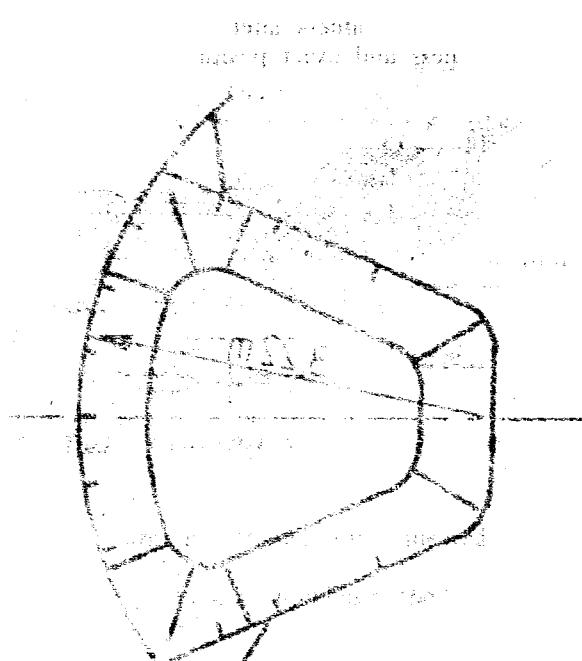


Orthotropic shells

The design of orthotropic hydraulic structures with the L.B.R.-3 software

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HYDRODYNAMIC
The most difficult part
of the analysis of orthotropic
shells and plates can be the
numerical treatment of the boundary
conditions. This is due to the fact
that the boundary conditions are
not necessarily compatible with the
numerical mesh. In order to overcome
this problem, a special technique
is used. It consists of dividing
the boundary into small segments
and applying different boundary
conditions to each segment. This
allows for a more accurate representation
of the boundary conditions.

The design of orthotropic hydraulic structures with the L.B.R.-3 software

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L.B.R.-3 is a specific software for hydraulic orthotropic structures. It allows optimization of locks, navigation dams, canal bridges, tidal surge barriers and ships, such as tanker boats, etc.

A navigation dam (or any other structure) can be computed with this software if the following conditions are respected: having a main direction for the development according to a Fourier series, being limited to the elastic field and being suitable for discretization into parts of orthotropic cylindrical shells. These shells can include 3 types of stiffening reinforcement ribs: stringers, rings and cross-bars.

The method is a harmonic analysis one and is founded on the Fourier series developments. It is based on an analytical resolution of the differential equations governing orthotropic cylindrical shells. These differential equations result from the D.K.J. method (Donnell, von Karman and Jenkins) and allow for torsional stiffness, lateral (tangent to the shell) bending stiffness and exact position of each stiffener. Calculations are not based on 'smeared-out' rib properties. Moreover, L.B.R.-3 gives displacements and stresses with the same accuracy (many other methods based on minimum of energy cannot give both with the same accuracy). With an analytical method, results can be obtained at all points of the structure (sheathing, web-flange junction, web-sheathing junction). New developments which enable the design of structures with many boundary conditions are also included.

The complete computing of a complex structure such as a navigation dam can be done within eight hours. This very short calculation time is of great importance to designers because in this way, they can get a quick confirmation of the behaviour, good or bad, of their projected structure.

INTRODUCTION

The most efficient methods for cylindrical orthotropic plates and shells can be grouped together under the name 'harmonic structural analysis'^{2,5}. The harmonic analysis methods include the finite strip, the Guyon-Massonet method, the Goldberg-Leve method, etc. None of these methods are specifically hydraulic. The stiffened sheathings method¹ is therefore proposed and it allows us to develop the stiffened sheathings software, *L.B.R.-3* (*Logiciel des Bordages Raidis – version 3*).

Orthotropic plates and shells are often used for navigation-dam gates (Fig. 1), navigation and maritime lock gates, tidal surge barriers (Fig. 2), bridge decks (Fig. 3) and tanker boats (Fig. 4 and Ref. 6).

In this paper, the stiffening sheathings method and the qualities of the L.B.R.-3 software are presented.

THE STIFFENED SHEATHINGS SOFTWARE L.B.R.-3

Our studies derived from the D.K.J. (Donald, von Karman, Jenkins) method of cylindrical shells. This method is a harmonic analysis one and is founded on the Fourier series developments. It is based on an analytical resolution of the differential equations governing

orthotropic cylindrical shells. These differential equations allow for torsional stiffness, lateral (tangent to the shell) bending stiffness and exact position of each stiffener.

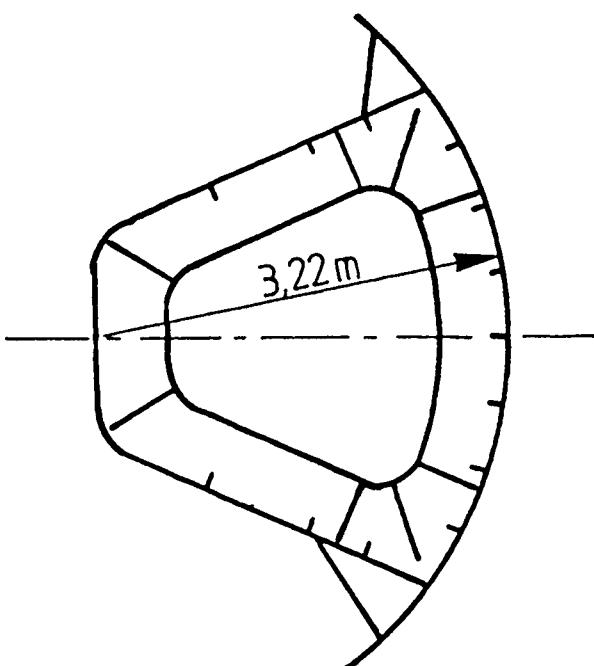


Fig. 1. A radial gate

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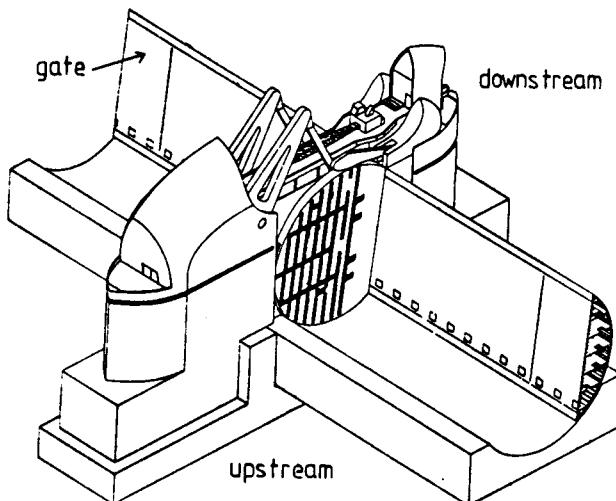


Fig. 2. A tidal surge barrier

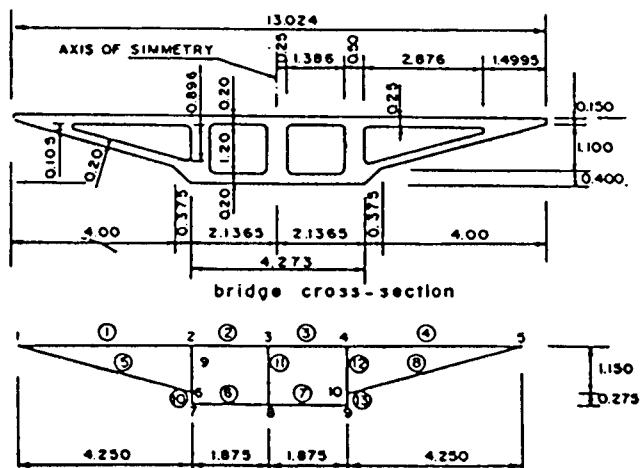


Fig. 3. A bridge deck

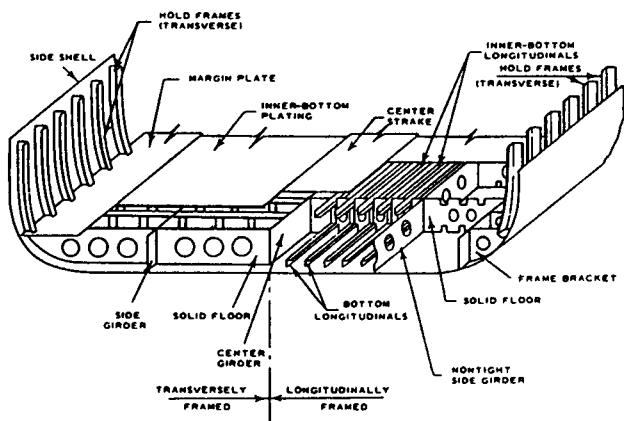


Fig. 4. Tanker boats (Ref. 6)

Calculations are not based on 'smeared-out' rib properties¹.

The originality of the method lies in the following two points:

- computation of box gates using orthotropic plates and shells; these structures being simply-supported along two vertical lines (for instance: a lifting lock gate),

- computation of box gates using orthotropic plates and shells with any boundary conditions along vertical lines (for instance: a radial gate which is elastically supported by two arms as in Fig. 8).

The basic element is a cylindrical orthotropic shell (Fig. 5) where the length is L and q is the radius. The cylindrical coordinates system is presented in Fig. 6, the axis OX along the cylindrical generator, the axis $O\phi$ along the circumference and OZ perpendicular to the shell¹. The displacements u , v and w are associated with the direction of the OX , $O\phi$ and OZ axis.

A structure can be computed with the L.B.R.-3 software if the following conditions are respected: having a main direction for the development according to a Fourier series, being limited to the elastic field and being suitable for discretization into parts of orthotropic cylindrical shells. These shells can include three types of stiffening reinforcement ribs: stringers, rings and cross-bars.

According to the Fourier series, the displacements have the following form:

$$w(x, \phi) = \sum_{n=1}^{\infty} w(\phi) \sin(n\pi x/L) \quad (1)$$

$$v(x, \phi) = \sum_{n=1}^{\infty} v(\phi) \sin(n\pi x/L) \quad (2)$$

$$u(x, \phi) = \sum_{n=1}^{\infty} u(\phi) \cos(n\pi x/L) \quad (3)$$

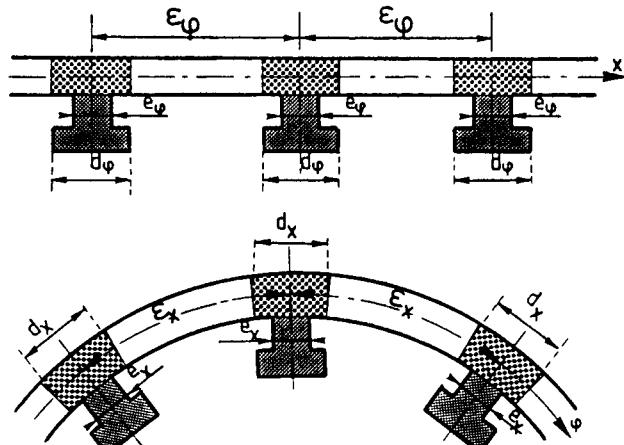


Fig. 5. Shell element with longitudinal and transversal stiffeners

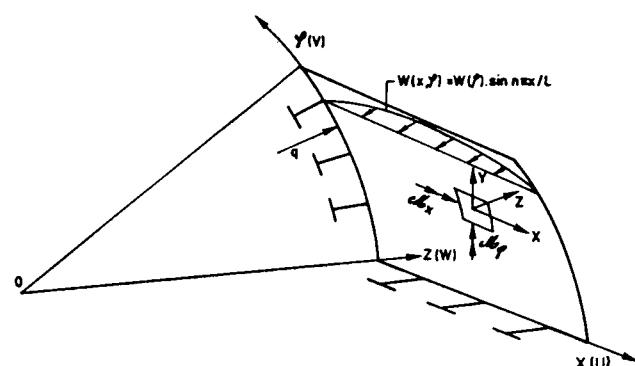


Fig. 6. Orthotropic cylindrical shell

Therefore, along the edges $x=0$ and $x=L$, the conditions related to the simply supported structure ($w=v=0$, $M_x=N_x=0$) are verified.

Moreover, L.B.R.-3 gives displacements and stresses with the same accuracy; many other methods based on the minimum energy cannot give both with the same accuracy. With an analytical method, results can be obtained at all points in the structure (sheathing, web, flange, web-flange junction, web-sheathing junction) and not only at the integration points (Gauss points) of the FEM.

In the last version of the L.B.R.-3 stiffened sheathings software, special external forces are included among the classical external forces X , Y , Z , M_x and M_ϕ (Fig. 6), which, in our case, are specific pressures (N/m^2) and specific moments ($N.m/m^2$). Among these external forces, there are also end-forces N_b and end-moments M_b which are applied at both ends of the plates and shells of the structures (Fig. 7). These end-forces and end-moments allow for the simulation of, for example, the forces and the moments that the supporting arms of a radial gate transmit to the box-girder (Fig. 8). Thus, it is now possible to compute structures with such end conditions as fixed, elastically supported, etc.

Figure 9 shows that the end moments development (equations (1)-(3)) requires an important number of terms of the Fourier series ($n=1, 2, 3, \dots$), especially if an exact load representation is necessary.

In practice, concentrated loads do not exist. They are always more or less spread over small areas located near the ends. Figure 10 therefore indicates the end forces N_b and the end moments M_b which are applied on small areas located near the ends. Successful results are obtained with only 7 terms of the Fourier series.

APPLICATIONS OF THE END FORCES AND MOMENTS

To present a good concept, it is necessary to recall the end conditions of a simply supported structure. The

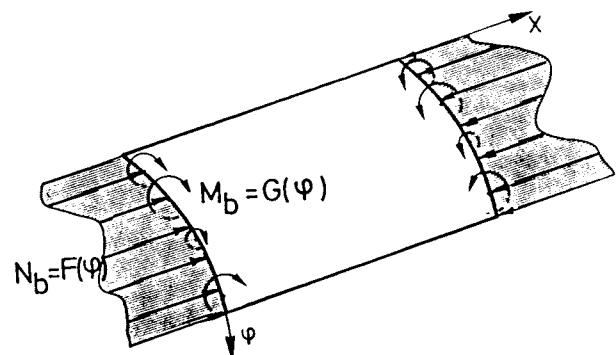


Fig. 7. End forces N_b and end moments M_b

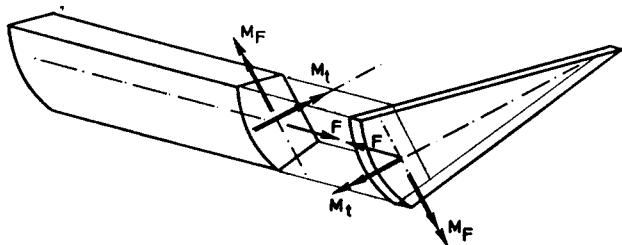


Fig. 8. Forces and moments at the junction between the box girder and a supporting arm

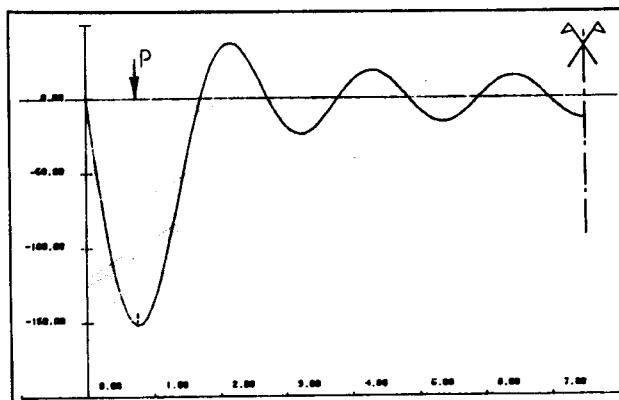


Fig. 9. Diagram of a concentrated load with seven terms of the Fourier series

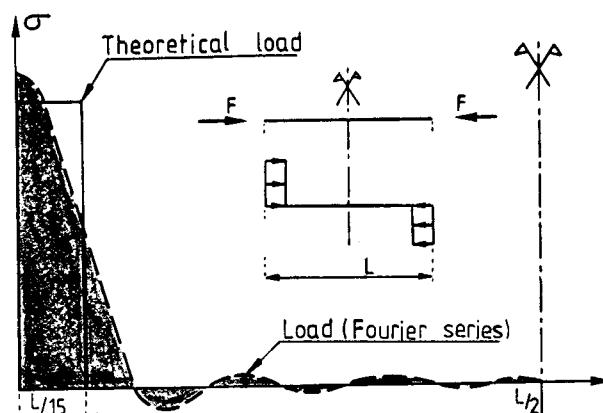


Fig. 10. Development of an asymmetrical load which is applied on the ends over small spaces $d * L / 15$

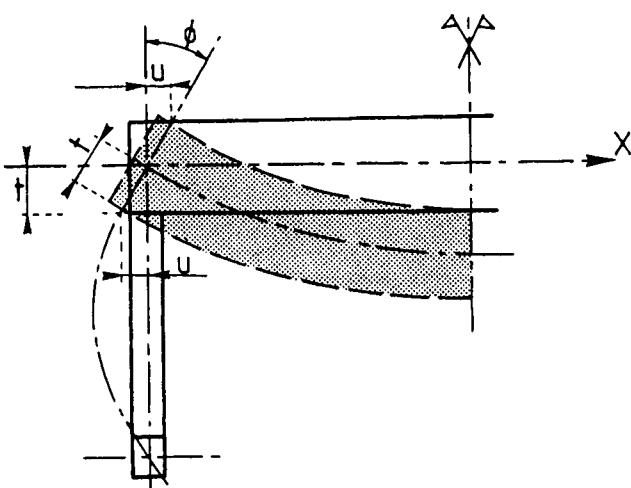


Fig. 11. End deformation of a box-girder

sine-series developments of $w(x, \phi)$ (equation (1)) are fixed. The other displacement variations are: $v(x, \phi)$ as $\sin(n\pi x/L)$ (equation (2)) and $u(x, \phi)$ as $\cos(n\pi x/L)$ (equation (3)). Thus at the ends ($x=0$ and $x=L$), the displacements w and v are nil but the longitudinal displacement u is not nil. The structure ends are therefore like flasks which are infinitely rigid in their plane. The perpendicular displacement to this plane is free, it is like the longitudinal displacements u .

Figure 11 shows the end deformation of the box-girder.

If the end of a supporting arm is subjected to a general rotation ϕ , the longitudinal displacements u and the rotations dw/dx at the *box-girder/arm* junction are automatically well known as a function of this variable ϕ . It is necessary to apply the end forces N_b and the end moments M_b at that junction, where they must coincide with the end displacements of the box-girder and those resulting by the arm rotation.

ESTABLISHING THE END EFFECTS

For simplicity and rationalization purposes, the third degree polynomial development (see equations (4) and (5)) has been chosen for the functions representing the end effects. Consider $F(\phi)$ to be the end forces function and $G(\phi)$ to be the end moments function

$$F(\phi) = a(q\phi) + b(q \cdot \phi)^2 + c \cdot (q \cdot \phi) + d \quad (4)$$

with a, b, c, d the unknown parameters of the end forces function.

$$G(\phi) = e(q\phi)^3 + f \cdot (q \cdot \phi)^2 + g \cdot (q \cdot \phi) + h \quad (5)$$

with e, f, g, h the unknown parameters of the end moments function. These parameters must be determined to have an equivalence between $F(\phi)$ and N_b (Fig. 12) and between $G(\phi)$ and M_b .

Practically, the coefficients a, b, c, d, e, f, g, h are obtained by fixing, point by point, displacement and rotation continuity between the box-girder and the supporting arms. If a structure was composed of n shells, $8n$ unknown parameters must be determined by fixing $8n$ conditions at some structural points.

The position of those points has been optimized in order to minimize the relative errors resulting from the choice.

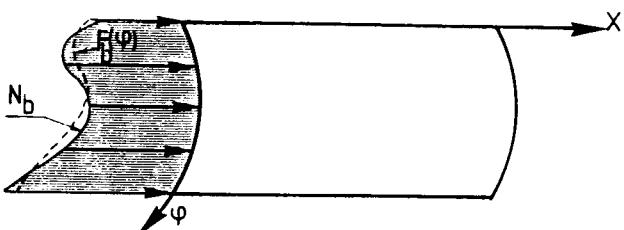


Fig. 12. Adjustment of the analytical end forces function $F(\phi)$ with the theoretical end forces N_b

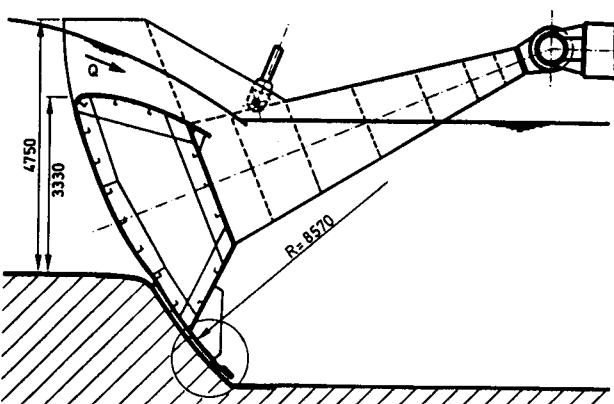


Fig. 13. Cross-section of the radial gate

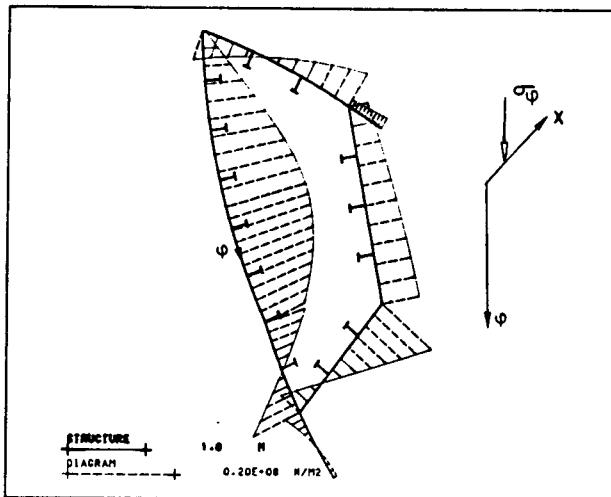


Fig. 14. Transversal stresses σ_ϕ diagram in the sheathing at mid-span ($x = L/2$)

THE RADIAL GATE OF A NAVIGATION DAM

The main piece of modern navigation-dam gates nowadays is an open or closed box-girder which is composed of plates and shells. This main piece can very easily be computed by our L.B.R.-3 software.

Presented here is the case of a radial gate (Fig. 13) for which the study is particularly delicate.

The first computation carried out with the L.B.R.-3 software has enabled us to control the pilot-study. These computations, carried out with one term and without the end effects (gate supposed simply-supported) demand only 72 data lines and 20 s CPU on an IBM 4381. The last checkings, realized with seven terms and the end forces N_b and the end moments M_b , require three more data lines and 7 min CPU.

The extremely reduced data and the very short computation time (CPU) confirm the advantages of using the L.B.R.-3 software in order to design such a structure. Moreover, L.B.R.-3 provides a varied range of numerous, reliable results. Thus, the complete computing of a complex structure like this radial gate, can be accomplished within eight hours (discretization, data correction, computing, printing and results analysis). This very short computation time is of great importance to a designer, as it enables him/her to get a quick confirmation of the good or bad behaviour of the projected structure. Owing to the characteristics previously mentioned, the diagrams of the displacements, the stresses and the forces acting in the structure can be obtained. Figure 14 shows the transversal stresses σ_ϕ diagram in the sheathing at mid-span.

CONCLUSION

The L.B.R.-3 software is a program for designing hydraulic orthotropic steel structures. It is the result of 20 years of experience and research in the field of hydraulic structures.

The basic developments, which have been presented, establish the L.B.R.-3 software as an efficient computational tool. To increase the application field of the method, end forces N_b and end moments M_b have been added to the classical external forces; N_b and M_b being

applied at both ends of the shells. So, hydraulic structures with many boundary conditions can now be computed with the Fourier series.

L.B.R.-3 allows optimization of locks, navigation dams, canal bridges, tidal surge barriers, ships such as tanker boats, etc. The structure can include 3 types of stiffening reinforcement ribs: stringers, rings and cross-bars.

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