

## ROLL MOTION OF A FLOATING STORM SURGE BARRIER.

by

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### SYNOPSIS

The purpose of this study is to analyse the roll motion of a huge storm-surge barrier during closing manoeuvres. This gate, designed for the Port of Rotterdam by Dehousse and his staff [1], is based on a new concept of storm-surge barrier : a gate designed like a tanker. The peculiarity of this project is the size of the floating gate : 390 m long, 54 m wide and 22 m high. For a 3 m draft during the closing operation of the gate by rotation, this floating gate has a deadweight of nearly 62,000 tons.

Owing to this new design of storm surge barrier, the analysis of the roll motion and several other verifications must be done, such as floating stability, efficiency of the rotation with propellers, as well as the design of the articulation between the gate and the bank. Analytical roll studies started in 1989, but results cannot be fully accepted. The added-mass (virtual mass) and the damping coefficient are two unknown parameters which give the results a large range of uncertainty.

This study shows how the added-mass and the damping coefficient were obtained through a 2D scale model of the gate. Tests made on the gate model gave a good and accurate estimation of the maximum roll angle.

With these two aforementioned coefficients, it should be possible to analyse in the future the global 3D behavior of the floating gate during the closure of the channel.

### INTRODUCTION

This study is related to the roll motion of a new system of storm-surge barrier (Fig. 1.a). This barrier is a floating gate designed like a tanker and its rotation movement (to close the channel) is similar to that of a ship around its anchor. During the rotation movement, a roll motion occurs due to wave action.

The aim of this study is to determine experimentally the maximum roll angle ( $\phi_{max}$ ), the added-mass (CMR) and the damping coefficient ( $K_1$ ) in order to understand the behavior of the gate during its roll motion. Owing to an unusual shape and the huge size of the barrier, the respective values of CMR and  $K_1$  cannot be found in literatures.

### THE FLOATING STORM SURGE BARRIER

The gate characteristics, which are necessary to understand the barrier closure operations and the analysis of the roll motion, are given below. The design of this floating storm-surge barrier concerns the construction of a barrier which could be built in the entrance channel of Rotterdam Harbour in The Netherlands [1]. Such a floating barrier is also envisaged to protect in the future the Antwerp estuary in Belgium.

In brief, the main features are : 390 m span ( 360 m + 2 x 15 m for the supports), 22 m height, 54 m width , 55 000 tons steel weight and 62 000 tons deadweight (Figs. 1.a-d and 2).

The conditions (span, head clearance, etc.) that were imposed in Rotterdam are the most restrictive ones ever met in the design of a moving gate. The Nieuwe Waterweg is actually the busiest maritime channel in the world with at least 60 000 ships passing through it every year. The actual channel is about 600 m wide, and 17 m deep under the datum level (NAP). The constraints include the requirement of maintaining an opening of at least 360 m wide which should allow navigation without hindrance. The highest water levels which must be considered are shown in Fig. 1.d . Furthermore, head clearance must remain unlimited.

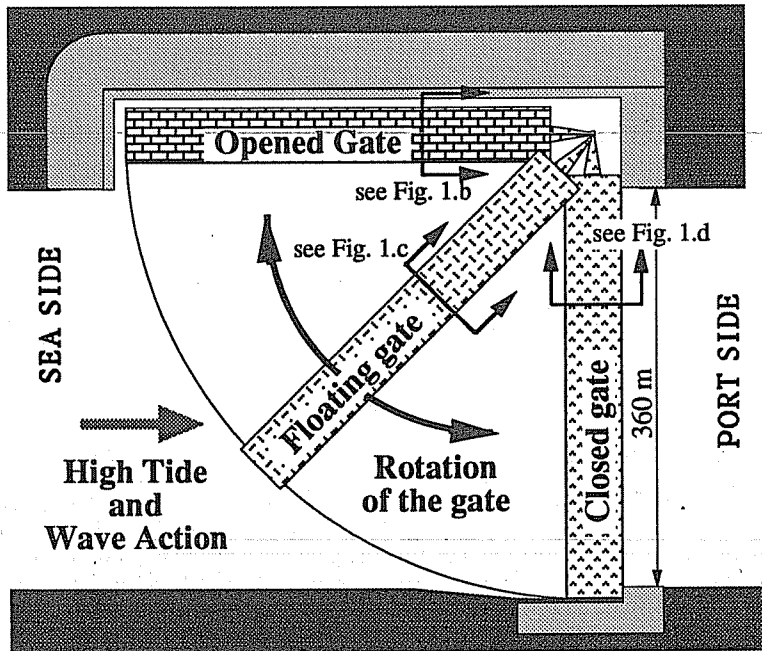


Fig. 1.a The floating storm surge barrier designed for the Port of Rotterdam.

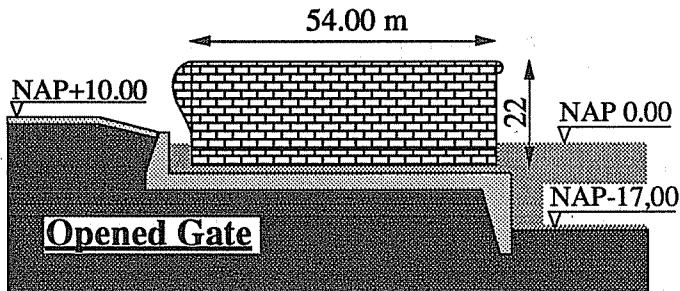


Fig. 1.b Opened Position : The gate is along the bank and rests on its end supports.

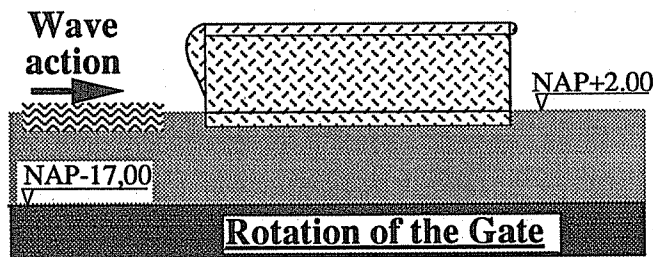


Fig. 1.c Rotation of the Gate : The gate floats with a 3 m draft and rotates under the control of propellers and cables.

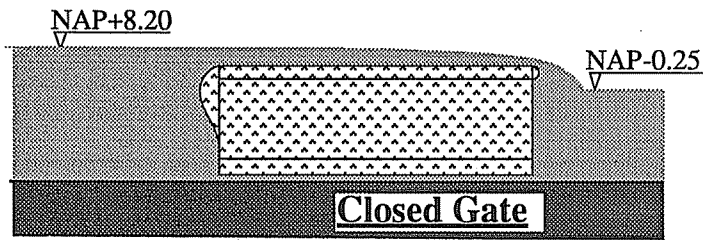


Fig. 1.d Closed Position : The gate is sunk to the -16m level by filling of ballasts.

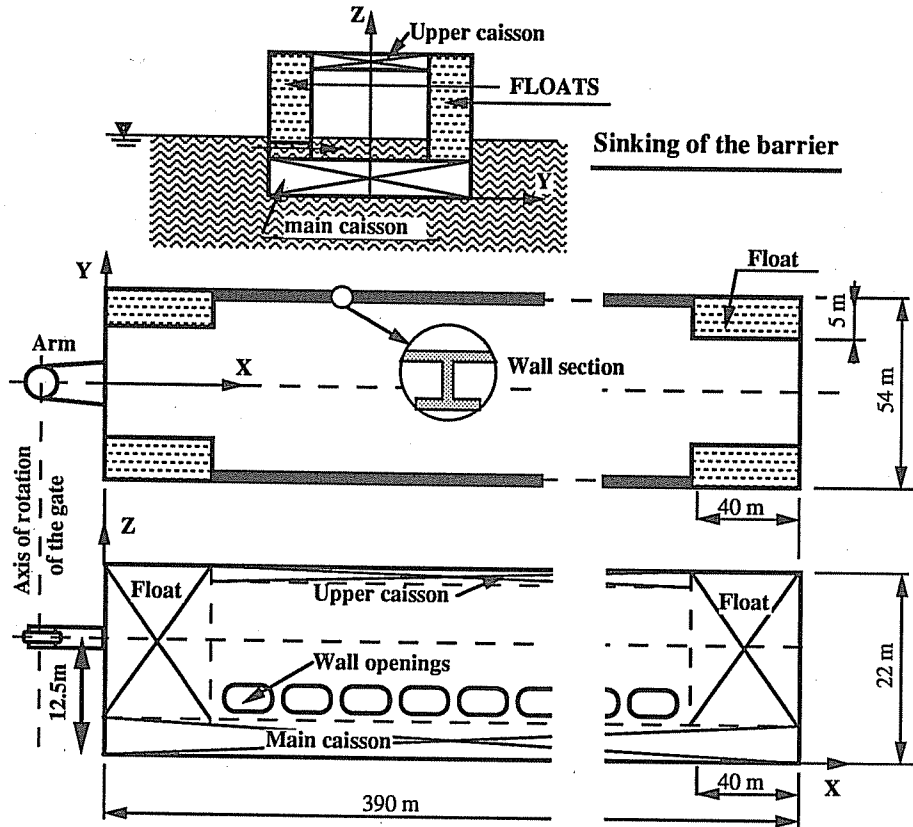


Fig. 2 Details of the floating barrier (cross section, plan view and front view).

The barrier under consideration [1] looks like a pivoting robot-pontoon and is submersible and easy to control during either immersion or emersion with the four floats located at the four corners of the gate (Fig. 2). We can sum up its main features as follow : a main watertight caisson allows the gate to float with a draft (draught) limited to 3 m . When the gate is sunk, walls, stiffeners and frames permit solicitations corresponding to the most important water-level differences between the two faces of the structure. Four floats provided to create a double catamaran effect lead to a very great general floating stability. A rotating device links the structure to the bank, while allowing 6 degrees of freedom (3 rotations and 3 displacements). At the top of the gate there is a double deck (upper caisson) acting as a float which is aimed at reducing stresses and deformations due to bending in the vertical plane when exceptional loads occur (Fig. 1.d). There are large openings in both walls of the barrier (Fig. 2) which can be shut with valves once the gate is sunk (Fig. 1.d).

According to the design, when the gate is in the open position (Fig. 1.b), it is parallel to the banks of the Nieuwe Waterweg, always allowing a 360 m clearance between the piers for the traffic. It is ballasted and sunk to the -4 m level resting on its end supports.

In case of emergency, it can be floated with a 3 m draft and a 19 m height free board (Fig. 1.c). When high water level is forecast, the gate is rotated around its hinge point (Fig. 1.a) under the control

of propellers and cables. Owing to wave action, roll motion will occur during the rotation. This movement will be maximum when the gate axis and the wave direction are perpendicular. The maximum roll angle ( $\phi_{max}$ ) is the most important characteristic in the design of the rotating device and the rigid arm connecting the gate to this device. Determining this maximum roll angle is one of the main purposes of this research.

After rotation, sinking is carried out by a partial filling of ballasts placed in the main caisson and side floats (Fig. 2). The gate sinks then to the -16 m level (Fig. 1.d). Through the aforementioned lateral openings in the walls, the water fills up the large chamber above the main caisson. After a storm has passed, the gate is set afloat again, using 4-bar compressed air, and moored at its original position.

### THE ROLL MOTION

The gate is rotated around its hinge point when high water level is forecasted (Fig 1.a). Owing to the usual simultaneous occurrence of high water level and the strongest waves created by high tide and storm, the roll motion cannot be neglected during the closure of the channel.

There are two ways of studying the roll motion: the analytical method [3] and experiments on hydraulic scale model. Both methods have their advantages and disadvantages. On the one hand, experiments on scale model reduce the number of assumptions and approximations but are limited by the scale effects (i.e. small scale such as 1/100) and by the required space for the model (i.e. large scale such as 1/10). On the other hand, the analytical method allows a 3D model as well as the opportunity to repeat the tests with many different wave characteristics. But it is restricted by empirical parameters as damping coefficient or added-mass. Moreover, mathematical models are limited by the wave theory limitations, and by the complexity and precision of the mesh model.

Laboratory tests were made for the following wave characteristics: 0.6m wave height (H), 87.6m wavelength (L), 8 sec wave period (T) and 19.5m water depth (d).

Since the barrier has a hinge point and since the rotation velocity of the closing gate is regulated by cables, the barrier motion cannot be considered as a full six-degree-of-freedom rigid body motion. Sway being a horizontal plane displacement which is limited by the cables of the barrier, it was not taken into consideration in this study. Due to the large value of the gate length-wavelength ratio, surge, yaw and pitch were also neglected.

Roll and heave will only be significant during the rotation process. Heave and roll only cause displacements in the transverse plane of the barrier and have no effect on the longitudinal motion. Therefore the barrier motion could be estimated by a 2D model. For any position of the barrier, it is possible to determine the wave components in the barrier-oriented coordinates (X,Y,Z - see Fig. 2). The longitudinal wave component along the X axis which causes surge and pitch were neglected and only the transverse component along the Y axis (roll and heave) were taken into account.

#### Analytical study of the roll motion.

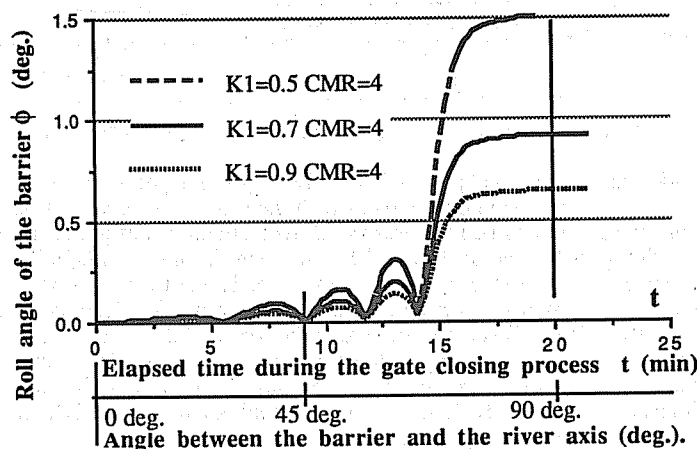


Fig. 3 Development of the roll angle during the rotation of the gate (Leeuwerck [3]).

For an analytical study of the roll motion of this floating gate, Leeuwerck [3] solved the one-degree-of-freedom equation of damped roll motion established for small roll amplitude using Finite Difference Method. He used the Strip Theory to integrate the wave action along the barrier. One of the

most difficult obstacle was to obtain realistic values for the added-mass (CMR) and damping coefficient ( $K_1$ ). Having no information about their values, Leeuwerck[3] could not make a definite and accurate estimation of the roll motion. He only gave qualitative results, i.e. roll angles obtained for CMR = 2, 4 and 7 and for  $K_1 = 0.5, 0.70$  and  $0.90$  (CMR and  $K_1$  will be defined hereafter). According to these ranges of values, the maximum roll angle for  $H=0.6\text{m}$  and  $T=8\text{sec}$  varied from  $0.3\text{deg}$  to more than  $2.5\text{deg}$ . Moreover, he obtained the development of the roll motion ( $\phi$ ) during the closure of the channel (Fig. 3) and could determine the position of the barrier when it came to its maximum roll angle ( $\phi_{\max}$ ). This maximum angle usually appears just before the closing of the gate when the wave direction and the gate itself form an angle of about  $90\text{degrees}$  (Fig. 3).

To develop Leeuwerck's promising analytical study, it is obvious that CMR and  $K_1$  have to be determined by an experimental method using a scale model of the barrier as presented in this study.

Once the values of the desired parameters will be available, Leeuwerck's 3D analytical study may produce more accurate results in the real 3D conditions (prototype).

### Experimental study of the roll motion.

To obtain the values of added-mass and damping coefficient, a scale model of the gate was used. The scale of the model being  $1/35$ , the wave characteristics were  $1.71\text{cm}$  ( $H$ ),  $1.35\text{sec}$  ( $T$ ) and  $2.5\text{m}$  ( $L$ ) and the cross section of the gate model was  $1.543\text{m}$  wide and  $0.626\text{m}$  high instead of  $54\text{m}$  and  $22\text{m}$  for the prototype. The weight of the gate was also modelled according to the scale:  $\lambda^3=1/42875$ .

The flume dimensions were  $10\text{m}$  in length,  $1\text{m}$  in width and  $0.80\text{m}$  in height.

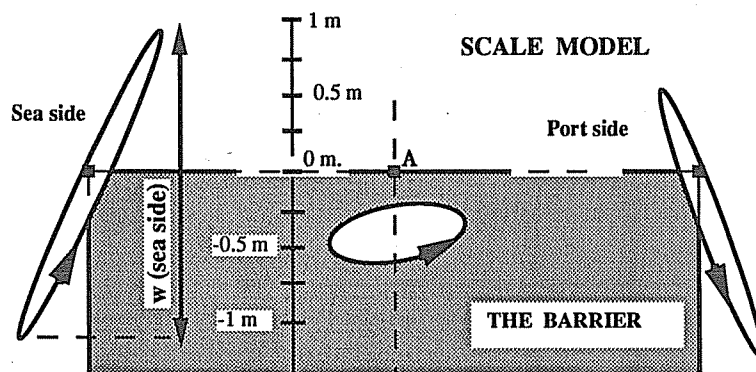


Fig. 4.a Roll motion of the upper part of the lateral walls (sea side and port side).

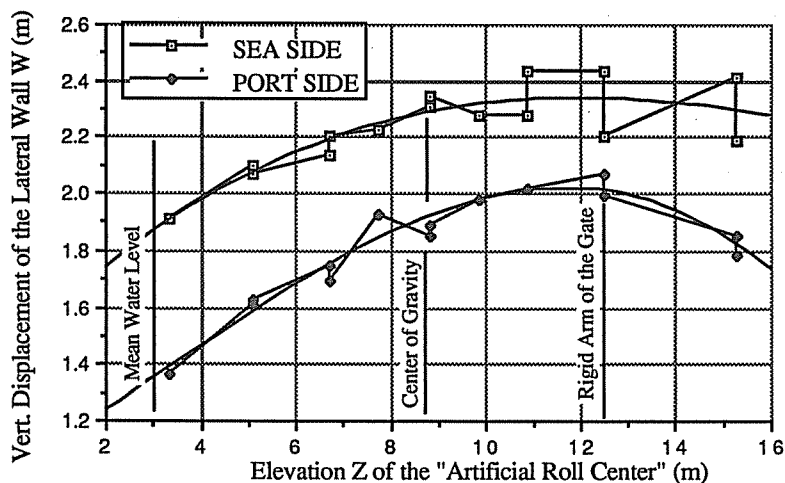


Fig. 4.b Vertical displacement of the lateral walls ( $W$ ) according to the elevation of the "artificial roll center".

The prototype size (390m long) and the selected scale (1/35) made it impossible to model the complete structure. A complete modelled gate would have to be 11.14m long (390m/35). However the flume being 1 m wide, only 9% ( 1/11.14) of the gate length was modelled. It was assumed that the added-mass and the damping coefficient remain nearly the same when the wave direction changes as it occurs during the gate rotation process. Therefore, a simplified model was used for the experiments, i.e. a 2D scale model was studied instead of a 3D model. Neither the channel width (600m), nor the gate length (390m) were taken into account in this study.

Another important assumption concerns the elevation of the "roll center" [2]. This center remains a vague and ill-defined concept. A common notion is that the "roll center" is the elevation  $Z$  where sway motion equals zero ( $Z=0$  was defined as the bottom of the gate - see Fig. 2)<sup>(1)</sup>. Hutchison [2] has recently improved this concept. However, in our study, we shall keep using the aforementioned definition. Moreover, we also accept the statement that the "natural roll center" lies between the center of gravity ( $Z=8.8\text{m}$ ) and the mean water level ( $Z=3.0\text{m}$ ); the elevation of the center of buoyancy  $Z=1.5\text{m}$ .

The barrier being connected to the river bank by a rigid arm, it can be considered that the barrier rolls around an "artificial roll center" located at the arm's elevation ( $Z=12.5\text{m}$ ).

Fig. 4.a shows the motion of the upper part of both lateral walls (sea side and port side). The diagram of the vertical displacements ( $W$ ) of these walls is shown in Fig. 4.b . The maximum value of  $W$  occurs when an "artificial roll center" is located between the center of gravity ( $Z=8.8\text{m}$ ) and the elevation of the arm ( $Z=12.5\text{m}$ ), the barrier being 22m high. Hereafter, the "artificial roll center" will be considered as lying at the elevation of the center of gravity.

(1) In the following explanations, all values and dimensions refer to the prototype.

## STUDY OF THE GATE MOTION - ROLL AND HEAVE

To analyse the roll motion of a barrier, it is necessary to determine the metacentric radius ( $MB = I/\nabla$ ) which is function of the inertia ( $I \text{ m}^4$ ) of the floating surface and of the displacement ( $\nabla \text{ m}^3$ ).

Even though the displacement ( $\nabla$ ) is known with a high accuracy, it is not the case for the floating surface. This surface changes according to the position of the barrier during its roll motion. While the gate is floating with its usual draft (3m) the top of the main caisson (Figs. 1.c and 2 ) lies at the mean water level (MWL) and can be partially submerged by the waves (Fig. 5).

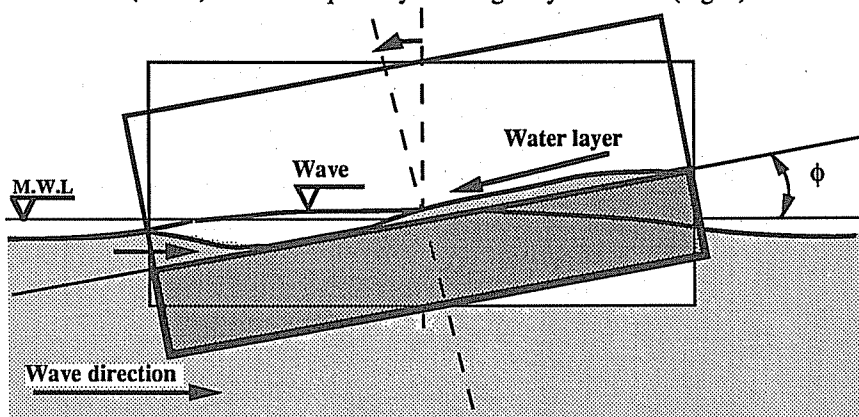


Fig. 5 Water layer moving on the main caisson during the roll motion.

In fact, observations showed that, during the roll motion, the main caisson remains completely immersed most of the time. Therefore it would be unrealistic to consider the submersion of this caisson. As far as the determination of the metacentric radius ( $MB$ ) is concerned, experiments show that  $MB$  computed for an immersed caisson ( $MB=81.0\text{m}$ ) is the most realistic assumption even in the case of wave action. In any case, in order to take into account the water above the main caisson (Fig. 5), the metacentric radius  $MB$  must be reduced. It has to be reduced as the displacement ( $\nabla$ ) increases with the overload of the water layer moving on the main caisson (10% of the deadweight) and as the inertia ( $I$ ) of the floating surface decreases (around 10% to 30%). The reduction of  $MB$  ranges then from 20% to 40%. Since the metacentric height ( $GM=MB-7.3 \text{ m}$ ) equals 73.7m before reduction (no wave action), we have :  $41.3 \text{ m} < GM < 57.5 \text{ m}$ .

Regardless of the added-mass, the natural roll period ( $T_{n_s}$ )<sup>(2)</sup> is expressed as:

$$T_{n_s} = \frac{2 \pi k_s}{\sqrt{g \cdot GM}} \quad (1)$$

where  $k_s$  ( $= 17.5 \text{ m}$ ) = is the gyradius (radius of gyration) of the mass of the structure (without added-mass).

Applying Eq. 1 with the reduced metacentric height ( $41.3\text{m} < GM < 57.5\text{m}$ ), the natural roll period ( $T_{n_s}$ ) (without added-mass) is included in the range of :  $4.63 \text{ sec} < T_{n_s} < 5.46 \text{ sec}$ .

(2) The subscript "s", included for instance in  $T_{n_s}$  and  $k_s$ , means that the structure is considered regardless of the added-mass.

#### DETERMINATION OF THE ADDED-MASS.

To establish an accurate analytical model, it is necessary to introduce the added-mass and damping coefficients. The added-mass CMR allows taking into account the water moving with the barrier. This "accompanying" mass of water have to be added to the mass of the structure to respect the motion equation. The energy dissipated during each oscillation is introduced through the damping coefficient  $K_1$ . Without damping ( $K_1=0$ ), a natural roll motion will never stopped. For  $K_1=1$ , the motion is fully damped after one oscillation ( $t=T_n/2$ ).

The CMR added-mass coefficient can be written as :

$$CMR = \frac{T_n}{T_{n_s}} \quad \text{since } T_n = \frac{2 \pi k}{\sqrt{g GM}} \quad \text{and } k = k_s + k_{ad} \quad (2)$$

where  $T_n$  = the natural roll period of the structure with added-mass - without damping ;  $k$  = the gyradius of the mass of the structure with added-mass; and  $k_{ad}$  = the gyradius of the added-mass alone.

The CMR coefficient defined by Eq.2 represents the factor ( $>1$ ) which multiplies the gyradius of the structure to take into account the added-mass. To obtain CMR, we have to determine the  $T_n$  value. This was obtained by two different methods, each method confirming the other one.

#### The characteristics of the roll motion for still water level (SWL).

In case of still water level (no wave action), the natural roll period with damping ( $T_n^*$ ) is almost the same as the theoretical one without damping ( $T_n$ ) [4]. The theoretical relationship between  $T_n^*$  and  $T_n$  is :

$$T_n^* = T_n \cdot \frac{1}{\sqrt{1 - \left(\frac{K_1}{\pi}\right)^2}} \quad (3)$$

where  $T_n^*$  = the theoretical natural roll period (structure and added-mass) with damping ; and  $K_1$  = the damping coefficient.

In case of still water, the oscillations were damped very quickly. No more than three oscillations were observed. This small number of cycles shows that the damping coefficient ( $K_1$ ) of the barrier is significant and implies that the roll measurements have to be made on a limited number of cycles (2 periods). The natural roll period was obtained by using a movie camera combined with a timer in order to analyse pictures one by one. The average value was  $T_n^* = 8.02 \text{ sec}$  (1.36 sec for the model).

Since  $0 < K_1 < 1$ , it is obvious from Eq. 3 that  $T_n^* > T_n > T_n^*/1.05$ . Therefore the value of  $T_n$  ranges from 7.64 sec to 8.02 sec. With respect to these estimations of the natural roll period of the structure without added-mass ( $4.63 \text{ sec} < T_{n_s} < 5.46 \text{ sec}$ ), the added-mass coefficient (CMR) is :  $1.40 < CMR < 1.73$  (Eq. 2).

**The resonance phenomenon.**

The analytical relationship (Eq. 4) of the roll angle ( $\phi$ ) results from the one-degree-of-freedom equation of damped roll motion [4]. This equation is composed of two terms : the roll motion which is induced by the wave action (first term) and the roll motion related to the natural roll period of the gate (second term) , i.e. roll in still water.

$$\phi = \text{WAVE ACTION (first term)} + \text{NATURAL ROLL MOTION (second term)}$$

$$\phi = \frac{\frac{\pi H}{L} \sin(\omega t - x)}{\sqrt{\left(1 - \frac{T_n^2}{T^2}\right)^2 + \frac{4 K_1^2}{\pi^2} \frac{T_n^2}{T^2}}} + \theta e^{-\left(2 \frac{K_1}{T_n} t\right)} \cdot \sin\left[\frac{2\pi t}{T_n} (\dots) + \dots\right] \quad (4)$$

where  $H, L, T$  = the wave characteristics ;  $\omega = 2\pi/T$  ;  $t$  = the time ; and  $x, \theta$  = constants.

The second term of Eq.4 , function of the damping coefficient ( $K_1$ ), decreases exponentially with time ( $t$ ). As far as experiments with wave action are concerned, the second term can be neglected.

The first term of Eq. 4. shows that the maximum roll angle ( $\phi_{max}$ ) is a function of  $\alpha_m$  ,  $T_n/T$  , and  $K_1$ . Then the  $\phi_{max}$  relationship is :

$$\phi_{max} = \frac{\frac{\pi H}{L}}{\sqrt{\text{fct}\left(\frac{T_n}{T}, K_1\right)}} = \alpha_m \cdot \text{fct}\left(\frac{T_n}{T}, K_1\right) \quad \text{with } \alpha_m = \frac{\pi H}{L} \quad (5)$$

Fig. 6 and Eq. 4 show that a resonance phenomenon ( $T_n=T$ ) occurs when the wave period is equal to the natural roll period of the gate (with added-mass). It is possible to obtain the graph of the function between  $\phi$  and  $T$  (Fig. 6) and therefore to determine the natural roll period ( $T_n$ ) when the highest roll angle occurs. To obtain this graph, maximum roll angle ( $\phi_{max}$ ) had to be measured for different values of the wave period ( $T$ ) with a fixed ratio of  $\alpha_m$  ;  $K_1$  being assumed constant.

Fig. 6 shows that the resonance phenomenon ( $T=T_n$ ) of the floating barrier occurs for a wave period ( $T$ ) ranging from 7.6 sec to 8 sec . With regard to Eq.2 , these values of  $T_n$  confirm that :  $1.40 < \text{CMR} < 1.73$  .

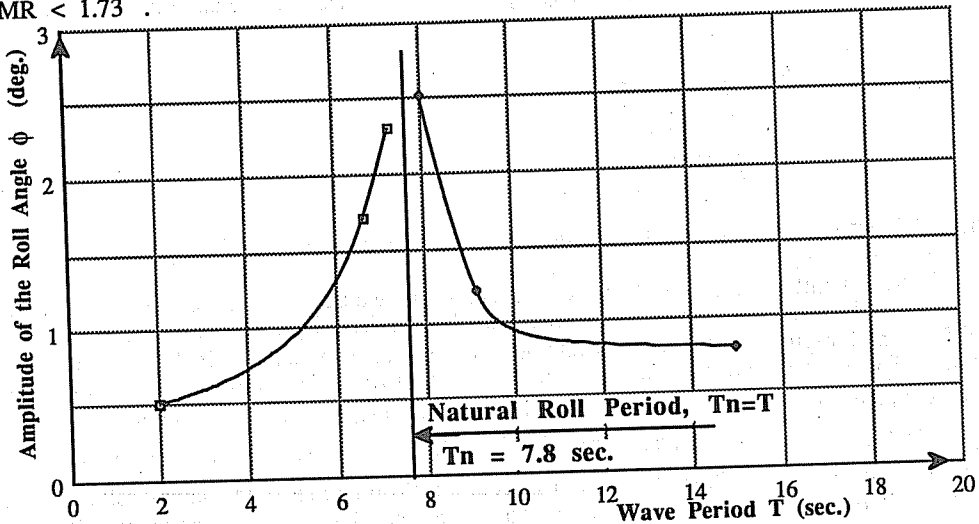


Fig. 6 The resonance phenomenon of the roll motion.



### DETERMINATION OF THE DAMPING COEFFICIENT $K_1$ .

In case of still water (no wave action) the second term of Eq. 4 must be considered. This term corresponds to the natural roll motion and contains two factors. One is a sine function which gives the periodic motion. The other one is an exponential term which contains the damping coefficient ( $K_1$ ). The value of  $K_1$  ( $0 < K_1 < 1$ ) determines the reduction of the roll angle between two successive oscillations.

The ratio  $q$  (Eq. 6) of the vertical displacement of a gate's corner (Fig. 7) which occurred during a half-period  $[t+T_n, t+1.5T_n]$  and the vertical displacement of the same corner which occurred during a previous half-period  $[t, t+0.5T_n]$  allowed us to obtain the value of the damping coefficient.

$$q = \frac{W_3 - W_4}{W_1 - W_2} = \frac{e^{-2K_1} - e^{-3K_1}}{1 - e^{-K_1}} = e^{-2K_1} \quad \text{then, } K_1 = -\frac{1}{2} \ln q \quad (6)$$

where  $q$  = the ratio of vertical displacements; and  $W_1, W_2, W_3, W_4$  = elevations of the gate's corner at the time  $t = t, t+0.5T_n, t+T_n$  and  $t+1.5T_n$ .

According to experimental results, the damping coefficient (Eq. 6) ranges from 0.63 to 0.78 with an average value of 0.70.

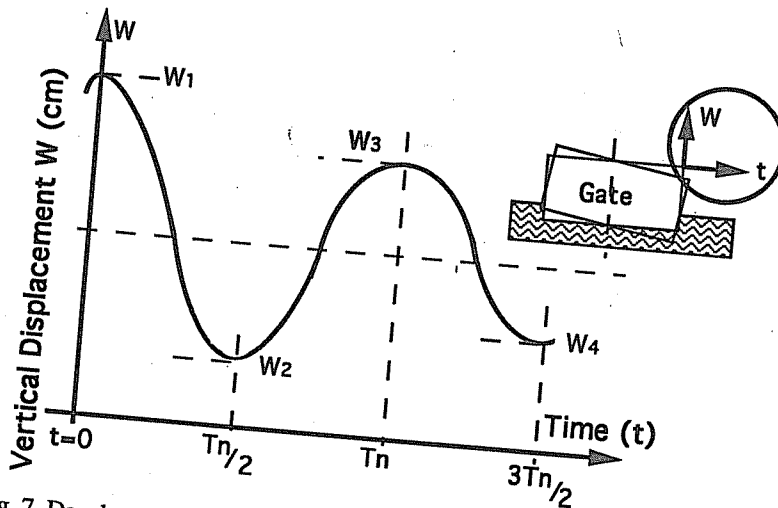


Fig. 7 Development of the natural damped roll motion with time (no wave action).

### CONCLUSIONS

In this study, a new system of storm surge barrier is proposed : a floating gate designed like a tanker. When the gate rotates during the closure process, a roll motion occurs due to the wave action.

The scale model technique was used to analyse this roll motion. The huge length of the gate (390m) and the large value on the scale (1/35) only allowed for building a 2D scale model. Nevertheless the accurate motion of every point of the barrier could be obtained.

For a wave height of 0.60 m and a wave period of 8 sec, the maximum roll angle ( $\phi_{max}$ ) was 2.52 deg which corresponds to a vertical displacement of the lateral wall of 1.18 m.

The natural roll period ( $T_n$ ) of the barrier (without added-mass) ranges from 4.63 sec to 5.46 sec. Moreover  $T_n = 7.7$  sec to 8 sec is the natural roll period with added-mass determined experimentally by two methods. Finally, the added-mass coefficient (CMR) ranges from 1.40 to 1.73.

The recommended damping coefficient ( $K_1=0.7$ ) was determined through experiments on roll motion in still water.

These two coefficients, the added-mass (CMR) and the damping coefficient ( $K_1$ ), will be useful in the future to continue the 3D analysis of the roll motion using the Leeuwerck's method [3].

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## APPENDIX - NOTATION

The following symbols are used in this paper:

CMR	= added-mass coefficient ;
d	= water depth ;
GM	= metacentric height (G is the center of gravity) ;
H,L,T	= wave characteristics (wave height, wavelength, wave period) ;
I	= inertia moment of the floating surface of the gate ;
K <sub>1</sub>	= damping coefficient (0 < K <sub>1</sub> < 1) ;
k	= gyradius (radius of gyration) around the center of gravity G of the mass of the structure + added-mass = k <sub>s</sub> * CMR = k <sub>s</sub> + k <sub>ad</sub> ;
k <sub>ad</sub>	= gyradius around the center of gravity G of the added-mass of the structure ;
k <sub>s</sub>	= gyradius of the mass of the structure alone (without added-mass) ;
MB	= metacentric radius = I/V (B is the center of buoyancy) ;
q	= ratio of vertical displacements ;
T <sub>nS</sub>	= natural roll period of the structure alone - without damping ;
T <sub>n</sub>	= natural roll period of the structure + added-mass - without damping ;
T <sub>n</sub> *	= natural roll period of the structure + added-mass - with damping ;
t	= time ;
X,Y,Z	= axis of the barrier-oriented coordinates ;
x	= constant ;
W	= vertical displacement of the top of the lateral walls (sea side and port side) ;
W <sub>1</sub> ,W <sub>2</sub> ,W <sub>3</sub> ,W <sub>4</sub>	= vertical elevation reached by the gate at the time t = t, t+0.5T <sub>n</sub> , t+T <sub>n</sub> and t+1.5T <sub>n</sub> ;
α <sub>m</sub>	= constant = πH/L ;
V	= displacement (m <sup>3</sup> ) ;
φ = φ(t)	= roll angle - function of time ;
φ <sub>max</sub>	= maximum roll angle ;
ω	= 2π/T ; and
θ	= constant.