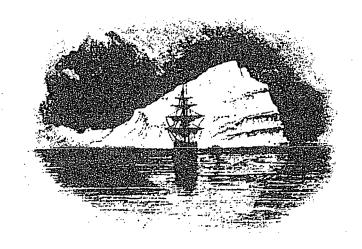


# ICETECET 2000 PROCEDINGS

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### SIXTH INTERNATIONAL CONFERENCE ON SHIPS AND MARINE STRUCTURES IN COLD REGIONS

## ICETECH'2000

# ШЕСТАЯ МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ ПО СУДАМ И МОРСКИМ КОНСТРУКЦИЯМ В ХОЛОДНЫХ РЕГИОНАХ

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## LEAST COST AND SCANTLING OPTIMIZATION AT THE INITIAL DESIGN STAGE OF SHIPS

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#### **ABSTRACT**

LBR-5 is a structural optimization tool for structures composed of stiffened plates and stiffened cylindrical shells. The LBR-5 optimization model is mainly composed of 3 basic modules (OPTI, CONSTRAINT and COST). The user selects the constraints (geometrical and structural constraints) in external databases. Standard constraint sets are proposed to users. Since the present optimization deals with least construction costs, unitary material costs, welding, cutting, ... and labor costs must be specified by the user to define an explicit objective function. Optimum solution is found with an optimization technique using convex linearization and a dual approach. Optimum analysis of a FSO unit (Floating Storage Offloading) is presented.

#### МИНИМАЛЬНАЯ СТОИМОСТЬ И ЧАСТИЧНАЯ ОПТИМИЗАЦИЯ НА НАЧАЛЬНОИ СТАДИИ ПРОЕКТИРОВАНИЯ СУДОВ

#### Аннотация

LBR-5 есть метод структурной оптимизации применительно к сложных конструкций, состоящих из плоских и криволинейных ортотропных плит. Метод оптимизации, положенный в основу LBR-5, состоит из трех главных модулей (OPTI, CONSTRAINT и COST). Пользователь выбирает ограничения (структурные и геометрические ограничения) используя начальные входные данные. Совокупность ограничений предлагаются пользователем. До, настоящего времени оптимизация относилась к оценки минимальной стоимости конструкций, стоимость материала с точки зрения экономической целесообразности, сварки, резки и ..., но и стоимость вложенного труда должно быть оценена пользователем для определения объективной, явной функции. В основе оптимального решения положен метод линеаризации выпуклости и двойственный принцип аппроксимации. Приводится анализ оптимального решения одной FSO (суда-хранилище).

#### 1. INTRODUCTION

The guidelines and major orientations of a structural design are always defined during the earliest phases of a project, i.e. the preliminary design stage or the first draft that corresponds in most cases to the offer. It is thus not difficult to understand why an optimization tool is attractive, especially one designed for use at the preliminary stage. This is precisely the way the LBR-5 optimization software for stiffened hydraulic and naval structures was thought through, created and developed [Rigo 1998, 1999].

The target is to link standard design tools (steel structure CAD, hull form, hydrostatic curves, floating stability, weight estimation...) with a rational optimization design module that, as of the first draft (or preliminary design), allows for:

- a 3D analysis of the general behaviour of the structure, or at least of the basic transverse cross-section (midship section);
- to explicitly take into account all the relevant limit states of the structure (service limit states

and ultimate limit states) thanks to a rational analysis of the structure based on the general rules of solid-mechanics and structure behaviour. By rational analysis, we mean a coherent and homogeneous group of analysis methods based on physics, solid mechanics, strength and stability treatises, etc. and that differ from empirical and parametric formulations:

- as of the first draft, an optimization of the sizing/scantling (profile sizes, dimensions and spacing) of the structure's constituent elements:
- integration of construction and manufacturing costs in the optimization process (through the cost objective function).

The advantages of this optimization module appear mainly at the preliminary stage. It is indeed during the first stages of the project that flexibility, modelling speed and method's easy use provide precious help to designers. At this moment, few parameters/dimensions have been definitively fixed and a coarse modelling by standard finite elements is often unusable for reasons of budget, modeling duration, available data,... and this, particularly for design offices and modest-sized yards (small and medium-sized enterprises).

This optimization module can also be used in finalstage of the project to perform a general verification or to refine the scantling.

Application fields of LBR-5 include hydraulic structures and naval structures. For the former, the application domain is clearly the ship's central parts (cylindrical and prismatic zones of cargo ships, passenger vessels, etc.). This zone is the most important in length for the big floating units. For smaller units (sailboats, small craft, etc.), the cylindrical zone is smaller, or even non-existent. In this case, the LBR-5 model can be used to perform transverse cross-section optimization (midship section).

The LBR-5 optimization tool is based on important know-how in the stiffened structures domain that was materialised in 1988 by the development of the LBR-4 linear analysis software for stiffened structures analysis [Rigo 1992a and b]. The scientific environment in which the optimization part was developed mainly concerns naval architecture. This work was made possible by unifying analysis methods and by using rational approaches to assess structure limit states. LBR-5 definitively favors a unified optimisation approach.

#### LBR-5 AND THE CONCEPT OF OPTIMIZATION-ORIENTED MODULES

The general problems to be solved can be summarised as follows:

- i = 1. N. $X_i$ the N design variables,
- the objective function to  $F(X_i)$ minimize,
- $C_j(X_i) \le CM_i$ i = 1,Mthe M structural and geometrical constraints,
- $X_{i \text{ min}} \leq X_{i} \leq X_{i \text{ max}} \text{ upper and lower bounds of }$ the X<sub>i</sub> design variables: technological bounds (also called side constraints).

The objective is to create a user-oriented optimization technique, in permanent evolution, i.e. that evolves with the user and his individual needs. We define these as "Programming-Oriented Modules". The LBR-5 optimization model is based on this new concept and is composed of several modules. Neither the module number nor their type is imposed. At the start, the whole model is made up of 3 basic modules (Figure 1) and forms the framework of the tool (COST, CONTRAINT and OPTI).

Figure 1 shows the basic configuration of the LBR-5 software with the 3 fundamental modules (COST, CONTRAINT and OPTI) and the "DATABASES" in which the user can do his "shopping", i.e. choose the relevant constraints and cost data. Figure 3 succinctly shows the LBR-5 software chart.

With regard to structural constraints, the user must first choose the types of constraints (yielding, buckling, deflection, etc.) then, for each type of constraint, select the method, the code or the rules to use and finally the points/areas/panels where these constraints will be applied.

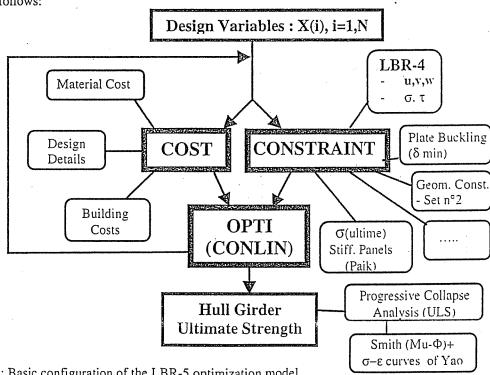


Figure 1: Basic configuration of the LBR-5 optimization model

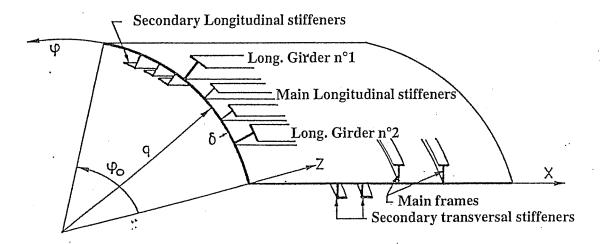


Figure 2: Basic stiffened panel (or basic element) used to model the structures.

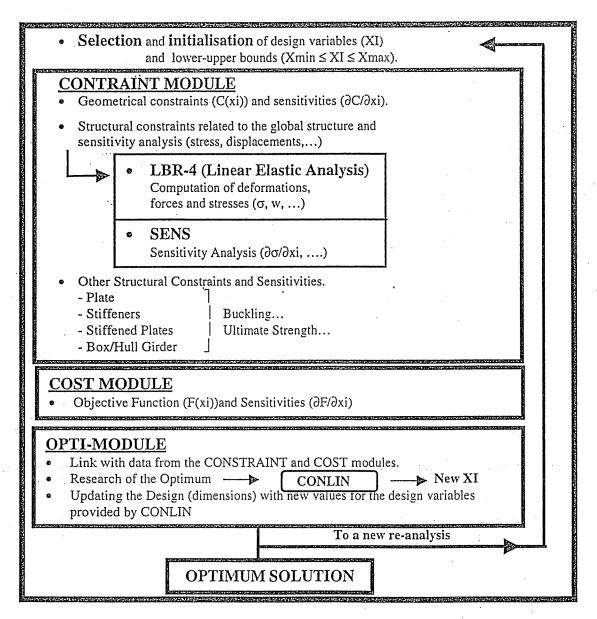


Figure 3: Chart of the LBR-5 model with CONSTRAINT, COST and OPTI modules

The structure is modelled with stiffened panels (plates and cylindrical shells) (Figure 2). For each panel one can associate, up to 9 design variables (XI). These 9 design variables are respectively:

- Plate thickness.
- For longitudinal members (stiffeners, crossbars, longitudinals, girders, etc.):
  - web height and thickness,
  - flange width,
  - spacing between 2 longitudinal members.
- For transverse members (frames, transverse stiffeners, etc.):
  - web height and thickness
  - flange width,

spacing between 2 transverse members (frames).

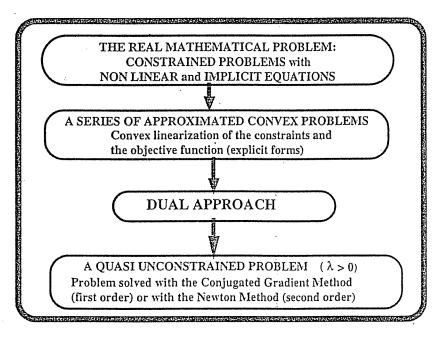
# 3. DESCRIPTION OF THE 3 BASIC MODULES: OPTI, CONTRAINT AND COST

The OPTI module contains the mathematical optimization algorithm (CONLIN) that allows solving non-linear constrained optimization problems. It is especially effective because it only

requires a reduced number of iterations. In general, fewer than 15 iterations (including a structure reanalysis) are necessary, even in presence of several hundred design variables (XI) (Figure 4). CONLIN is based on a convex linearization of the non-linear functions (constraints and objective functions) and on a dual approach [C. Fleury, 1989 and 93]. This module uses as inputs the results/outputs of the two other basic modules, i.e. CONTRAINT for the C(XI) constraints and COST for the F(XI) objective function.

The main difficulty in solving a dual problem is dealing with the non-linear and implicit constraints. In order to avoid a large number of time-consuming re-assessments of these non-linear and implicit functions, Fleury suggests applying convex approximations. At each iteration, all the functions (objective function and constraints) are replaced by an approximation called 'convex'. In a word, the complex initial optimization problem decomposed in a sequence of more simple convex optimization problems (obtained through a convex linearization) that can be easily solved using a dual approach (Figure 4).

Figure 4:
The CONLIN model:
convex approximations
and
dual approach.



The CONTRAINT module helps the user to select relevant constraints within constraint groups at his disposal in a databank (Figure 1). In fact, the user remains responsible for his choice. However, in order to facilitate this selection, several coherent constraint sets are proposed to the user. These sets are based on national and international rules/codes (Eurocodes, ECCS Recommendations, Classification Societies, etc.).

To date, only a limited number of modules are available (in general 1 or 2 for each constraint type). It is up to the user to complete, adapt and add new modules according to his specific requirements (type of structure, codes and regulations to be followed, technical and scientific level, available hardware, etc.). The objective is to enable the user himself to build the tool he needs.

Constraints are linear or non-linear functions, either explicit or implicit of the design variables (XI).

These constraints are analytical "translations" of the limitations that the user wants to impose on the design variables themselves or to *parameters* like displacements, stresses, ultimate strength, etc.

#### So, one can distinguish:

- <u>Technological constraints (or side constraints)</u> that provide the upper and lower bounds of the design variables.

For example:  $Xi min = 4mm \le Xi \le Xi max = 40 mm$ .

with: Xi min a thickness limit dues to corrosion, etc;

Xi max a technological limit of manufacturing or assembly.

Geometrical constraints impose relationships between design variables in order to guarantee a functional, feasible, reliable structure. They are generally based on "good practice" rules to avoid local strength failures (web or flange buckling, stiffener tripping, etc. ), or to guarantee welding quality and easy access to the welds. For instance, welding a plate of 30 mm thick with one that is 5 mm thick is not recommended.

Example:  $0.5 \le X2 / X1 \le 2$ 

with X1, the web thickness of a stiffener and X2, the flange thickness.

Structural constraints represent limit states in order to avoid yielding, buckling, cracks etc. and to limit deflection, stress, etc. These constraints are based on solid-mechanics phenomena and modeled with rational equations.

Thus, these constraints can limit:

- Deflection level (absolute or relative) in a point of the structure,
- Stress level in an element ( $\sigma_x$ ,  $\sigma_y$  and  $\sigma_c = \sigma_{von\ Mises}$ ),
- Safety level related to buckling, ultimate resistance, tripping, etc. (example: σ / σ<sub>ult</sub> ≤ 0.5).

For each constraint (or solid-mechanics phenomenon), the <u>selected behaviour model</u> is especially important since this model fixes the quality of the constraint modeling.

The list of constraints included in the LBR-5 model is intimately bound to the types of structures targeted by this research. Let's recall that these are mainly metallic, prismatic (box girders) and stiffened (orthotropic) structures used for hydraulic and marine structures. These structures are composed of stiffened panels that are either cylindrical or plane. The panels are joined one to another by generating lines (edges of the prismatic structure) and are stiffened longitudinally and transversely (Figure 5).

When going from the "local" to the "general" components (Figure 9), one differentiates three types of constraints:

- constraints on panels and components,
- constraints on frames and transversal stiffening,
- constraints on the general structure.

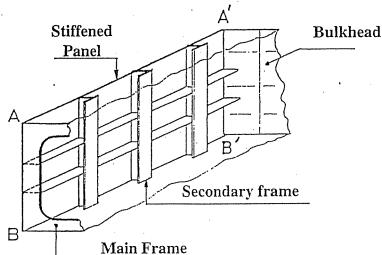


Figure 5: A stiffened panel

When going from the "local" to the "general" components (Figure 5), one differentiates three types of constraints:

- -constraints on panels and components,
- -constraints on frames and transversal stiffening,
- -constraints on the general structure.
- Constraints on stiffened panels (Figure 5).

  Panels are limited by their lateral edges (junctions with other panels, AA" and BB")

either by watertight bulkheads or transverse frames. These panels are orthotropic plates and shells supported on their four sides, laterally loaded (bending) and submitted, at their extremities, to in-plane loads (compression/tensile and shearing).

The global buckling of panels (including the local transverse frames) must also be considered.

Supports of panels and, in particular those corresponding to the reinforced frames, are

assumed infinitely rigid. This means that they can distort themselves significantly only after the stiffened panel collapse.

- Constraints on the transverse frames (Figure 5)

  The frames take the lateral loads (pressure, dead-weight, etc.) and are therefore submitted to combined loads (large bending and compression). The rigidity of these frames must be assured in order to respect the hypotheses on panel boundary conditions (undeformable supports).
- Constraints on the global structure (box girder/hull girder).

The ultimate strength of the global structure or a section (block) located between two rigid frames (or bulkheads) must be considered as well as the elastic bending moment of the hull girder (against yielding).

The limit states that will be considered are:

- A service limit state that corresponds to a situation where the structure can no longer assure the service for which it was conceived (examples: excessive deflection cracks).
- An *ultimate limit state* that corresponds to collapse/failure.

The COST module: In 2000, even for a first draft, a least weight optimization process can no longer be justified and should be replaced by a least construction cost or, even better, by a minimum global cost (including operational costs).

Up to now, the objective function of the LBR-5 software has considered both construction costs (COST module) and weight (example: 60% of the cost and 40% of the weight). In order to link the objective function (Euro) to the design variables (Xi), the unit costs of raw materials (Euro/Kg), the productivity rates for welding, cutting, assembling, ... (man-hours/unit of work = m-h/unit) and labour costs (Euro/m-h) must be specified by the user.

These unit costs vary according to the type and the size of the structure, the manufacturing technology (manual welding, robots,...), the experience and facilities of the construction site, the country, etc. It is therefore obvious that the result of this optimization process (sizing optimization) will be valid only for the specific economic and production data under consideration. Sensitivity analysis of the economic data on the optimum scantling can also be performed, providing the manager with valuable information for improving the yard.

## 4. OPTIMIZATION OF A FSO BARGE

The present optimization with the LBR-5 model concerns a FSO barge of 336 m with a capacity of 2

500 000 barrel capacity, designed to serve as floating reservoir (provisory storage area) in view to receiving crude oil before being transferred on board tankers (FSO = Floating Storage Offloading). It is a moored barge, without its own propulsion system. The anchorage, independent of the barge, permits an almost free motion. Thus, the FSO barge always moves facing the current.

The barge filling is achieved using a pipeline connected to the shore. The small discharge of the pipeline induced uniform and slow loading. On the other hand, the discharge of the FSO unit that corresponds to the filling of a VLCC (Very Large Crude Carrier) is very fast and not uniform. The main characteristics of the barge are given on Table 1.

Figure 6 presents general views of the barge studied. The optimization of a 46-m hold composed of two center tanks of 24 m x 30 m x 46 m and two lateral ballast tanks of 6 m in width was performed.

The maximal hull girder bending moment (without waves) has been valued at 670 000 t.m (6.7 Mio kN.m) and the shear force at 25 000 t (250 000 kN). This bending moment is particularly high by comparison to standard VLCC bending moment (±3 Mio kN.m). In addition, to take into account the wave bending moment, the optimum scantling was performed for a maximum bending moment of 10 Mio kN.m (hogging and sagging).

Based on structure symmetry, only half of the structure is modeled for structure optimization with the LBR-5 model. The two loading cases are considered.

In order to model the strong rigid bracket at each extremity of the tanks' transverse girders, the bottom panel of theses center tanks (24 m in width) is modeled with three stiffened panels of 8-m wide in order to simulate a variable rigidity of these transverse members (Figure 6). Similarly, the longitudinal bulkheads and the deck are also modeled with three elements each. Since the central bulkhead is on the symmetry axis, only half of its rigidity is taken into account in the model.

At the end of the optmization procedure, the recommended scantlings are (Table 2):

- for least cost (C = 100%, P = 109%):
  - $\delta \leq 40 \text{ mm with 7 frames } (\Delta = 5.75 \text{ m})$
  - cost per kilo: 2.17 Euros
- for least weight (C = 106%, P = 101%):
  - $\delta \le 40 \text{ mm}$  with 8 frames ( $\Delta = 5.11 \text{ m}$ )
  - cost per kilo: 2.42 Euros

Concerning the cost per kilo or unitary cost (Euro/kg), least cost optimization leads to unitary costs 10 to 15% lower than for least weight optimization (2.17 Euro/kg instead of 2.42 Euro/kg).

Lpp	(length between perpendiculars)		336 m (10 + 6 x 46 + 50 m)
В	(width)		60 m (6 + 24 + 24 + 6 m)
H	(depth)		30 m
T	(draft)	• •	20.5 m
C <sub>b</sub>	(bloc coefficient)		0.95

Table 1: Main characteristics of the FSO barge

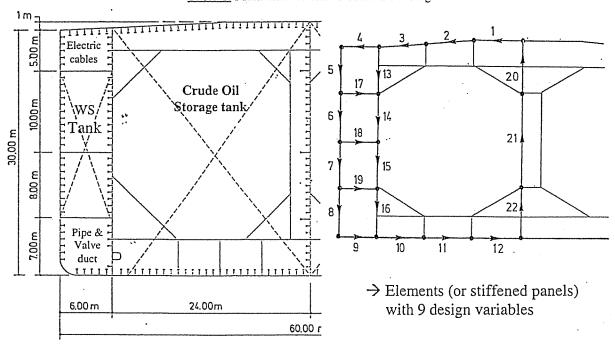


Figure 6: General view of the FSO barge and Mesh modeling

The state of the s								
Configurations	Weight	Cost	Cost per kg	' '	$\Delta$ (center tanks)			
	KN (%)	10 <sup>6</sup> Euro (%)	Euro/kg	+ N(*)	+ N(*)			
δ≤40 mm								
<u>Least Cost</u>								
$C1: \Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	29280	6.34	2.17	5.75 m	5.75 m			
•	(109 %)	(100 %)		3N = 7	N = 7			
C2: $\Delta_{\text{side tank}} = \frac{1}{2} \Delta_{\text{center tanks}}$	29740	6.63	2.23	6.57 m	3.285 m			
	(111%)	(105 %)		N = 6	N = 13			
Least weight	·							
C3: $\Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	27150	. 6.70	2.42	5.11 m	5.11 m			
	(101%)	(106%)		N=8	N = 8			
C4: $\Delta_{\text{side tank}} = \frac{1}{2} \Delta_{\text{center tanks}}$	26850	7.13	2.61	5.75 m	2.875 m			
	(100 %)	(113 %)	٠	N = 7	N = 15			
δ≤30 mm	-				-			
Least Cost	38870	8.52	2.19	3.07 m	3.07 m			
$C5: \Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	(145 %)	(134 %)		N = 14	N = 14			
Least weight	38500	9.64	2.50	3.07 m	3.07 m			
C6: $\Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	(143 %)	(114 %)		N = 14	N = 14			
Initial Scantling	39370	9.74		7.66 m	7.66 m			
(Start of the Opt. Process)	(147 %)	(154 %)	2.47	N = 5	N = 5			

(\*) N = Number of frames for a 46-m long hold, N =  $(46/\Delta)$  –1

Most advisable scantlings (design)

<u>Table 2:</u> Comparison between the different optimum (after 10 iterations)

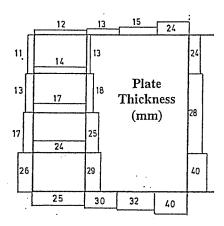


Figure 7: Optimal scantling of the FSO barge (Least cost -  $\delta \le 40 \text{ mm}$ ,  $\Delta = 5.75 \text{ m}$ )

Figure 7 presents the plate thickness' of least cost optimum design for the FSO midship section. Similar figures are obtained for the others design variables. After standardization of the frame size and adding brackets, Figure 8 shows the midship section provided by LBR-5.

#### 5. CONCLUSIONS

LBR-5 is a structural optimization tool for structures composed of stiffened plates and stiffened cylindrical shells. It is an integrated model to analyze and optimize naval and hydraulic structures at their earliest design stages: tendering and preliminary design.

Initial scantling is not mandatory. Designers can directly start by an automatic search for optimum sizing (scantling). Design variables (plate thickness, stiffener dimensions and their spacing) are freely selected by the user.

LBR-5 is composed of 3 basic modules (OPTI, CONSTRAINT and COST). The user selects the relevant constraints (geometrical and structural constraints) in external databases. Standard constraint sets are proposed to users. Since the present optimization deals with least construction costs, unitary material costs, welding, cutting and labor costs must be specified by the user to define an explicit objective function. Using all this data (constraints, objective function and sensitivity analysis), an optimum solution is found using an technique based optimization on convex linearizations and a dual approach. Independently of the number of design variables and constraints, the number of iterations requiring a complete structural re-analysis is limited to 10 or 15.

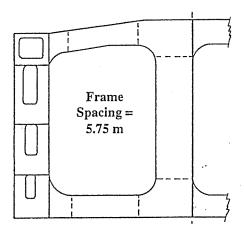


Figure 8: Optimum midship sections for the FSO barge. (Least cost -  $\delta \le 40$  mm,  $\Delta = 5.75$  m)

Optimum analysis of a FSO barge (Floating Storage Offloading) is presented as an application of the LBR-5 optimization model.

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