curve based on the hot spot stress approach, to see experiences of fatigue cracking or no cracking. The procuntered in this case is associated with the effect of variability or uncertainty on the results.

The basic SN curves are modified to account for:

- environmental conditions (air, cathodic protection, exposure to free corrosion)
- effect of variable amplitude loading
- thickness effect

The SN curves beyond N >  $10^7$  are commonly modified due to the effect of corrosion and variable amplitude loading. Although, under constant amplitude loading, welded steel joints exhibit a fatigue limit, this is not necessarily the case under variable loading, since cracks initiated by high stresses may subsequently propagate under low stress levels. This may be taken into effect by suitably extending the SN curve beyond the endurance level.

Another aspect of variable amplitude loading is stress interaction, i.e., influence of previous load cycles on subsequent damage. This effect is not accounted for by the Miner-Palmgren approach.

The basic SN-curves in BV-O /107/, DEn/HSE /16/, and DnV-S /110/ refer to joints in air or with cathodic protection, with a m=3 and m=5 for N <  $10^7$  and N >  $10^7$ , respectively. In case of free corrosion, the fatigue endurance is reduced by a factor of 2 and no reduced slope for N >  $10^7$  is applied. NPD and DnV-CNF use a similar approach, except that for joints with cathodic protection, the curve with m=3 applies to N =  $10^8$ , at which a fatigue limit is introduced. API /67/ recommends a corrosion limit at N =  $10^7$  or  $2 \cdot 10^7$  for the X and X' curves in atmospheric service, while no endurance limit is used for free corrosion. ABS-S /109/ seems to refer only to one set of SN curves.

It is noted that for ships it is normally required that the fatigue analysis is carried out by reducing the nominal plate thicknesses with the corrosion allowance. The thickness effect is generally accounted for by a correction factor  $(t_{\rm ref}/t)^{\rm m/4}$  on the fatigue life; however, the reference thickness  $t_{\rm ref}$  varies slightly in different codes. The API approach for tubular joints accounts for thickness and weld profile, as explained in /118/.

## 6.3 Model uncertainties

## 6.3.1 General

The limit state is described by SN data and the Miner-Palmgren approach. However, since stress concentration factors for nominal stresses are intimately related to the fatigue resistance, uncertainties in SCF's are briefly commented upon.

The BS 5400/DEn/HSE curves are given with uncertainty measures (bias, COV) for K, while m is considered constant (typically equal to m = 3). Uncertainty measures of the ANSI/AWS/API data are given in /119/. The uncertainty in the fatigue endurance, N may be modeled by Weibull or lognormal

distributions /120/. While the Weibull model is theoretic preferable, the lognormal model is frequently used.

If the joints are classified according to characteristic fatigue strength, it should be noted that will imply that each class will include joints with diffe mean value and standard deviation, if only the characteri value (mean minus two standard deviations) is the same. would increase the scatter of the joint category.

The model uncertainty is determined by endurance data the range of  $10^5$  to  $10^7$  cycles, and the uncertainty representative for this range. Extrapolation, e.g., beyond  $10^7$ , is obviously uncertain, especially because corros variable amplitude loading and other phenomena affect this of SN-curves. This fact is also implied by the difference used for this area, and should be reflected in a lamodel uncertainty for that range.

# 6.3.2 Stress concentration factors

Fatigue design approaches based on the nominal st approach, incorporate the effect of weld and other 1 geometry in the SN curves. Hence, only more "glo geometrical effects need to be accounted for in the effect. However, assigning (new) welded joints to one a several categories without using experimentally determined data, would imply a certain (model) uncertainty.

The hot spot approach (used so far for tubular joint offshore platforms and for ships, e.g., by DnV) is based or or a few basic SN-curves and carefully determined structure concentration factors. Parametric formulae generated systematic experiments and finite element analyses, are for common cases, while otherwise an ad hoc determination SCF by experimental or numerical methods is required. Take compares model uncertainties for some parametric formulae SCF's in tubular joints achieved in /121/. It should be not that the uncertainty varies with the SCF level, indicating the formulae are not fully optimized. Also, it is noted most design codes accept the use of different parameter formulae, which obviously will increase the uncertainty important by the code. No measures of the uncertainty in the SCF's steel-plated joints in ships /110/ seem to be available.

For the cases when, say, finite element methods applied to determine the SCF, it is necessary to have a measure for the uncertainty of the calculation procedure. Obvious this is only possible if the method is clearly specified of element, mesh size, extrapolation of stress to hot setc.).

Table 7 Measured to predicted SCF's in tubular T- and Y-joints, based on data given in /121/

Load		Method					
SCF range	Uncertainty	Gibstein	Kuang	Smedley	UCL		
Axial load	Bias	0.97	0.92	0.85	0.79		
SCF: 3.3 to 13.7	St. dev.	0.22	0.17	0.11	0.13		
24 specimens							
In-plane b.	Bias	0.89	0.96	0.81	0.70		
SCF: 1.1 to 4.9	St. dev.	0.19	0.20	0.19	0.14		
12(8) specimens							
Out-plane b.	Bias	0.98	1.01	0.91	0.81		
SCF: 1.6 to 10.6	St. dev.	0.18	0.25	0.16	0.11		
25 specimens							

### 6.3.3 Miner-Palmgren hypothesis of cumulative damage

The uncertainty in using the MP model can be assessed by calculating  $D_i$  (Eq. (6.1)) at failure for specimens (i) subject to variable amplitude loading, based on the actual load spectra and SN-curves for the actual joints. The model uncertainty may be determined by the sample statistics of Di and the assumption that the random uncertainty  $(V_D)$  in D is composed of the model uncertainty of the MP approach  $(V_{\Delta})$  and the uncertainty in the constant amplitude (SN) data  $(V_{SN})$ , as follows:  $V_D^2 = V_{\Delta}^2 + V_{SN}^2$ /119/. The data summarized in /119/ indicate a mean  $\Delta$  in the range 0.8 to 1.6 and a COV in the range of 20-30% for each data set. However, most of these data refer to block loading which yields systematically higher damage ratios than random loading and is, hence, not representative for random loading /122,123/. It should be observed that the model uncertainty of the MP approach, is directly linked to how the SN curve is modeled outside the range of available data (say, >107); and how the number of cycles are counted in broad-band loading. Broad-band loading seems to give low damage ratios at failure /122,123/. However, no explicit measures of the model uncertainty of the Miner Palmgren approach is given in these references.

#### 7. CONCLUSIONS AND RECOMMENDATIONS

Ultimate and fatigue limit states for establishing de equations, and failure functions for reliability analysis been reviewed, with a focus on limit states for marine structures.

While experimental results traditionally have been for validating or calibrating analytical limit s formulations, recent developments in numerical methods them a useful supplement. Numerical methods agree well carefully executed experiments and can be efficiently use systematically study the influence of geometrical and mater parameters.

It is (still) customary to estimate the uncerta associated with limit state equations by assuming that experiments represent the "truth". However, this is not a the case - uncertainties in the experiments will often as the estimated model uncertainty and make it conservative.

The adequacy of limit state functions should be asserbly calculating the bias and COV in relation to experiment numerical data, possibly as a function of slenderness or correction to the model, it is particularly important achieve models with a low COV,  $V_R$ . The resistance factor a direct function of  $V_R$ , e.g.,  $\gamma_R \simeq \exp\left[\left(\alpha_R\beta_T - k_R\right)V_R\right]$  where is the importance factor of the resistance, and  $\beta_T$  is target reliability index (typically in the range 3.2 to While  $\alpha_R$  may be relatively small  $(\alpha_R \leq 0.4)$  for beam-column jackets, it may be 0.7 to 0.8 for steel-plated panels shells in floating structures.

Several accurate formulations exist for axially lestiffened flat panels, with a bias and COV of 1.1 and Accurate prediction of stiffener tripping by strength materials type formulations, still remains unsolved. Custofrmulations for interaction between axial and lateral load stiffened panels imply very large uncertainties, and fur experimental/numerical studies should be carried out to retain problem.

For merchant ships, resistance formulations base (linear) buckling analysis with a correction for plasticit still applied in some codes. Such formulations shoul replaced by the more modern approaches used for off structures. This will yield more rational design, and rethe likelihood of errors (by limiting the number formulations used) and in general ease the transfetechnology between these areas.

In the last decade, ultimate strength formulations stiffened plate girders subject to in-plane bending and have emerged in the civil engineering community. The uncertainty of the most accurate approach corresponds to a and COV of 1.0 and 7%, respectively. Such models also h potential use in marine structures, which should be explored

While adequate ultimate strength formulations are available for cylindrical columns and beam columns, the formulation for the effect of hoop compression on beam-column behavior used in existing codes, should be improved.

It is particularly important to have accurate formulations for stiffened, large diameter cylinders because the uncertainty in load effects for such components is relatively small compared to that of the resistances, and the uncertainty has a significant influence on the partial safety factor for the resistance.

The API codes LRFD, WSD and 2U contain ultimate strength criteria for cylindrical shells subjected to basic and combined loads. Formulations are more empirical fits to data than desirable, and the difference between LRFD and WSD codes is unnecessary. Among other codes, the DnV-CNB and NPD offer advantages, by applying a generalized Merchant-Rankine approach, which facilitates interaction between elastic and plastic failure, and between different load types. However, the model uncertainties in this formulation should be reduced by introducing appropriate knock-down factors.

With regard to unstiffened cylinders, ISO activities aim at harmonizing formulations (API, NPD) for an international offshore fixed steel structures code by the end of 1994. venues for improvement of existing formulations for stiffened shells are indicated when the ISO harmonization The largest uncertainties for unstiffened/ring continues. stiffened shells are associated with axial tension and hoop as well as bending and hoop compression. compression Improvement in collapse loads for orthogonally stiffened shells with radial, and radial and axial loading, is also desirable. The treatment of tripping failure of stiffeners also has to be resolved. However, it may be necessary to include detailed information about imperfections and residual stresses and also systematically use nonlinear FEM to reach such a goal for stiffened thin-walled cylindrical shells.

A major step in developing ultimate strength formulations for tubular joints took place when the results of Yura's reassessment of experimental data were introduced in the API codes in the early 1980's. Several other codes: CSA, DnV, DEn/HSE and NPD have since then emerged, proposing different expressions for ultimate strength, with different range of validity. New experimental data and the quality and systematic use of nonlinear finite element analyses make it appropriate to establish a more universal code for the static strength of tubular joints, with a minimum of uncertainty. Besides rectifying some deficiencies in limit states for simple, unstiffened joints, a more explicit evaluation uncertainty in the determination of ultimate capacity of complex, stiffened joints is required.

Limit states for fatigue design of marine structures subject to wave loading, are specified in terms of SN curves and the Miner-Palmgren approach. For offshore structures, formulations based on nominal stress are applied for plated

structures, while the hot spot stress method is used tubular joints, and exceptionally for other components. We Germanischer Lloyd for a long time has been applying nominal stress approach for welded joints in ships, the spot stress approach has also been adopted in at least one for ship structures. It is, therefore, desireable to harmon approaches for fatigue design. This would imply agree in when nominal and hot spot stress approaches should be used well as joint classification in the nominal stress approached at and procedures for calculating SCF's in the hot stress approach.

The uncertainties associated with SN curves, to a l extent, relate to geometrical stress concentration. explicit uncertainty measures are available for approa based on the nominal stresses (and typically nine or more joint classes), less information is available for hot stress approaches - in which the major uncertainty transferred to the SCF (which is normally associated with load effect). It should be noted that the approach base nominal stress and joint classes, imply an uncerta associated with selecting weld joint class. SN-curves sho as far as possible, be based on nominal stresses experimentally justified SN curves. It is necessary to use spot stress approaches for new joints, but an estimate of uncertainties involved should be obtained and used. Due to limited amount of experimental data and a complex interaction between variable amplitude loading and corrosion effects the range of 107 cycles and beyond, a larger model uncerta should be assigned to this part of the SN curve.

When using the limit state equations in reliable analyses, an estimated model uncertainty may have to adjusted to reflect a different level of geometrian and other parameters in actual structure components compared to that in the specimens or numer models. Also, model uncertainties for interaction equationed to be carefully implemented.

Hopefully, (all) codes in the future will inclucommentary which will be useful in future revisions of codes.

A benchmark study concerned with the ultimate streng stiffened panels conducted by this committee, demonstrated the implementation of a method for ultimate strength predi in a computer program, and its use, may lead to e (departure from the intended procedures).

To reduce such errors, methods and computer codes ne be validated against benchmark tests, e.g., by ISSC. addition, quality assurance of actual calculation tasks to be exercised. Yet, ultimate strength and fatigue estimates will still be affected by such "human factors".

#### ACKNOWLEDGEMENT

The committee appreciates very much the contribution from Dr. M. K. Chryssanthopoulos and Mr. C. Tolikas to the benchmark study, and Chapters 3.6-3.7, respectively. We also would like thank Bureau Veritas, Det norske Veritas and Lloyd's Register of Shipping for kindly and actively participating in the benchmark study.

#### REFERENCES

- /1/ ISO 2394, "General Principles on Reliability for Structures," Int. Standards Organization, First ed., 1973 (Second ed., 1986: Draft, revision 1994).
- /2/ ISSC IV.1, "Report of Committee IV.1 Design Philosophy," Copenhagen, 1988, Wuxi, 1991, St. John's, 1994.
- /3/ Chryssanthopoulos, M.K. et al., "Imperfection Modeling for Buckling Analysis of Stiffened Cylinders," J. Struct. Engng., ASCE, vol. 117, no. 7, July 1991.
- /4/ Ang, A.H.-S., "Structural Risk Analysis and Reliability-Based Design," J. Struct. Div., ASCE, vol. 99, no. ST9, 1973, pp. 1891-1910.
- /5/ Benjamin, J.R. and Cornell, C.A., "Probability, Statistics and Decision for Civil Engineers," McGraw Hill, New York, 1970.
  /6/ Faulkner, D., "Criteria and Guidance for Good Strength Models," Dept. Naval Arch. and Ocean Engng., Report NAOE-91-15, Univ. of
- Glasgow, July 1991.

  /7/ Moan, T. et al., "Limit States for Tendon and Production Riser Bodies. Selection and Implementation of Limit States as Failure Functions and Design Equations," Report STF70 F93071, SINTEF Trondheim, April 1993.
- /8/ API Bul. 2V, "Bulletin on Design of Flat Plate Structures," American Petroleum Institute, Bulletin 2V, First Edition, May 1, 1987.
- /9/ BS5400, "Code of Practice for Design of Steel Bridges, Part 3" British Standards Institution, London, 1982.
- /10/ BV, "Reglement du Bureau Veritas pour la classification des navieres de longeur superieure a 65m; édition de 1987."
- /11/ DnV-CNB, "Buckling Strength Analysis," Classification Note 30.1, May 1992 (Refer previous version, "Buckling Strength Analysis of Mobile Units," June 1984, revised Oct. 1987)
- /12/ DnV-S, "Rules for Classification of Ships," Det norske Veritas, Oslo, 1992.
- /13/ ECCS, "Recommendations for the Design of Longitudinally Stiffened Webs and of Stiffened Compression Flanges," 1st edition, ECCS Technical Group 8.3 - Structural Stability, Publ. No. 60, European Convention for Constructional Steelwork, 1990.
- /14/ EC3, "Design of Steel Structures, Part 1.1 General Rules and Rules for Buildings," Eurocode 3, 1992, with National Application Documents, Draft for Development, DD ENV 1993-1-1.
- /15/ GL-S, "Rules and Regulations, I Ship Technology, Part 1 Seagoing Ships, Chapter 1 - Hull Structures, " Hamburg, 1992.
- /16/ DEn/HSE, "Offshore Installations: Guidance on Design, Construction and Certification," UK Department of Energy, London, 4th Edition, 1990.
- "Regulations for the Structural Design of Fixed Offshore /17/ NPD, Structures on the Norwegian Continental Shelf," Norwegian Petroleum Directorate, Stavanger, 1993.

- Dubas, P., Gehri, E. (eds.), "Behaviour and Design of Steel P Structures, " Publication No. 44, ECCS, Brussels, Jan. 1986.
- /19/ Galambos, T.V. (ed.), "Guide to Stability Design Criteria for Structures, " 4th Edition, John Wiley, New York, 1988.
- Dowling, P.J., Harding, J.E. and Bjornovde, A. "Constructional Steel Design: An Int. Guide," Elsevier Ap Science, London, 1992.

  Beedle, L.S., "Stability of Metal Structures A World V
- /21/ Beedle, L.S., "Stability of Metal Structures A wor Struct. Stability Research Council (Second Edition), 1991. Rigo, P.H. et al., "Benchmark Study of Ultimate Strength Predic
- for Stiffened Panels," to appear. Smith, C.S., Davidson, P.C., Chapman, J.C. and Dowling /23/
- "Strength and Stiffness of Ship's Plating under In-Plane Compre and Tension, "Trans. RINA, vol. 130, 1988. /24/ Davidson, P.C., Chapman, J.C., Smith, C.S. and Dowling, P.J.,
- Design of Plate Panels Subject to In-Plane Shear and Bi Compression, Trans. RINA, vol. 132, 1990.
- /25/ Davidson, P.C., Chapman, J.C., Smith, C.S. and Dowling, P.J., Design of Plate Panels Subject to Biaxial Compression and La Pressure, "Trans. RINA, vol. 134, 1992. J.C., Davidson, P.C. /26/ Smith, C.S., Anderson, N., Chapman,
- Dowling, P.J., "Strength of Stiffened Plating under Con Compression and Lateral Pressure, Trans. RINA, vol. 134, 1992. /27/ Chapman, J.C., Smith, C.S., Davidson, P.C. and Dowling, "Recent Developments in the Design of Stiffened Plate Structu F
- Structures, Elsevier, Advances Marine in Dunfermline, May 1991, pp. 529-548.

  /28/ Davidson, P.C., "Design of Plate Panels under Biaxial Compres
- Shear and Lateral Pressure," Ph.D. Thesis, Imperial Col University of London, 1989. /29/ Gordo, J.M. and Guedes Soares, C., "Approximate Load Short Curves for Stiffened Plates under Uniaxial Compression," Proc Integrity of Offshore Structures, Glasgow, EMAS, London, 1993.
- /30/ Ueda, Y. and Yao, T., "Fundamental Behavior of Plates and Stir Plates with Welding Imperfections," ISMS '91, Shanghai, 1991 377-388.
- /31/ Bonello, M.A., Chryssanthopoulos, M.K. and Dowling, P.J., "Ult Strength Design of Stiffened Plates under Axial Compression Bending, "Marine Structures, vol. 6, no. 5&6, 1993, pp. 533-552 /32/ Bonello, M.A., "Reliability Assessment and Design of Sti:
- Compression Flanges," Ph.D. Thesis, Imperial College, University London, 1992. /33/ Dowling et al., "Design of Flat Stiffened Plating: Phase 1 Rep
- CESLIC Report SP 9, Dept. of Civil Engineering, Imperial Co. London, Dec. 1991.
- Vayas, I., "Torsional Rigiditis of Open Stiffeners to Compre Flanges," J. Constr. Steel Research, vol. 20, 1991, pp. 65-74. /35/ Panagiotopoulos, G.D., "Ultimate Torsional Strength of Flanges," J. Constr. Steel Research, vol. 20, 1991, pp. 65-74.
- Stiffeners Attached to Flat Plating under Axial Compression Marine Structures, vol. 5, 1992, pp. 535-557.

  /36/ Hindi, W.A., "Behaviour and Design of Stiffened Compression F.
- of Steel Box Girder Bridges," Ph.D. Thesis, University of St January 1991.
- /37/ Ueda, Y., Rashed, S.M.H. and Paik, J.K., "Buckling and Ul Strength of Plates and Stiffened Plates under Combined Loads" published in Marine Structures).
- /38/ Mansour, A.E., "Approximate Formulae for Preliminary Desi Stiffened Plates," Proc. of OMAE '86, Tokyo, 1985.
- /39/ Smith, C.S., "Compressive Strength of Welded Steel Ship Grill Trans. RINA, vol. 117, 1975.
- /40/ Rutherford, S.E., "Ultimate Longitudinal Strength of Ships: Study, "SNAME Transactions, vol. 98, 1990, pp. 441-471.
- /41/ Rutherford, S.E., "Stiffened Compression Panels. The Anal Approach," Technical Report HSR No. 82/26/R2, Lloyd's Regist Shipping, London, 1981.

- Bonello, M.A. and Chryssanthopoulos, M.K., "Report on Benchmark Test
- to ISSC Committee V.1" (private communication), 1993.

  Dowling, P.J. and Harding, J.E., "Box Girders," Chapter 2.7 in /20/. Dowling, P.J. and Harding, J.E., "Box Girders,
- /44/ Maquoi, R., "Plate Girders," Chapter 2.6 in /20/.
  /45/ Rockey, K.C., "The Design of Web Plates for Plate and Box Girders -A State of the Art Report," Proc. Int. Symp. Steel Plated Structures," (ed. P.J. Dowling et al.), Imperial College, London, 1976. Crosby Lockwood Staples, London, 1977.

  /46/ Sheer, J. and Pasternak, H., "Zum Nachweis von Vollwandtragern mit
- dunnen Stegen und Quer-sowie auch Langssteifen in Regelwerken," Bauingenieur, vol. 64, 1989, pp. 121-133.
- /47/ Cooper, P.B., "The Ultimate Bending Moment for Plate Girders," Proceedings of IABSE Colloquim, London, ed. Massonnet et al., IABSE, Zurich, 1972, pp. 291-297.
- /48/ Porter, D.M., Rockey, K.C. and Evans, H.R., "The Collapse Behaviour of Plate Girders Loaded in Shear," The Structural Engineer, vol. 53, no. 8, 1975.
- Rockey, K.C., Evans, H.R. and Porter, D.M., "A Design Method for Predicting the Collapse Behaviour of Plate Girders," Proc. Instn. Civ. Engrs., Part 2, vol. 65, March 1978, pp. 85-112.
- /50/ AISC, "Load and Resistance Factor Design (LRFD), Specification for Structural Steel Buildings," AISC, Chicago, 1986.
- /51/ BS5950, "The Structural Use of Steelwork in Buildings, British Standards Institution, London, 1985.
- /52/ DASt Richlinie 0.15, "Trager mit Schlanken Stegen," Deutcher Ausschuss für Stahlbau, 1990.
- "European Recommendations for Steel Construction," EG77-2E, ECCS, March 1978.
- /54/ Jetteur, Ph., Maquoi, R., Massonnet, C. and Skaloud, M., "Calcul des Ames et Semelles Raidies des Ponts en Acier," Construction
- Metallique (4), 1979, pp. 41-53.

  /55/ Sen, R., "Evaluation of BS5400 Plate Girder Rules," Proc. Instn. Civ. Engrs., Part 2, vol. 81, Sept. 1986, pp. 335-352.
- Tolikas, C., "Report to ISSC Committee V.1" (private communication). /57/ Ostapenko, A., "Shear Strength of Longitudinally Stiffened Plate Girders," Proceedings, Structural Stability Research Council, New
- York, 1980, pp. 36-40.

  /58/ "Zur Berechnung der Tragfahigkeit augesteifter Vollwandtrager mit schlanken Stegen under Schub-und Biegebeanspruchung in Eurocode 3-Bericht 6057," T.U. Braunschweig, Institut fur Stahlbau, July 1988.
- /59/ Narayanan, R. and Der Avanessian, N.G.V., "An Equilibrium Method for Assessing the Strength of Plate Girders with Reinforced Web Openings," Proc. Instn. Civ. Engrs., Part 2, vol. 77, June 1984, pp. 107-137.
- /60/ Narayanan, R. and Darwish, I.Y.S., "Strength of Slender Webs having Non-Central Holes," The Structural Engineer, vol. 63B, no. 3, Sept. 1985, pp. 57-62.
- Narayanan, R., "Ultimate Shear Capacity of Plate Girders with Openings in Webs," in Plated Structures: Stability and Strength, /61/ Narayanan, R.,
- ed. R. Narayanan, Applied Science Publ., London, 1983.

  Evans, H.R., "Assessment of Eurocode 3 Determination of Model Factors for Plate Girders," Report CAR/BRE, Civil Engineering Department, University College, Cardiff, 1985.

  Evans, H.R., Moussef, S., "Design Aid for Plate Girders," Proc. Instn. Civ. Engrs., Part 2, vol. 85, March 1988, pp. 89-104.

  Harding, J.E. and Hobbs, R.E., "The Ultimate Load Behaviour of Box Girder Web Papels "The Structural Engineer vol. 578, 1979, pp. 49-
- Girder Web Panels," The Structural Engineer, vol. 57B, 1979, pp. 49-
  - /64/ Singer, J., "Buckling Experiments on Shells A Review of Recent Developments," Solid Mechanics Archives, vol. 7, 1982, pp. 213-313.
  - /65/ Kendrick, S.B., "Structural Design of Submarine Pressure Vessels," Naval Construction Research Est., Dunfermline, Rep. No. R483, March 1964.
  - /66/ API (RP2A) WSD, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms," American Petroleum

- /67/ API (RP2A) LRFD, "Recommended Practice for Planning, Designin Constructing Fixed Offshore Platforms - Load and Resistance F Design," American Petroleum Institute, Washington, First Edi July 1, 1993.
- /68/ API, Bulletin 2U, "Bulletin on Stability Design of Cylind RP2T)," American Petroleum Insti Shells (For use with Washington, April 1987.
- "ASME Boiler and Pressure Vessel Code Nuclear Components," Case N-284, ASME, New York, 1983.
- "Specification for Unfired Welded Pressure Vess 5500, British Standards Institute, London, 1976 (Amended April 1981. /71/ BS 5950-1, "Structural Use of Steelwork in Buildings, Part 1.
- Code of Practice for Design in Simple and Continuous Construc Hot Rolled Sections," British Standards Institution, London, 19
- /72/ DnV-OS, "Rules for the Design, Construction and Inspection Offshore Structures. Appendix C Steel Structures," Det n Veritas, Oslo, 1977 (Revised 1982).
- /73/ ECCS, "European Recommendations for Steel Construction. Section Buckling of Shells," European Convention of Construct Steelwork, Publication 29, Second Edition, 1983 (ECCS Bull., E Ed., 1988).
- /74/ "Buckling of Offshore Structures," J.P. Kenney and Partners London, Granada, 1984.
- Member Strength Formulations "Tubular /75/ Frieze, P.A., International Harmonization," Report OTO/94/102, Health and S Executive, London, August 1993.
- Loh, J.T., "A Unified Design Procedure for Tubular Members," OTC 6310, Offshore Technology Conference, Houston, Texas, May 1 /77/ Miller, C.D. and Saliklis, E.P., "Analysis of Cylindrical Database and Validation of Design Formulations," API PRAC Programment of Design Formulations,
- 90-56, Report, CBI, February 1993.
- P.C., "Beam-Column and Birkemoe, Behavio /78/ Prion, H.G.L. Fabricated Steel Tubular Members," J. Struct. Engng., ASCE, 118, no. 5, May 1992, pp. 1213-1232.
- /79/ Hellan, Ø., Moan, T. and Drange, S.O., "Use of Nonlinear Pu Analysis in Ultimate Limit State Design and Integrity Assessment Jacket Structures," Proc. 7th BOSS Conf., MIT, July 1994.
- /80/ Odland, J., "Buckling of Unstiffened and Stiffened Ci: Cylindrical Shell Structures," Norwegian Maritime Research, vo no. 3, 1978.
- /81/ Cho, P.A., "Strength Formulation for S.-R. and Frieze, Stiffened Cylinders under Combined Axial Loading and Pressure," J. Construct. Steel Research, vol. 9, 1988, pp. 3-3
- /82/ Moan, T. et al., "Limit States for the Ultimate Strength of Tu Subjected to Pressure, Bending and Tension Loads," J. Struct., to appear, 1994.
- /83/ Conoco/ABS TLP Rule Case Committee, "Model Code for Stru Design of Tension Leg Platforms," ABS, New York, February 1984 "Model Code for Stru
- /84/ Das, P.K., Faulkner, D. and Guedes da Silva, A., "Limit Formulations Mad Modelling for Reliability-Based Analyse Orthogonally Stiffened Cylindrical Shell Structural Compon Dept. of Naval Architecture and Ocean Engineering, University Glasgow, Rep. No. NAOE-91-26, August 1991.
- /85/ Das, P.K., Faulkner, D. and Zimmer, R.A., "Efficient Reliab Based Design of Ring and Stringer Stiffened Cylinders under Co Loads," Proc. 6th BOSS Conf, Univ. College, London, 1992.
- /86/ CIDECT, "Design Guide for Circular Hollow Section (CHS) Joints Predominantly Static Loading," by J. Wardenier et al., Verla
- Rheinland, GmbH, Köln, 1991.
  /87/ CSA, "Code for the Design Construction and Installation of Offshore Structures," Preliminary Standard S473-M1989, Ca Standards Association, December 1989.
- /88/ DnV-OFS, "Rules for Classification of Fixed Offshore Installat Det norske Veritas, Oslo, July 1989.

/89/ Paul, J.C. et al., "Ultimate Behavior of Multiplanar Double K-joints of Circular Hollow Section Members," Int. J. Offshore and Polar Engng., vol. 3, no. 1, March 1993, pp. 43-50

Marshall, P.W., "Design of Welded Tubular Connections:

Use of AWS Code Provisions, " Elsevier, Amsterdam, 1992.

/91/ Wimpey Offshore, "Background to the New Static Strength Guidance for Tubular Joints in Steel Offshore Structures," Report OTH 89 308, Department of Energy, HMSO, London, 1990.

Yura et al., "Ultimate Capacity Equations for Tubular Joints," Paper No. OTC 3690, Offshore Technology Conference, Houston, Texas, 1980.

- Ochi, K. et al., "Basis for Design of Unstiffened Tubular Joints under Axial Brace Loading," Proc. IIW Conf. on Welding of Tubular Structures, Doc. XV-561-84, Boston.
- /94/ Underwater Engineering Group, "Design of Tubular Joints for Offshore Structures," UEG/CIRIA Report UR33, EUG Offshore Research, London, 1985.
- /95/ MSL Engineering, "Static Strength of Tubular Joints on Offshore Structures, " Report to Health & Safety Executive, 1992.
- /96/ Ellinas, C.P., Lalani, M. and Sharp, J.V., "Tubular Joint Strength Formulations. The Implications for Harmonisation," Proc. SUT Int. Conf. on API RP 2A-LRFD, Its Present and Future Role in Offshore Safety Cases, London, 24 Nov. 1993.

/97/ Healey, B.E. and Zettlemoyer, N., "In-Plane Bending Strength of Circular Tubular Joints," Proc. Fifth Int. Symp. on

Structures, Nottingham, U.K., August 1993. /98/ "Report to ISO TC 67SC7 on Harmonization of Static Strength Standard of Offshore Tubular Joints and Members" (M. Birkinshaw, Convener), 22 March 1993.

- /99/ Paul, J.C., Valik, C.A.C. v/d, Wardenier, J., "The Static Strength of Circular Multi-Planar X-Joints," Proc. Int. Symp. on Tubular Structures, Lappearanta, Finland, September 1989, Elsevier, 1990.

  /100/ ISSC III.2, "Report of ISSC Committee III.2. Fatigue and Fracture,"

  Copenhagen, 1988; Waxi, 1991; St. John's, 1994.
- /101/ "Fatigue (ed.), Tapir publishers, Almar-Ness Handbook," A.
- Trondheim, 1985. /102/ BSI 5400, "Steel, Concrete and Composite Bridge," Part 10, Code of Practice for Fatigue, " British Standards Association, 1980.
- /103/ ECCS, "Recommendation for the Fatigue Design of Steel Structures," Convention for Constructional Publication No. 43, European Steelwork, Brussels, 1985.
- /104/ "Design Recommendations for Cyclic Loaded Welded Steel Structures," Welding in The World, vol. 20, no. 7/8, 1982.
- "Background to New Fatigue Design Guidance for Steel Welded Joints in Offshore Structures," Department of Energy, HMSO, London, U.K.,
- DnV-CNF, "Fatigue Strength Analysis for Mobile Offshore Units," Det norske Veritas, Classification Note No. 30.2, August 1984.
- /107/ BV, "Cyclic Fatigue of Nodes and Welded Joints on Offshore Units," Guidance Note N1 199, Bureau Veritas, March 1987.
- /108/ BV-S, "Cyclic Fatigue on Welded Joints on Steel Ships," Guidance Note N1 188, Bureau Veritas, March 1988.
- /109/ ABS-S, "Guide for the Fatigue Strength Assessment of Tankers," A Hull Rule Restatement Project Report, American Bureau of Shipping, NJ, 1992.
- "Fatigue Assessment of Ship Structures," Report DNVC 93-/110/ DnV-CNFS, 0432, Det norske Veritas, Oslo, 1993.
- /111/ Fricke, W., "Fatigue Control in Structural Design of Different Ship
- Types," Proc. VIth Congress IMAM '93, Varna, 1993.
  "American Welding Society, Structural Welding Code, Steel," ANSI/AWS D1.1-92, AWS, Miami, 1992,
- /113/ AASHTO, "Standard Specification for Highway Bridges," 14th ed., The American Assoc. of State Highway Transp. Officials, Washington, D.C., 1989.

/114/ Keating, P.B. and Fisher, J.W., "Evaluation of Fatigue Tests and Design Criteria on Welded Details," NCHRP Report 286, National

/115/ Munse, W.H. et al., "Fatigue Characterization of Fabricated Details for Design," Report SSC - 318, Ship Structures Commi Washington, D.C., 1982.

- /116/ Stambaugh, K.A. et al., "Reduction of S-N Curves for Ship Struc Commi Details," Draft Report SR-1336, Ship Structures Washington, D.C., 1992.
- /117/ Yoneya, T. et al., "Hull Cracking of Very Large Ship Structu Proc. Fifth Integrity of Offshore Structures Conf., University Glasgow, EMAS Science Publ., London, 1993.
- Marshall, P.W., "API Provisions for SCF, SN and Size-Pr Effects," Paper No. OTC 7155, Offshore Technology Confer Houston, 1993.
- /119/ Wirsching, P.H., "Probability Based Fatigue Design Criteria Offshore Structures, "Final Report, API PRAC Project 81-15, Ja 1983.
- /120/ "Fatigue Reliability: A State of the Art Review. A Four Series," ASCE J. Str. Div., ASCE, vol. 108, no. ST1, January 19
  /121/ Hellier, A.K., Conolly, M.P. and Dover, W.D., "Stress Concentr Factors for Tubular Y and T Joints," Int. J. Fatigue, vol. 12
- 1, 1990.
- /122/ Tubby, P.J. and Razimjoo, G.R., "Fatigue of Welded Joints Variable Amplitude Loading," G.S.P. 5573/10A/91, April 1991.
  /123/ Gurney, T.R., "Comparative Fatigue Tests on Fillet Welded Junder Values Types of Variable Amplitude Loading in Air," OMAE, vol. III-B, ASME, New York, pp. 537-542.