Toward stochastic multi-scale methods in continuum solid mechanics

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Corrections

- Equation (44) currently reads

$$\begin{aligned}&\mathbb{E}\left[\boldsymbol{Q}(\boldsymbol{x}^{(1)})\right] = \mathbb{E}\left[\boldsymbol{Q}(\boldsymbol{x}^{(2)})\right]; \quad \text{and} \\ &\mathbb{E}\left[\left(\boldsymbol{Q}(\boldsymbol{x}^{(1)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(1)})]\right)\left(\boldsymbol{Q}(\boldsymbol{x}^{(1)}+\tau\boldsymbol{n}^{(1)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(1)}+\tau\boldsymbol{n}^{(1)})]\right)^T\right] = \\ &\mathbb{E}\left[\left(\boldsymbol{Q}(\boldsymbol{x}^{(2)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(2)})]\right)\left(\boldsymbol{Q}(\boldsymbol{x}^{(1)}+\tau\boldsymbol{n}^{(1)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(1)}+\tau\boldsymbol{n}^{(1)})]\right)^T\right],\end{aligned} \tag{44}$$

  should read

$$\begin{aligned}&\mathbb{E}\left[\boldsymbol{Q}(\boldsymbol{x}^{(1)})\right] = \mathbb{E}\left[\boldsymbol{Q}(\boldsymbol{x}^{(2)})\right]; \quad \text{and} \\ &\mathbb{E}\left[\left(\boldsymbol{Q}(\boldsymbol{x}^{(1)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(1)})]\right)\left(\boldsymbol{Q}(\boldsymbol{x}^{(1)}+\tau\boldsymbol{n}^{(1)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(1)}+\tau\boldsymbol{n}^{(1)})]\right)^T\right] = \\ &\mathbb{E}\left[\left(\boldsymbol{Q}(\boldsymbol{x}^{(2)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(2)})]\right)\left(\boldsymbol{Q}(\boldsymbol{x}^{(2)}+\tau\boldsymbol{n}^{(2)}) - \mathbb{E}[\boldsymbol{Q}(\boldsymbol{x}^{(2)}+\tau\boldsymbol{n}^{(2)})]\right)^T\right],\end{aligned} \tag{44}$$

- On page 25, below Eq. (64), "$\int_{\delta\mathcal{S}_{\boldsymbol{H}}} d\boldsymbol{h} = d(\mathcal{S}^{(i)}_{\boldsymbol{H}})/N^{\boldsymbol{h}}$" should read "$\int_{\delta\mathcal{S}^{(i)}_{\boldsymbol{H}}} d\boldsymbol{h} = d(\mathcal{S}_{\boldsymbol{H}})/N^{\boldsymbol{h}}$"

- Equation (A.122) currently reads

$$\begin{aligned}S_{r\,s}\left(\boldsymbol{\kappa}^{(m_x\,m_y\,m_z)}\right) &= \sum_{n_x=0}^{2N_x-2}\sum_{n_y=0}^{2N_y-2}\sum_{n_z=0}^{2N_z-2} \tilde{R}_{r\,s}\left(\boldsymbol{\tau}^{(n_x\,n_y\,n_z)}\right) e^{-2\pi i\boldsymbol{\kappa}^{(m_x\,m_y\,m_z)}\cdot\boldsymbol{\tau}^{(n_x\,n_y\,n_z)}} \\ &= \sum_{n_x=0}^{2N_x-2}\sum_{n_y=0}^{2N_y-2}\sum_{n_z=0}^{2N_z-2} \tilde{R}_{r\,s}\left(\boldsymbol{\tau}^{(n_x\,n_y\,n_z)}\right) e^{-2\pi i\left(\frac{m_x n_x}{2N_x-1}+\frac{m_y n_y}{2N_y-1}+\frac{m_z n_z}{2N_z-1}\right)},\end{aligned} \tag{A.122}$$

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but it should read

$$
\begin{aligned}
&S_{r\,s}\left(\boldsymbol{\kappa}^{(m_x\,m_y\,m_z)}\right) \\
&\quad = \Delta\tau_x\Delta\tau_y\Delta\tau_z \sum_{n_x=0}^{2N_x-2}\sum_{n_y=0}^{2N_y-2}\sum_{n_z=0}^{2N_z-2} \tilde{R}_{r\,s}\left(\boldsymbol{\tau}^{(n_x\,n_y\,n_z)}\right) e^{-2\pi i \boldsymbol{\kappa}^{(m_x\,m_y\,m_z)}\cdot\boldsymbol{\tau}^{(n_x\,n_y\,n_z)}} \\
&\quad = \Delta\tau_x\Delta\tau_y\Delta\tau_z \sum_{n_x=0}^{2N_x-2}\sum_{n_y=0}^{2N_y-2}\sum_{n_z=0}^{2N_z-2} \tilde{R}_{r\,s}\left(\boldsymbol{\tau}^{(n_x\,n_y\,n_z)}\right) e^{-2\pi i\left(\frac{m_x n_x}{2N_x-1}+\frac{m_y n_y}{2N_y-1}+\frac{m_z n_z}{2N_z-1}\right)} ,
\end{aligned}
\tag{A.122}
$$

- On page 216, Appendix A.6.2, the sentence "generate the Gaussian pseudo-samples $\{\boldsymbol{q}^{\eta^p}(\boldsymbol{x}^{(i)})\}$ of the Gaussian field $\boldsymbol{Q}^{\eta}(\Omega)$ from Eq. (A.124) using as spectrum $\mathbf{S}^{\eta^{(k)}}(\boldsymbol{\kappa})$ (and not the continuous form);" should read "generate the Gaussian pseudo-samples $\{\boldsymbol{q}^{\eta^p}(\boldsymbol{x}^{(i)})\}$ of the Gaussian field $\boldsymbol{Q}^{\eta}(\Omega)$ from Eq. (A.124) using as spectrum $\mathbf{S}^{\text{cont}\,\eta^{(k)}}(\boldsymbol{\kappa})$ (in the continuous form);"

- Equation (A.129) currently reads

$$
\mathbf{S}^{\text{cont NG}}(\boldsymbol{\kappa}) = \frac{1}{N^{\boldsymbol{x}}}\bar{\hat{\boldsymbol{Q}}}^{\text{NG}}(\boldsymbol{\kappa})\left(\hat{\boldsymbol{Q}}^{\text{NG}}(\boldsymbol{\kappa})\right)^{T} , \tag{A.129}
$$

but it should read

$$
\mathbf{S}^{\text{cont NG}}(\boldsymbol{\kappa}) = \frac{\Delta\tau_x\Delta\tau_y\Delta\tau_z}{N^{\boldsymbol{x}}}\bar{\hat{\boldsymbol{Q}}}^{\text{NG}}(\boldsymbol{\kappa})\left(\hat{\boldsymbol{Q}}^{\text{NG}}(\boldsymbol{\kappa})\right)^{T} , \tag{A.129}
$$