Null-Subspace Analysis for Structural Damage Monitoring in Fatigue Testing of Luminaires

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Abstract

A damage detection method of mechanical system based on output measurements only and on the subspace identification concept is presented. The method uses the definition of null subspace analysis of the Hankel matrix. It is demonstrated to be sensitive to small-size structural damages and suitable for continuous on-line monitoring. The measured response records of structures under environmental or artificial vibrations are assembled in the Hankel matrix that is further factorized into special subspaces. The active subspace defined by the first principal components is shown to be orthonormal to the null subspace defined by the remaining principal components. Therefore, any evident increase of the residues signifies possible structural damages. The method is illustrated by an application to fatigue tests of a street lighting device (luminaires). Experimental methods and numerical calculations on luminaires are simply described and discussed. The test results show that the developed methods may be easily used to effectively monitor the damage evolution during the fatigue tests.

1 Introduction

Many engineering devices are subjected, in operation or during transportation, to various types of excitation such as harmonic rotation, pulsating or oscillating forces, or random forces (e.g. wind, transportation, etc). These vibrations may cause structural damages. For example, fatigue effects due to ambient vibrations of long duration are the main cause of failures in street (or highway) lighting devices (luminaires) [1]. The manufacturers are very concerned with vibration testing of the prototypes in order to ensure that the structures can withstand the expected vibration environments during their operation lifetime.

Structures are tested by different kinds of excitation according to the qualification standards, such as sine sweep, sine dwell, random, shock or combined excitations. The vibration signals of the structure are recorded during the test using accelerometers or strain gauges. Structural damages can then be monitored by comparing the vibration features identified at different testing stages. Usually, in the classical testing approach frequency spectra are used as vibration features. In practice, this methodology has some disadvantages such as the difficulty to manage the large number of data and the fact that spectra comparison are generally performed after the qualification test.

The aim of this paper is to propose a means to monitor the structural integrity during the qualification test. The proposed damage indicators should be able to state if structural damages are occurring, and then if necessary, one may abort the test before its completion to reduce the testing cost. The developed method is based on the subspace identification concept. The responses of the structure that is excited by a shaker (or by a natural vibration source in a general application), are continuously or periodically measured by several sensors (e.g. accelerometers, strain gauges, etc.) installed at appropriate locations. The data are used to construct the Hankel matrices. It is assumed that if no structural damage occurs, the orthonormality of the subspaces of the Hankel matrices between different data sets should approximately remain valid with small residues that may serve as damage-sensitive features. Training of the residues in health state but possibly in different levels of excitation provides a critical limit of damage assessment. Subsequent data are examined to detect if the features deviate significantly from the norm.

In developing an on-line monitoring tool, both efficiency and simplicity are pursued. There are large amounts of researches on the subject. Here we cite only ref. [2-12] as examples. In fact the present work may be seen as a continuity and development of these previous works and consists of a practical application of [2] in structural fatigue testing.

2 Damage detection by null subspace analysis of Hankel matrix

Let us consider that structural vibration responses are periodically measured by m accelerometers or strain gauges, leading to a series of output records of N data points. Each data set is collected in matrix $\mathbf{Y}=\{\dots\ \mathbf{y}_k \dots\}\in\Re^{m\times N}$, k=1…N. In the case of continuous measurement, one may separate the records into a series of data sets in a chosen time interval. Starting from any data set, we construct the Hankel matrix $\mathbf{H}_{p,q}$ filled up with p block rows and q block columns of the output covariance matrix $\boldsymbol{\Lambda}_i$:

$$\mathbf{H}_{p,q} = \begin{bmatrix} \Lambda_0 & \Lambda_1 & \dots & \dots & \Lambda_{q-1} \\ \Lambda_1 & \Lambda_2 & \dots & \dots & \Lambda_q \\ \dots & \dots & \dots & \dots & \dots \\ \Lambda_{p-1} & \Lambda_p & \dots & \dots & \Lambda_{p+q-2} \end{bmatrix} ;\quad \mathrm{q} \geq \mathrm{p} \tag{1}$$

where p and q are user-defined parameters (in this work we take $p=q$). The output covariance matrices $\boldsymbol{\Lambda}_i$ are approximately estimated in the following way:

$$\Lambda_i \approx \frac{1}{N-i} \sum_{k=1}^{N-i} \mathbf{y}_{k+i}\, \mathbf{y}_k^{\mathrm{T}} \tag{2}$$

As known from the control theory, the Hankel matrix constructed by covariance matrices, contains all the modal information of the structure, from which the state matrices may be extracted and then modal parameters (natural frequencies, damping ratios and mode shapes) may be identified. From the point of view of damage detection, we are not concerned with the precise values of modal parameters or other structural features. Instead, only relative change of the features is necessary to provide structural damage information. The proposed damage detection method is based on the null subspace concept [13] of the Hankel matrices.

Performing the singular value decomposition (SVD) of the weighted Hankel matrix $\overline{\mathbf{H}}$ leads to :

$$\overline{\mathbf{H}} = \mathbf{W}_1\ \mathbf{H}_{p,q}\mathbf{W}_2 \ \approx\ \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix} \begin{bmatrix}\mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix} \begin{bmatrix}\mathbf{V}_1 & \mathbf{V}_2\end{bmatrix}^{\mathrm{T}} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^{\mathrm{T}} \tag{3}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ are invertible weighting matrices (both are chosen as identity matrix in this work); diagonal matrix $\mathbf{S}_1$ contains $n \approx 2N_m$ non-zero singular values sorted in decreasing order (N_m is the number of modes). Equation (3) allows to write the following relations :

$$\mathbf{U}_1^{\mathrm{T}}\overline{\mathbf{H}}\mathbf{V}_1 \ \approx \mathbf{S}_1 \tag{4}$$

$$\overline{\mathbf{H}}\mathbf{V}_2 \approx \mathbf{0} \ \text{ or }\ \overline{\mathbf{H}}\mathbf{V}_2 = \zeta_v \tag{5}$$

$$\mathbf{U}_2^{\mathrm{T}}\overline{\mathbf{H}} \approx \mathbf{0} \ \text{ or }\ \mathbf{U}_2^{\mathrm{T}}\overline{\mathbf{H}} = \zeta_u \tag{6}$$

It is shown that performing SVD on the weighted Hankel matrix $\overline{\mathbf{H}} \in\Re^{r\times c}$, where $r=m\times p$ and $c=m\times q$, leads to four fundamental subspaces: $\mathbf{U}_1$ contains the maximum number (i.e. n) of independent column vectors that span the column space of $\overline{\mathbf{H}}$; $\mathbf{V}_1^{\mathrm{T}}$ contains the maximum number (n) of independent row vectors that span the row space of $\overline{\mathbf{H}}$; $\mathbf{U}_2$ contains the maximum number (c-n) of independent column vectors that span the column null space of $\overline{\mathbf{H}}$; $\mathbf{V}_2^{\mathrm{T}}$ contains the maximum number (r-n) of independent row vectors that span the row null space of $\overline{\mathbf{H}}$. In almost all practical cases, eq.(3) may only be approximately applied, so do eqs.(4-6), due to the fact that the exact order n of the system is difficult to determine. Generally, the residue matrix ζ is mainly due to the noise effects and the weakly-excited high modes that are neglected by cutting off with a chosen order n. Sometimes, its variation from different data sets may be large enough to mask the variation due to small structural damages. Therefore it is difficult to directly use this residue for damage detection.

On the other hand, due to orthonormality property of matrices $\mathbf{U}$ and $\mathbf{V}$, we have always exactly in mathematical sense the following equations for any data set :

$$\mathbf{U}_2^{\mathrm{T}}\mathbf{U}_1 = \mathbf{0} \tag{7}$$

or

$$\mathbf{U}_2^{\mathrm{T}}\left(\mathbf{U}_1\mathbf{S}\mathbf{V}_1^{\mathrm{T}}\right) = \mathbf{0} \tag{8}$$

$$\mathbf{V}_1^{\mathrm{T}}\mathbf{V}_2 = \mathbf{0} \tag{9}$$

or

$$\left(\mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^{\mathrm{T}}\right)\mathbf{V}_2 = \mathbf{0} \tag{10}$$

From the numerical point of view, we are using the active parts (with the chosen order n) of column or row spaces in the above calculations instead of whole Hankel matrix. Unlike (5-6), equalities (7-10) do not depend on the precise determination of system order n.

In the following, we will consider only the left kernel relations (7-8) of the Hankel matrix in the damage detection analysis**.** It may be shown that $\mathbf{U}_1$, containing the first n active principal components, represents a generalized subspace (also called hyperplane), around which the response data locate [2, 9]. Without damage or environmental variation, this subspace of the Hankel matrix should remain unchanged (i.e. no rotation of subspace occurs) and the orthonormality (7-8) apply between different data sets. Therefore, observing the rotation of this subspace or changes in orthonormality between different data sets may obtain damage information of structures. For convenience of description, hereafter we will call $\mathbf{U}_1$ and $\mathbf{U}_2$, respectively, column active subspace and column null subspace of the weighted Hankel matrix $\overline{\mathbf{H}}$.

The same procedure may be considered when the Hankel matrix is constructed by the response data block instead of data covariance matrices:

$$\mathbf{H}_{1,i} = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_j \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_{j+1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_i & \mathbf{y}_{i+1} & & \mathbf{y}_N \end{bmatrix} \tag{11}$$

where i is the user-defined number of row blocks, and j the number of columns taking practically j=N-i+1. By (11), it may be demonstrated that we are using a dynamic model that contains richer information than the classical model used in principal component analysis. This issue has been discussed in our previous work [2,9]. In the present context, dynamic modeling means using a time-shift output record in the analysis: not only looking at the vibration signal at the same instant but also at other time instants. Using eq.(1) consists of another type of dynamic modeling.

As only the left kernel of the Hankel matrix will be considered, SVD is performed on the covariance of the Hankel matrix:

$$\overline{\mathbf{H}}_i\overline{\mathbf{H}}_i^{\mathrm{T}} \approx \begin{bmatrix}\overline{\mathbf{U}}_1 & \overline{\mathbf{U}}_2\end{bmatrix} \begin{bmatrix} \overline{\mathbf{S}}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix}\overline{\mathbf{U}}_1 & \overline{\mathbf{U}}_2\end{bmatrix}^{\mathrm{T}} = \overline{\mathbf{U}}_1\overline{\mathbf{S}}_1\overline{\mathbf{U}}_1^{\mathrm{T}} \tag{12}$$

where $\overline{\mathbf{H}}_i = \mathbf{W}_1\mathbf{H}_{1,i}\mathbf{W}_2$; (both weight matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are chosen as identity matrix in this work). As the dimension of $\overline{\mathbf{H}}_i\overline{\mathbf{H}}_i^T$ is much smaller than original $\overline{\mathbf{H}}_i$, this modification reduces much the SVD computational cost. By a same analysis as with the COV-driven Hankel matrix, we have following the expressions similar to (7-8):

$$\overline{\mathbf{U}}_2^{\mathrm{T}}\overline{\mathbf{U}}_1 = \mathbf{0} \tag{13}$$

$$\overline{\mathbf{U}}_2^{\mathrm{T}}\left(\overline{\mathbf{U}}_1\overline{\mathbf{S}}_1\overline{\mathbf{U}}_1^{\mathrm{T}}\right) = \mathbf{0} \tag{14}$$

Again, equalities (13-14) do not depend on the chosen system order *n*. As (7-8) and (13-14) have completely similar form, the following discussion is suitable to both types of Hankel matrix.

When two different reference data sets are examined: one for column active subspace $\mathbf{U}_1$ or active Hankel matrix ($\mathbf{U}_1\mathbf{S}_1\mathbf{U}_1^{\mathrm{T}}$), another for column null subspace $\mathbf{U}_2$, equalities (7-8) or (13-14) no longer strictly apply due to the noise effects and other error sources (e.g. variation of the excitation). Therefore a series of tests in structural healthy state should be performed to provide a reliable reference residue. It is expected that tests may be performed in different excitation levels because in most practical situations, the natural exciting forces may change from time to time although they may be thought approximately stationary during a short acquisition period. Assuming that column null subspace $\mathbf{U}_{2,0}$ is determined by reference data (indicated by index 0), and column active subspace $\mathbf{U}_{1,r}$ (and $\mathbf{S}_{1,r}, \mathbf{V}_{1,r}$) by *r*-th current data set, the residue matrices corresponding to (7-8) or (13-14) are

$$\delta_r = \mathbf{U}_{2,0}^{\mathrm{T}}\mathbf{U}_{1,r} \tag{15}$$

$$\Delta_r = \mathbf{U}_{2,0}^{\mathrm{T}}\left(\mathbf{U}_{1,r}\mathbf{S}_{1,r}\mathbf{V}_{1,r}^{\mathrm{T}}\right) \tag{16}$$

When using the Data-driven Hankel matrix, $\mathbf{V}_{1,r}$ is replaced by $\mathbf{U}_{1,r}$. The residue matrix δ_r (15) represents an orthonormality change between the subspaces of responses due to the noise effects and/or mainly due to structural damages if any. In contrast with (15), the residue matrix Δ_i (16) represents similar orthonormality change but weighted by active singular matrix $\mathbf{S}_{1,r}$ and right kernel matrix $\mathbf{V}_{1,r}^{\mathrm{T}}$. Both residue matrices (15-16) may be candidates as damage-sensitive features. From the numerical point of view, however, the former seems better than the latter due to the fact that, although the effect of $\mathbf{V}_{1,r}^{\mathrm{T}}$ is not clear, $\mathbf{S}_{1,r}$ decreases generally with damages and this may reduce the sensitivity of residue indicator to damages (if the measured data are not well normalized). Nevertheless, it may be shown that when (16) is adopted for damage detection, the present method may be related to the method proposed in [3].

From the orthonormality of subspaces in (15), we may propose to take the complementary angle between subspace $\mathbf{U}_{2,0}$ and $\mathbf{U}_{1,r}$, defined as below, as damage indicator:

$$\alpha_r = \sin^{-1}\left[\mathrm{norm}(\delta_r)\right] \tag{17}$$

where norm(.) is an operator giving the maximal singular value of a matrix. Obviously, the value of α remains in the range (0-90°). Without the orthonormality property in (16), we may use a corresponding norm value as damage indicator.

$$\sigma_i = \mathrm{norm}\left(\Delta_i\right) \tag{18}$$

Additionally, we have proposed in [2] another damage indicator represented as x^2 for characterizing (7) and (13), or x^2_H for characterizing (8) and (14), which is somehow more sensitive to local variations in the residue matrices.

3 Vibration testing methods of luminaires

During their lifetime, luminaires are subject to environmental excitations due to traffic and wind turbulences. Fatigue effects due to ambient vibrations of long duration are the main cause of structural failures in outdoor pole-mounted luminaires.

Up to now, qualification tests are performed according to different standards which are not specific to street lighting devices. These standards, summarized below, do not have the same severity so that the choice of one standard rather than another is not obvious.

- **The International Electro-technical Commission (IEC) 68-2-6 Standard**

The object of this standard is to provide a standard procedure to determine the ability of components, equipment and other articles to withstand specified severities of sinusoidal vibration over a given frequency range or at discrete frequencies for a given period of time [14]. The requirements for the vibration motion, choice of severities including frequency ranges, amplitudes and endurance times are specified; these severities represent a rationalised series of parameters. The relevant specification writer is expected to choose the testing procedure and values appropriate to the specimen. An example of test following IEC 68-2-6 requirements is given in Table 1.

- **The American National Standard for Roadway Lighting (ANSI) C 136-31 Standard**

The USA ANSI C 136-31 standard (2001) proposes that a requirement for a minimum vibration withstanding capability be considered for luminaires for road and street lighting. According to the proposal, there are factors that may cause externally induced vibration effects which may not be adequately covered by the application of a static load test. For this reason, a vibration test might serve as a more appropriate and suitable substitute. This new standard suggests that :

- the fundamental resonant frequency must be determined for each of the three perpendicular directions and must be in the frequency range between 5 and 30 Hz;
- the luminaire must be vibrated at or near his natural frequency;
- the acceleration intensity measured at the luminaire centre of gravity must be 1.5 g for normal roadway applications and 3 g for bridge and overpass applications;
- the lighting device must be capable of withstanding the described vibration for 100 000 cycles in each plane.

- **A Belgian standard**

In March 1999, the Ministry of the Flemish Community made a test proposal following the IEC 68-2-6 prescriptions to qualify lighting devices [15]. The proposal suggests that :

- the natural frequencies f_0 and quality factors Q should be identified by a sine sweep (sweep rate 1 oct./min. and amplitude 0.5 g) in the frequency range [5-25] Hz. The quality factor Q associated with the natural frequency f_0 is given by

$$Q = \frac{f_0}{\Delta f\Big|_{\frac{|H|_{max}}{\sqrt{2}}}} \tag{19}$$

  with $\Delta f=f_2-f_1$ measured at the point of half power of the Frequency Response Function H;
- if several natural frequencies are identified, only the one with the most important quality factor should be taken into account;
- if Q>2, the luminaire should be tested at the retained natural frequency during 30 minutes with a sinusoidal amplitude of 0.5 g;
- if Q<2, the luminaire should be tested during 1 hour in the sinusoidal frequency range [5-25] Hz with a sweep rate of 1 oct./min. (the proposal does not define the amplitude of the sine sweep);
- the control point should be as close as possible to the fixing point;
- the test should be performed along each structural axis.

Table 1 summarizes the standards used for the vibration testing of luminaires.

Standard	Excitation	Amplitude	Duration
IEC 68-2-6	sine sweep [10-55-10] Hz	0.15 mm fixing point	100 sweeps
ANSI C 136-31	sine $f_0 \in$ [5-30] Hz	1.5 (3) g centre of gravity	100 000 cycles
Belgian project	sine $f_0 \in$ [5-25] Hz	0.5 g fixing point	30 minutes

Table 1 : Possible standards and parameters for the vibration testing of luminaires

A methodology has been developed in [16-17] in order to simulate and quantify the severity of different vibration environments. To this end, different severity criteria were defined in the simple case of a one-degree-of-freedom reference system (e.g. maximax response spectrum, fatigue damage spectrum or dissipative damage spectrum) before being generalized to multi-degree-of-freedom systems using a more representative finite element approach [0], [17]. To validate the methodology and perform experimental tests, the luminaire 'Super Saturne 400 Watt' shown in Figure 1. 1has been chosen.

The first step of the methodology consists to generate a finite element model (Figure 1) using a computer aided design. In the second step, the modal damping ratio is first identified by a low-level-excitation modal analysis and then optimized to fit the computed FRF (Frequency Response Functions) with the experimental ones at the excitation level prescribed by the standard. The third step consists in computing the dynamic response of the luminaire with each of the presented standards. The obtained Von Mises stress distribution in the structure allows the designer to locate the region of stress concentration for the excitation frequency. Figure 2 shows that the maximum stresses occur near the attachment point. It is in this area that cracks are expected to initiate when a fatigue test is performed.

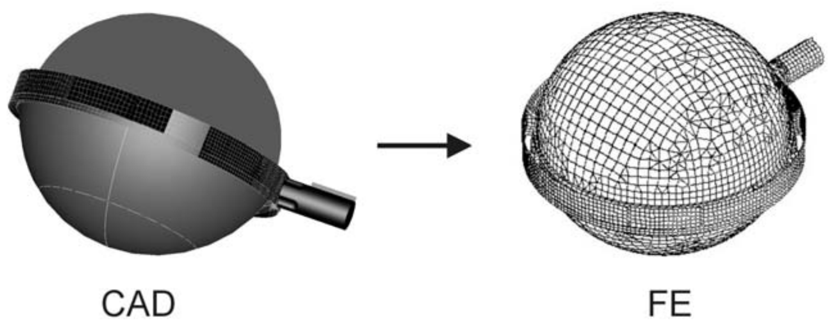

Figure 1 : CAD to FE model of the luminaire 'Super Saturne 400 Watt'

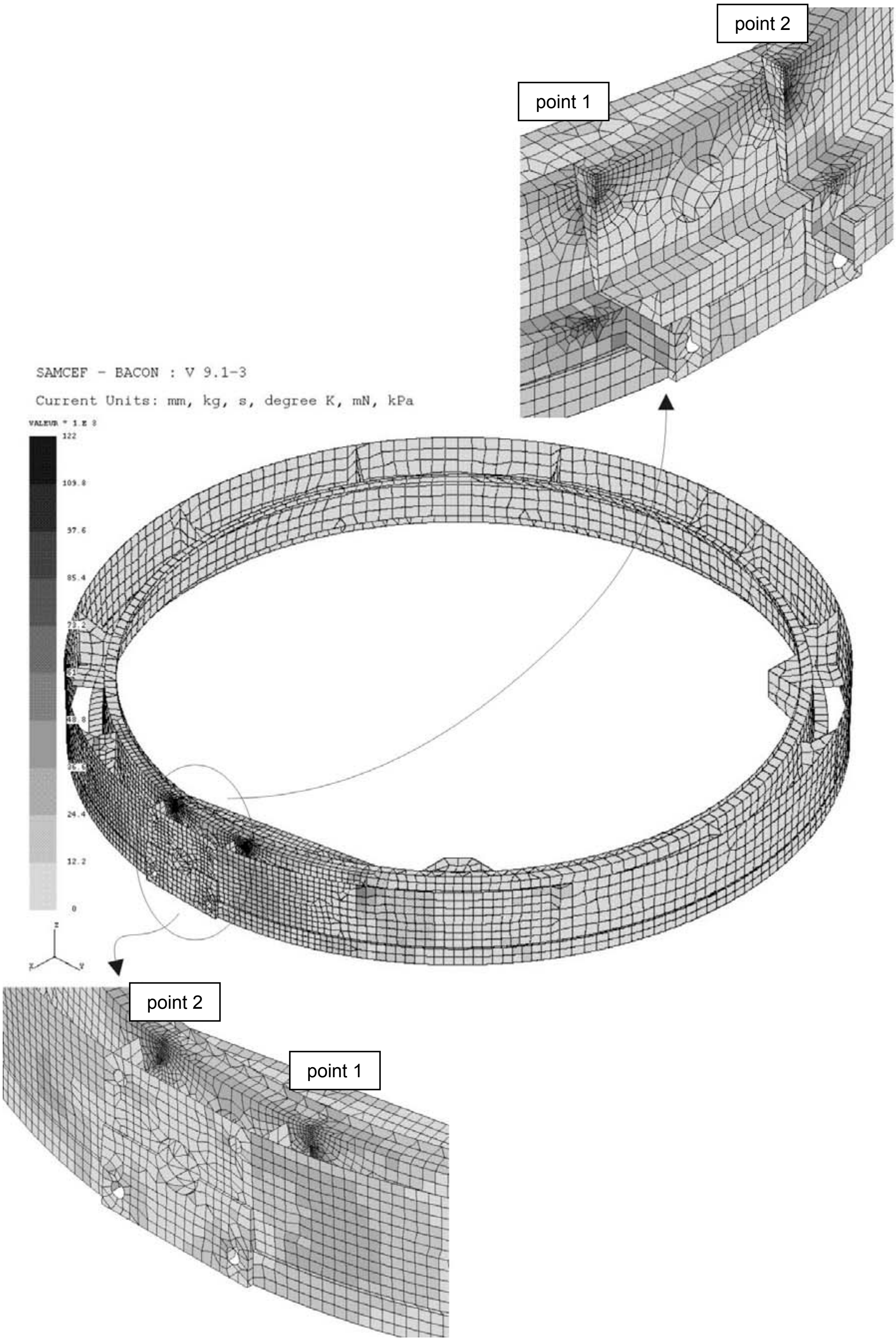

Figure 2 : Von Mises stress distribution in the main part of the luminaire (Belgian standard – vertical axis – excitation at f_{01} = 12.8 Hz)

4 Application to fatigue testing of luminaires

Fatigue tests were performed on the luminaire "Super Saturne 400 Watt" using an electro-dynamic shaker. The test set-up is shown in Fig. 3. The developed null subspace analysis (NSA) method is applied to monitor its damage evolution during tests. In this work, we report only tests with the Belgian standard project, i.e. the luminaire is vibrated at (in fact close to) its first natural frequency with a sine excitation. In this case, we may easily compare the damage monitoring indicators with the recorded natural frequencies during the test. Note that during the test, the natural frequencies may vary, and it decreases due to gradually growing damages; the control system of the shaker allows to follow this variation.

In order to experimentally validate the NSA-based damage detection method, the limunaire was over-tested with respect to the Belgian standard. Accordingly, the acceleration level at the attachment (4.905 m/s^2) was replaced by a higher value (6 m/s^2) so that accumulated damages lead to failure. Seven accelerometers and four strain gauges were installed at appropriate locations to continuously measure structural vibration with a sampling frequency of 256 Hz. After about 4 hours of test, the first observable damage occurs at point 1. The test stopped after 32 hours of test when a second main crack at point 2 passed over all the thickness of the ring (Fig. 4). The NSA was performed usually every 30 minutes with each data set of 8192 sampling points.

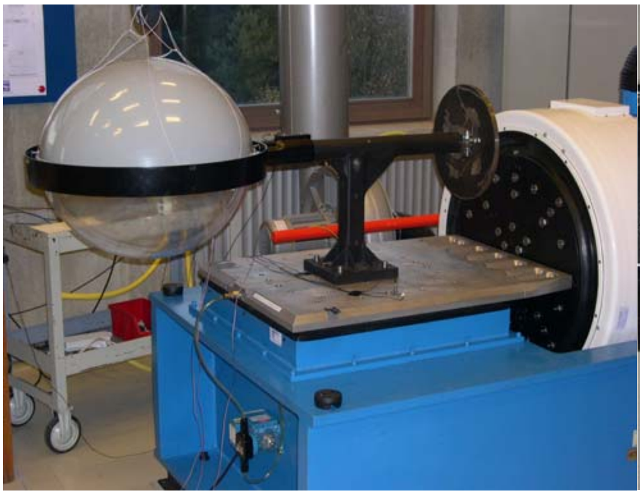

Fig. 3: Vibration fatigue test set-up of luminaire

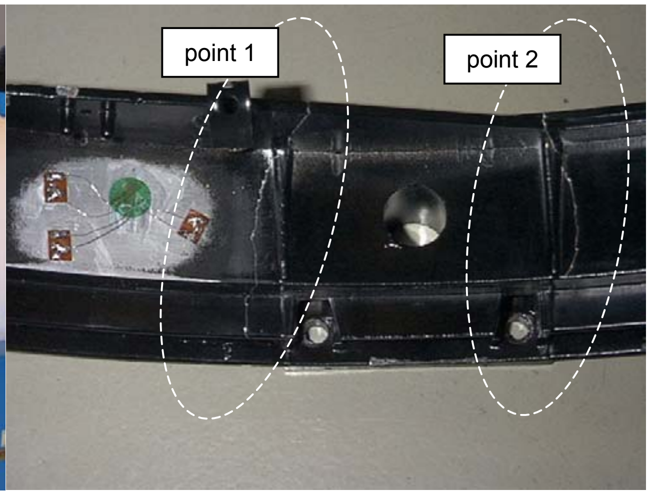

Fig. 4 : Fatigue failure occurring near attachment

Calculations with different data types (acceleration or stress) and different calculating parameters give similar monitoring results. Here, we present first the monitoring calculation using the measured data of 4 strain gauges. The calculating parameters are given in Fig. 5 where a detailed comparison of damage monitoring by two ways is shown. The top part of the figure shows the time evolution of the identified natural frequency along the fatigue test. The lower part of the figure presents the monitoring results obtained by the developed NSA method. The test was divided into 5 runs for a total duration of about 32 hours. The natural frequency of the first mode decreases continuously from the beginning of the test. The first event happens after about four hours: a crack appears at point 2 and the natural frequency goes down suddenly. After about 14 hours, the crack at point 2 propagates and goes through the inside face. A new decrease of the natural frequency is observed. At the last stage of the test, a crack appears at point 1 after 28 hours, and the natural frequency decreases quickly and continuously. When the natural frequency reaches 10 Hz, the test is aborted. It is very interesting to note that damage monitoring by NSA is very consistent with the evolution of the first natural frequency of the luminaire, and both agrees with the real observation on damages (cracks).

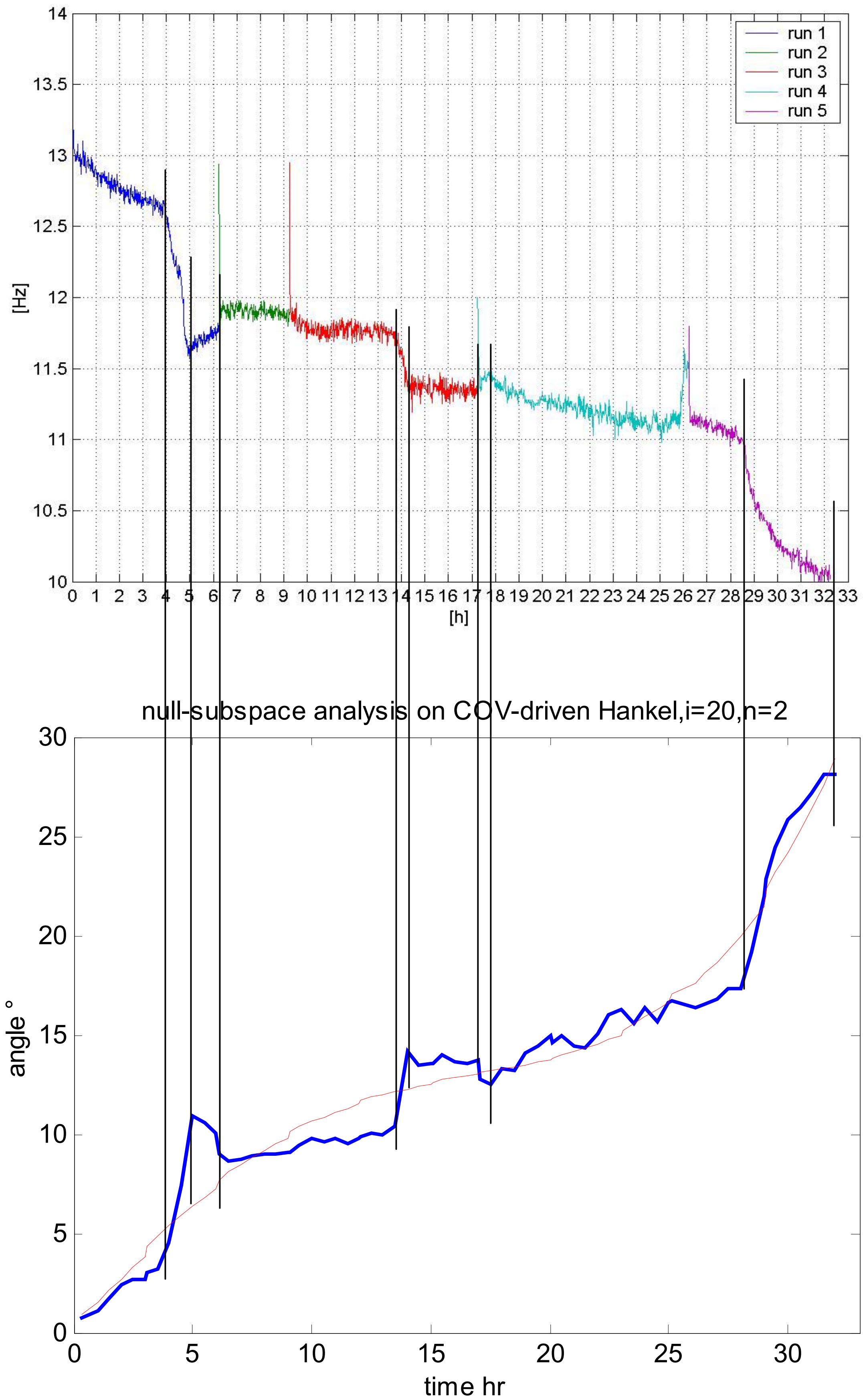

Fig. 5 : Comparison of damage monitoring

5 Conclusions

In this paper, a damage detection method based on subspace analysis of the Hankel matrices of measured vibration data has been presented. It is demonstrated that the column active subspace of a Hankel matrix defined by some first principal components is orthonormal to the column null subspace defined by the remaining principal components. An obvious increase of the residues of the orthonormality by different data sets (i.e. between the active subspace of reference data and the null subspace of current data) signifies possible damages in the structure. The angle between subspaces defined as damage indicator is well normalized in a finite value range (0-90°). The method uses time domain data (i.e. accelerations, stresses, etc.), and neither input measurement or finite-element model is needed. It may be applied to damage monitoring of structures under various environmental or artificial vibrations.

The method was applied to vibration fatigue tests of a street lighting device in laboratory. Failures (cracks) occur near the attachment of the luminaire, where the maximum stress and strain is located. Two methods were used to monitor the damage evolutions during the tests leading to very consistent results. Because of its simplicity and efficiency, the proposed NSA-based damage detection method is expected to be suitable for on-line health monitoring of structures.

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