MODELING OF THE WAVE PROPAGATION, THE COASTAL CURRENTS AND THE INDUCED TOPOGRAPHIC CHANGES

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Abstract: This paper models the sea bottom evolution. This model is built up on three main parts. The first one describes the wave propagation, taking simultaneously into account the refraction, the diffraction and the reflection but also the energy dissipation caused by wave breaking. This model is solved by the numerical method of the finite volumes, using an iterative resolution (the GMRES method). Several validations of this model have been carried out on many examples such as harbors of various geometries and depths, beaches,… The wave induced currents are computed in the second part based on the concept of the radiation stresses describing the excess of the flux momentum caused by the waves. A model based on the complete Navier-Stokes equations integrated on the depth and associated with appropriated forcing and friction terms is used. This model is computed as an update of WOLF software (HACH, ULg) and is applied to some examples such as beaches with or without coastal structures. Finally, the third part is about the sediment transport. It exhibits the topographic modifications of the coastal sea bottom caused by the waves and their currents. A loop execution of these three models ends this work. It gives us an operational tool to describe the topographic evolution of the coastal areas caused by the incident waves, the induced currents and then the resulting sediment transport, fulfilling the objectives of this paper.

Keywords: Wave propagation, Coastal currents, Sediment transport, Topographic modifications

1 . INTRODUCTION

The natural coastal limit between sea and ground is a fragile and often moving entity. Currently the socio-economic requirements involve sometimes irreversible modifications which it would be necessary to forecast and study. The main objective of the present paper is to suggest a model describing the behavior and the modifications of the coastal sea-beds. This modeling is based on three specific successive models: the first one describes wave propagation, the second is about wave induced currents and the last one deals with sediment transport.

The first part computes the wave propagation, taking simultaneously into account the refraction, the diffraction and the reflection but also the energy dissipation induced by wave breaking along the coast. A finite volume scheme is used for the spatial discretization combined with an implicit scheme of resolution. An iterative method is selected to avoid the explicit inversion of the system matrix. The successive linearized systems are solved with the powerful GMRES algorithm, which is advantageously coupled to a preconditioner. For this purpose an incomplete LU decomposition is applied.

The wave induced currents are modeled in the second part using the concept of radiation constraints describing the excess of flux momentum caused by waves. A complete model based on the Shallow-Water Equations (SWE) is used, combined with appropriate forcing and friction terms determined by the results of the first model. This model has been implemented as an update of WOLF software (Laboratory of Applied Hydrodynamics and Hydraulic Constructions (HACH), University of Liège).

The stability of a beach strongly depends on the balance between the deposit of sediments and their redistribution by the coastal currents. The third and last part of this paper proposes a study of the sea-beds erosion and exhibits the topographic modifications subsequent to the presence of the waves and their currents.

Many examples validate each of these three models which are then been applied to large-scales studies such as beaches with or without coastal structures. Finally, a complete example treating at the same time calculation of wave propagation, coastal currents and sediment transport has been be carried out in order to show the efficiency of the looping of the three software and to illustrate the potentialities of the program to determine the variation of sea-beds resulting from the three simultaneous phenomena.

2 . WAVE PROPAGATION

The principal goal of the first part of this paper is the determination of a plan of waves, i.e. the calculation of the wave amplitude and the wave number, on a wide horizontal zone. Considering the complexity of the phenomenon of studying the wave propagation under all its aspects, it seems difficult to obtain a mathematical formulation taking simultaneously into account each effect. For this reason, some simplifications will be brought in order to formulate an adequate mathematical description of the mechanism. The model that will be established must be integrated in a global process of resolution and quickly provide an exchange of information with other models, such calculation of currents and sediment transport.

2.1 Mild-Slope equation

Within the framework of linear wave theory, Berkhoff (Berkhoff 1974; Berkhoff 1983) proposed a two-dimensional theory that can deal with large regions of refraction and diffraction. The underlying assumption of the theory is that evanescent modes are not important for waves propagating over a slowly varying bathymetry, except in the immediate vicinity of a three-dimensional obstacle. For a monochromatic wave with frequency ω, Berkhoff proposes that the three-dimensional velocity potential function ϕ has the form:

$$\phi(x,y,z) = f(z)\varphi(x,y)e^{i\omega t} \tag{1}$$

where

- f represents the effect of the depth and of the variation in the vertical direction z:

$$f = \frac{\cosh k(z+h)}{\cosh(kh)} \tag{2}$$

the wave number $k(x,y)$ and the depth $h(x,y)$ vary slowly in the horizontal directions x and y according to the linear frequency dispersion relation

$$\omega^2 = gk\tanh(kh) \tag{3}$$

and g is the gravitational acceleration.

- φ represents the velocity potential at the horizontal surface ($z=0$)
- $e^{i\omega t}$ is a harmonic time function.

By use of this expression and integration of the potential equations over the water depth, we obtain the following equation:

$$\partial_i(cc_g\partial_i\varphi) + \frac{\omega^2}{c}c_g\varphi = 0 \tag{4}$$

with the celerity c and the group velocity c_g. This elliptic-type partial differential equation (4) is valid for sufficiently small bottom slope and is known as the Mild Slope Equation (MSE). For long waves in shallow water the limit of (4) at $kh << 1$ reduces to the well-known linear shallow-water equation. On the other hand, if the depth is a constant or for short waves in deep water ($kh >> 1$), (4) reduces to the Helmholtz equation where k satisfies the relation dispersion (3).

The MSE do not take into account the energy dissipation caused by wave breaking. However, many observations confirmed the dominant part played by the breaking in the process of sediment transport. It is precisely in the surf zone, located between the breaking line and the coast, that the highest current velocities are measured and that the near total of sediment transport takes place. A description of wave breaking in this area is therefore necessary. A term is then added in the MSE to take this phenomena into account (Booij 1981):

$$\partial_i (cc_g \partial_i \varphi) + \frac{\omega^2}{c} c_g \varphi + i\omega\alpha\varphi = 0 \tag{5}$$

where α is the ratio between the dissipated and total energy.

The MSE takes simultaneously into account wave refraction, reflection, diffraction and breaking. The domain of resolution is divided into three parts (Fig. 1): Ω_1 infinite zone representing the open sea, Ω_2 the complex area of interest and Ω_3 the ground. Adapted boundary conditions have to be imposed on each different boundary of the domain.

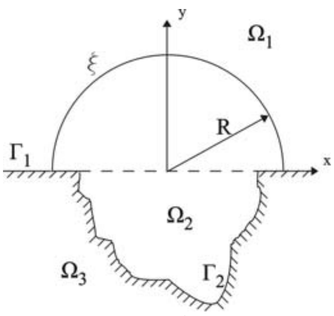

Fig. 1: Domain of resolution.

2.2 Numerical computing

The MSE has then to be numerically computed to obtain the potential in all the domain of calculation. This potential has a real and an imaginary part; we have then to resolve a system of two equations (one for the real part and one for the imaginary one) with two distinct variables.

A finite volume method is used for the spatial discretization with uniform grid made up of rectangular meshes. For the inversion of the system, a method of resolution entirely implicit is selected. Because the methods of direct resolution of great systems are very expensive in computing time, an iterative type method of resolution is employed, the GMRES method (Saad 1996). This one presents the advantage of appreciably reducing the cost of calculation for the large systems and of introducing a criterion to stop the process as soon as the precision obtained for the current iteration is sufficient. The implementation of the algorithm for the resolution of the model of wave propagation is integrated as a module in the WOLF software developed by the HACH Laboratory. This presents the advantage of disposing of the graphic interface for the pre- and post-processing of the data, allowing an easy visualization of the results.

2.3 Applications

Many applications (study of wave-induced oscillations in harbors of various geometries, wave amplification along beaches, large-scale areas such as the Calvi Bay in France,…) have been carried out in order to validate the model of diffraction-refraction proposed and solved by the method of finite volumes (Fraikin 2003). The results are in good agreement with analytical, experimental or numerical results presented by different authors in the literature (Hwang and Tuck 1970; Lee 1971; Raichlen and Naheer 1976; Chen 1986; Lee and Park 1998).

A useful application of the model of wave propagation developed in this paper is the determination of the waves field around a maritime work. We treat here the case of a uniform beach with a breakwater situated perpendicularly to the coast and calculate the amplitude of the resulting waves. The depth increases linearly with the distance to the coast. The reflection on the breakwater is supposed to be total, while the coastline is supposed to be absorbing.

The characteristics of the incident wave are : wave height $H = 0.8$ m, period $T = 8$ s and angle of incidence of 250°. The wave heights obtained are represented on the Fig. 2. A surf zone is present along the coast on a distance of about fifteen meters, the breakwater is thus active on the major part of this one. We remark wave amplification at the head of the breakwater caused by reflection.

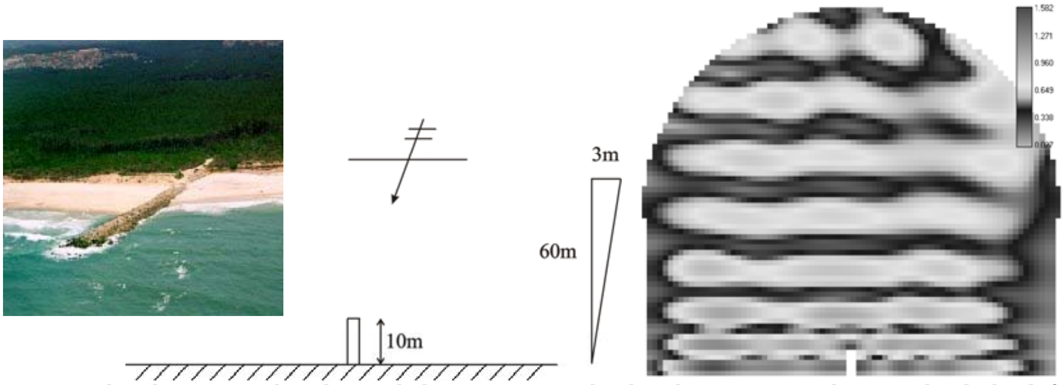

Fig. 2 : Wave heights (m) (right) obtained along a coast with a breakwater situated perpendicularly (left).

3 . WAVE INDUCED CURRENTS

3.1 Theoritical Aspects

It is well known that when sea waves or swell approach a straight coastline at an oblique angle a mean current tends to be set up parallel to the coastline. Such longshore currents and the associated longshore transport of sand or other sedimentary material are of prime importance in coastal engineering.

Many hypotheses have been advanced to account for this phenomenon. One of the more satisfactory explication is based on the concept of the radiation stress as developed by Longuet-Higgins (Longuet-Higgins and Stewart 1964; Longuet-Higgins 1970) which estimates the excess of flux momentum due to waves. This estimate has already proved remarkably successful in the prediction of several wave phenomena, particularly the setup, or change in mean level of the sea surface in the breaker zone. In regions where no breaking occurs these stresses can be obtained theoretically as a second-order quantity from the first-order small-amplitude wave theory. In the surf zone where breaking and turbulence prevail, no theory exists for describing either the oscillatory waves or the radiation stresses. However, by hypothesizing some closure relations between the radiation stresses and mean-flow quantities, a semi-empirical framework is obtained for calculating the mean flow in the surf zone.

The mathematical model used to represent wave induced currents is obtained from the time and depth averaged Navier-Stokes equations, the Shallow-Water Equations (SWE) (Mei 1983):

$$\frac{\partial \overline{\xi}}{\partial t} + \partial_i \left[\left(\overline{\xi} + h \right) U_i \right] = 0 \tag{6}$$

$$\frac{\partial\left(\overline{\xi}+h\right)U_j}{\partial t}+\partial_i\left[\left(\overline{\xi}+h\right)U_iU_j\right]+g\left(\overline{\xi}+h\right)\partial_j\overline{\xi}=\frac{1}{\rho}\left[\partial_iT_{ij}+F_j-\tau_j^B\right] \qquad (7)$$

where :

- $\overline{\xi}$ is the time mean free-surface displacement
- h the water depth
- U_i ($i=1,2$) the mean velocity
- T_{ij} the turbulent stresses

$$T_{ij}=\rho\mu\left(\overline{\xi}+h\right)\partial_jU_i \qquad (8)$$

- τ_j^B ($j=1,2$) the bottom friction
- F_j ($j=1,2$) the wave induced stresses, modeled by the radiation stresses concept

$$F_j=-\partial_iS_{ij}=\frac{D}{\omega}k_j \qquad (9)$$

where the tensor S_{ij} characterizes the waves effect on the mean flow and is named radiation stresses tensor. It can be modelled by introducing the energy dissipation D.

3.2 Numerical aspects

The numerical computing of the mathematical model is realized by use of the logiciel WOLF2D (HACH), an efficient analysis and optimization tool, which has been completely developed at the University of Liège (Archambeau, Dewals et al. 2003). WOLF 2D is a part of WOLF free surface flows computation package, which includes in the same development environment the resolution of the 1D Saint-Venant equations, the 2D SWE as well as a physically based hydrological model, and powerful optimization capabilities based on Genetic Algorithms. The spatial discretization of the 2D conservative SWE is performed by a finite volumes method. This ensures the mass and momentum properties to be conserved, even across discontinuities such as hydraulic jumps. Flux treatment is here based on an original flux-vector splitting technique developed for WOLF. The hydrodynamic fluxes are splitted and evaluated partly downstream and partly upstream according to the requirements of a Von Neumann stability analysis. Optimal agreement with non-conservative and source terms as well as low computational cost are the main advantages of this original scheme. An accurate and non-dissipative explicit temporal scheme has been chosen, the Runge-Kutta algorithm. This well-known algorithm brings a rate of dissipation adapted for an acceptable time of resolution and is usually preferred with the other schemes commonly suggested. Moreover, this scheme is easily parameterized for the search of a stationary solution to cause a higher rate of dissipation that accelerates the convergence process. Implicit schemes are however available to manage this last type of case.

3.3 Application

In order to validate the wave induced currents model, a simple application has been carried out in this section: the case of a beach with a breakwater situated perpendicularly to the coast as treated in the previous section. Using the model of propagation, we can evaluate forcing and friction terms for the SWE (no turbulence is taken into account in this example). The boundary conditions are the imposition of a free surface elevation at the open sea and a condition of impermeability on the beach. In the direction parallel to the beach (on the lateral boundaries), an implicit boundary condition is imposed in WOLF, ensuring the uniformity of the flow in this direction. The results are represented on the Fig. 3. Far from the breakwater, we obtain a uniform flow which is that of a beach without structure. Important velocities are observed at the side upstream because the flow is forced to by-pass the obstacle. On the downstream side, a recirculation area develops, creating a currents protected zone, interesting phenomenon for the impact study of the coastal structures on the littoral.

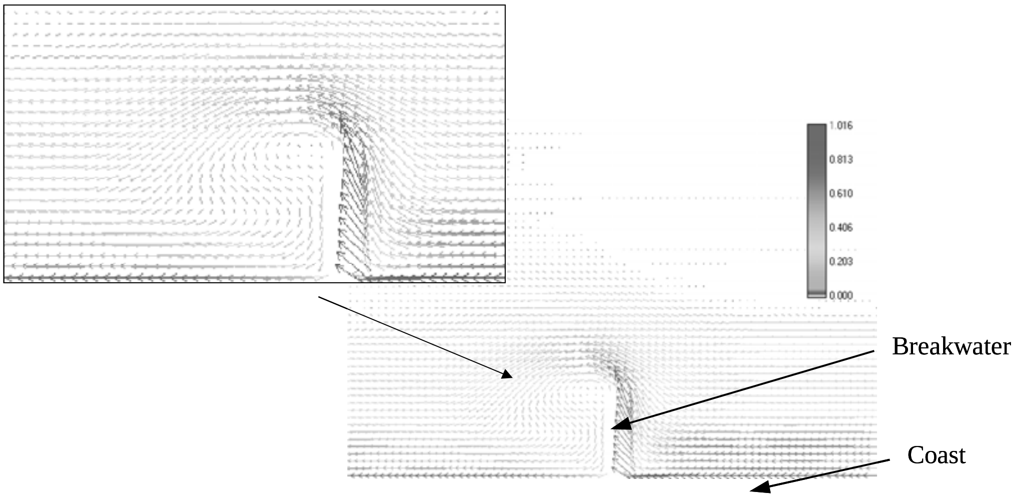

Fig. 3: Wave induced currents velocities (m/s).

4 . SEDIMENT TRANSPORT

4.1 General principle

The formation of a beach results mainly from the sediment amount transported by the rivers or transported by the marine erosion of the rock coasts. The balance between the sediment deposition and their redistribution by the coastal currents determines the stability of a beach, while the imbalance in favour of one of these two factors causes respectively the fattening or the erosion of a littoral. Currently the majority of the beaches goes through a phase of erosion. This problem represents a threat for the tourist development, but also a strong damage for the permanence of the infrastructures. It consequently seems useful to be interested more specifically in the phenomenon of sediment transport and the resulting topographic variation.

We use the idea that the topographic variation is proportional to the difference between:

- shear stresses developed by the fluid in a reference state,
- and shear stresses developed by the fluid for a beach with a breakwater:

$$\Delta z_b = \gamma(\tau_* - \tau_{*cr})^{\xi} \tag{10}$$

with z_b the bottom elevation and γ a proportionality coefficient. This principle was proposed by many authors (e.g. Paquier (Paquier 2002)).

This sediment part completes the global scheme of resolution for the calculation of topographic variation caused by the presence of waves. This one is based on a quasi-steady approach including the three parts described previously (Fig. 4). Wave parameters are obtained by the first model of wave propagation and allow the evaluation of source terms for the SWE. Their solution gives the hydrodynamic solution, which is employed to determine sediment transport and then the subsequent topographic variation. This scheme is applied until a steady state is reached.

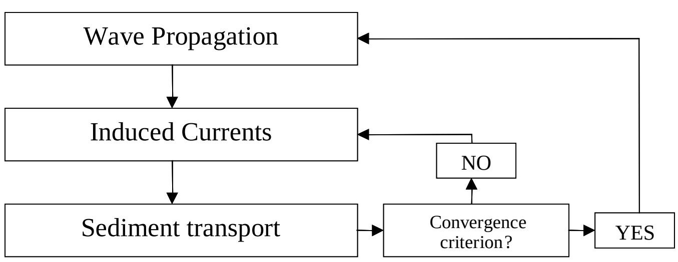

Fig. 4: Global scheme of resolution to evaluate coastal topographic modifications

4.2 Global application of the model

The application presented here consists in determining, by the successive calculation of the wave parameters, the induced currents and the sediment transport, the variation of the funds around a maritime work, and more particularly around a breakwater perpendicular to the coast. The depth increases with the distance to the coast. The Fig. 5 represents the bottom topography initially and after the fifth sediment transport computation. We can observe a sediment deposition behind the structure in the area of currents recirculation. An important area of erosion is formed at the head of the breakwater. This one can represent a danger for the stability of the beach in case of strong currents. In addition, a channel of erosion by-passing the structure is formed, representing the principal way of the currents. With the number of iterations fixed, the maximum topographic variation decreases with time until to reach a steady-state.

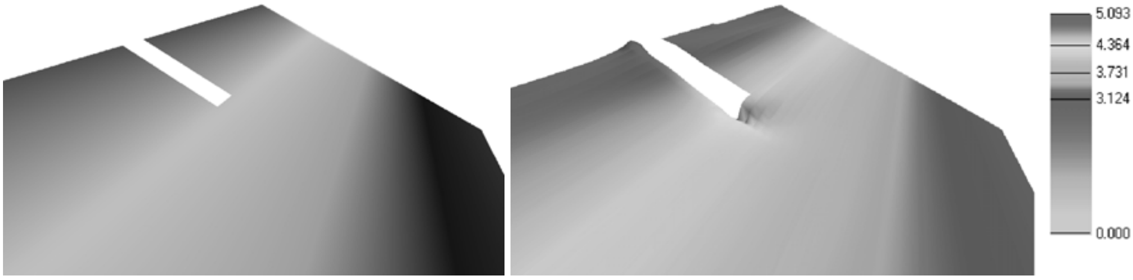

Fig. 5: 3D-view of the bottom topography of a beach with a breakwater perpendicular to the coast initially (left) and after convergence of the process (right).

5 . CONCLUSION

The natural phenomenon of the evolution of the coastal sea-beds depends on a complexity and an extraordinary variety, considering the multitude and the diversity of the independent variables concerned. This paper allowed, on the basis of adequate simplifications in agreement with the description of the studied system, to release a model of behavior of these marine environments which is based on three principal modelings. The first provides a description of the swell during its movement, the second allows the evaluation of the wave induced littoral currents, the third treats of the evolution of the coastal sea-funds following the evaluation of sediment transport.

The mathematical developments carried out in the first part made it possible to set up a model of wave propagation taking simultaneously into account refraction and diffraction. The evolution of the waves close to the beaches and along maritime works also suggested us to introduce reflection. The equation of propagation as initially established did not take into account the energy dissipation and more particularly that caused by the wave breaking. However, to study the erosion of the beaches without having a model providing a rational description of the wave characteristics along the coast would not be of any interest. By using the idea of an energy equivalence between the phenomenon of wave breaking and the hydraulic jump, we then established the function of energy dissipation due to breaking according to the wave characteristics and improved the model so that it became dissipative. The numerical resolution of this mathematical model was carried out with an approach by finite volumes. An implicit scheme of resolution was selected; however, the methods of direct resolution of great linear systems being extremely expensive in computing time, an iterative method was privileged, the GMRES method. The complete model was implemented in the FORTRAN language as a module in the software WOLF. This model was tested and largely validated on various examples with analytical solutions or treated by authors in the recent literature, such as harbors of various geometries and depths, beaches with or without maritime works and proved its efficiency in the more complex case of the Calvi bay in Corsica.

The observation of coastal phenomena reveals the existence of a major current in the littoral zone, parallel to the coast, induced mainly by the wave breaking. The second part sets up a mathematical model for the evolution of the littoral flow subjected to an incident wave. This one is based on the principle of the radiation stresses which express the excess of flux momentum generated by the presence of waves. A complete model was used, based on the time and depth averaged Navier-Stokes equations completed with adequate forcing terms coming from the first model. The computing of this hydrodynamic model was carried out by completion of the computation software WOLF2D in full adequacy with this phenomenon. The graphic interface for the pre- and post-processing constitutes an useful tool for the data processing and the visualization of the results. This model was applied successfully to some academic cases, in particular a uniform beach with or without maritime structures to ensure its protection against erosion.

The third and last part of this paper proposes a study of the erosion undergone by sea-beds and of the topographic variation which results from it. A general formula expressing the sediment transport as a function of the tension developed by the flow was used, it allows a qualitative description of the sediment erosion and deposition. An adapted sediment model thus completes the scheme of resolution.

The use in loop of the three models proposes an operational tool to highlight the evolution of sea-beds at the neighbourhoods of a beach subjected to an incident wave, involving littoral currents and from there, a transport of sediments. The global approach consists in a quasi-stationary resolution, i.e. in a sequential calculation of the wave characteristics, the currents and the resulting erosion.

The structuring in three distinct parts also opens the way to multiple extensions, as well on the level of more sophisticated waves description as on the hydrodynamics by the implementation of turbulence or on the sediment part by taking into account the suspension

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