Accelerated Fatigue Testing Methodology of Luminaires on Electro–Dynamic Shaker*

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Abstract

During their lifetime, luminaires are subject to excitations induced by their environment. Traffic and wind are common sources of vibration for street–lighting structures. Fatigue effects of long time ambient vibration are the leading cause of structural failures for outdoor pole mounted luminaires.
The concern of manufacturers is to test prototype designs in order to determine if they can support the vibration environment expected in service without being damaged. At the present time, qualification tests are performed according to standards of varying severity so that the choice of one rather than another is not well defined.
In comparison with the common standards, the advantage of the developed methodology is that it enables the manufacturer to optimise the luminaires design as it allows to guarantee the integrity of the structure during all its lifetime.

Keywords : luminaire, shaker, accelerated fatigue testing

1 Introduction

The objective of the paper is to present a complete methodology to determine accelerated fatigue testing of luminaires based on a model of the vibration environment. The first step consists in computing the spectral response of the pole–luminaire system to random excitations taking into account the effects of turbulent wind and Von Karman vortices. The second step deals with the definition of equivalence criteria : maximax response spectrum, fatigue damage spectrum or dissipative damage spectrum. To choose the most appropriate criterion, the identification and knowledge of failure processes are of prime importance. Finally, an iterative method, based on an optimisation process, minimises the difference between the reference spectrum and the spectrum of the test to be performed in laboratory. The design variables of the optimisation process can be the type of excitation, the test duration or the amplitude levels.

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2 Excitation of street–lighting structures

The object of this first step is to compute the dynamic response of the pole–luminaire system subject to random aerodynamic excitations (traffic effect is not taken into account). The mathematical model used for the simulation is linear and based on the finite element method [3]. The assumptions related to the modelling are the following :

- the pole behaves as a beam;
- the luminaire is considered as a rigid body and is modelled by a concentrated mass at its center of gravity.

The pole–luminaire system is described as shown in Fig.1, where the linear vibrations of the structure are governed by the differential equation system (2.1).

$$\mathbf{M}\,\ddot{x} + \mathbf{C}\,\dot{x} + \mathbf{K}\,x = f \tag{2.1}$$

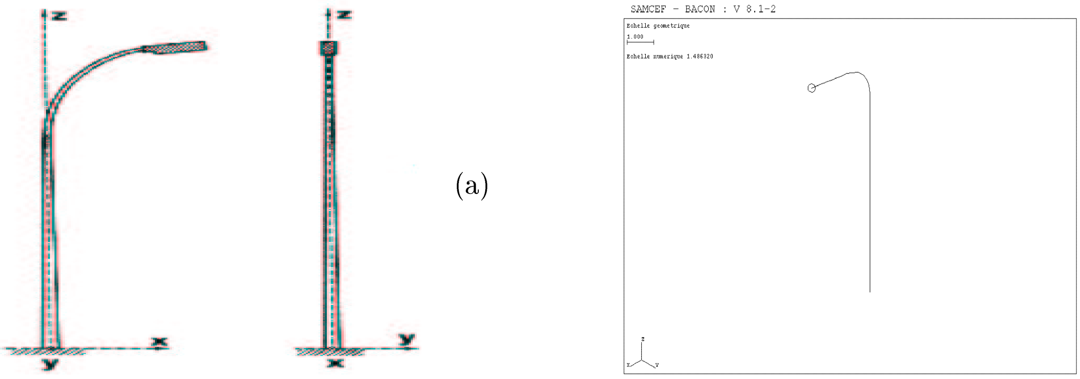

Figure 1: Structural axes. (a) Common system; (b) Finite element model.

M, **C** and **K** are respectively the mass, damping and stiffness matrices. The last one is the sum of the structural and geometric stiffness matrices [3]. The vectors x and f represent, in structural axes, the displacement and force acting on each degree of freedom of the finite element model.

Once the independant random aerodynamic excitations are known, the supposed elastic linear behaviour of the structure easily allow to solve separately the problems in the frequency domain [6]. If r is a response quantity (e.g. a stress component or a displacement) linearly related to the modal amplitudes y of a structure exposed to a field of random forces, its *Power Spectral Density (PSD)* function can be computed according to the following steps (Fig.2) :

- compute the modal excitation *PSD* matrix $\mathbf{\Phi_P}(\omega)$ from the excitation *PSD* matrix $\mathbf{\Phi_F}(\omega)$ and the mode shape matrix $\mathbf{\Xi}$;
- compute the modal response *PSD* matrix $\mathbf{\Phi_y}(\omega)$ from the excitation *PSD* matrix $\mathbf{\Phi_F}(\omega)$ and the modal transfer matrix $\mathbf{H}(\omega)$;
- compute the response *PSD* function $\Phi_r(\omega)$.

As one can see, the procedure is computationally simple as soon as the damping data, transfer matrix are available and the excitation has been defined. This latter point turns out to be the most difficult one for most physical problems.

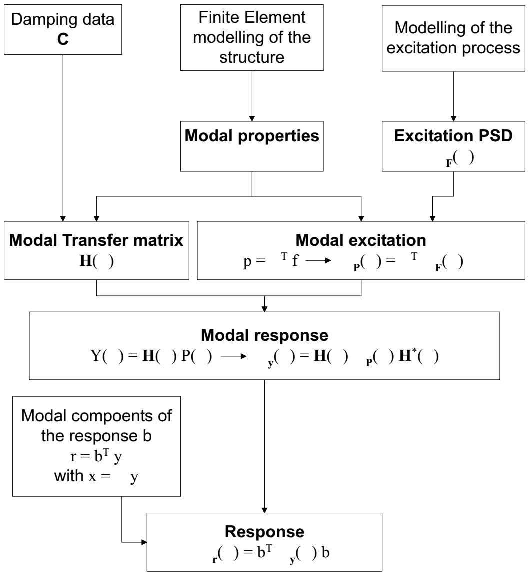

Figure 2: Spectral analysis.

2.1 Effect of turbulent wind

Near the ground, a boundary layer is mainly generated by the friction forces. The analysis of the wind frequency distribution reveals that an important part of the energy is concentrated on a period of time of one minute which corresponds to turbulent movements. If one defines wind relative axes, represented in structural axes by the azimuth (α_1) and elevation (α_2) angles, the instantaneous speed at time t for a point p of the space may be represented by the sum of the mean wind velocity and of a stochastic process which represents the along wind speed fluctuations around its mean value.

$$V(p,t) = u(p) + u'(p,t) \tag{2.2}$$

If the mean wind speed is fixed, one obtains a short term model of the instantaneous speed which accounts for the fluctuations due to the atmospheric turbulence. This process considered as stationary on a period of time T, of about ten minutes, is defined by its *Power Spectral Density* matrix [6]. The turbulent wind model used in this study is based on *Davenport*'s spectrum [2].

$$\Phi_{\xi_i \xi_j}(\omega) = \frac{4\,\kappa\,u_{10}^2}{u_i\,u_j\,\omega}\,\frac{(\frac{600\,\omega}{\pi\,u_{10}})^2}{[1+(\frac{600\,\omega}{\pi\,u_{10}})^2]^{4/3}}\,\exp(-\frac{\omega}{2\,\pi\,u_{10}}\,C_z\,|z_i - z_j|) \tag{2.3}$$

with $0 < \omega < +\infty$ and $\xi_i = u'_i/u_i$.

Equation (2.3) depends on the roughness of the ground (κ), the reference meteorological speed (u_{10}) measured at 10 m above the ground, the wind mean velocity (u_i) depending on the altitude of point i [6], the pulsation (ω) and a term of spatial coherence function of the vertical correlation constant (C_z) and of the distance between two points i and j along the vertical axis.

When a structure is subject to wind effect, aerodynamic forces are generated. In this approach, only the effect of the wind speed turbulent component is considered (the static component does not produce vibrations).

If one assumes that

- the structure does not modify the incident turbulent flow;
- the aerodynamic coefficients of the structure are measured in the stationary flow $u(p)$;
- $O(u'^{\,2}) \ll O(u^2)$;

the wind instantaneous velocity can be linearised and the *PSD* matrix of forces $\mathbf{\Phi}_{\mathbf{C_i C_j}}(\omega)$ is obtained from equation (2.3) by introducing the profile aerodynamic properties ν_i [4], [6] :

$$\mathbf{\Phi}_{\mathbf{C_i C_j}}(\omega) = \nu_i\, \nu_j^T\, \Phi_{\xi_i \xi_j}(\omega) \tag{2.4}$$

$$\begin{cases} \nu_i = \rho\, u_i^2\, (S_i\, C_{F_i})_{(\alpha_1,\alpha_2)} & \text{force coefficient} \\ \nu_i = \rho\, u_i^2\, (V_i\, C_{T_i})_{(\alpha_1,\alpha_2)} & \text{torque coefficient} \end{cases}$$

where ρ is the air density, S_i and V_i are the area and volume associated with point i, C_{F_i} and C_{T_i} are the aerodynamic force and torque coefficients at i.

2.2 Effect of Von–Karman vortices

A second type of excitation induced by the wind is the action of the wake on the structure. For a particular rate of flow, characterised by its *Reynolds* number (R_e), vortices are shed in the wake inducing across–wind vibrations on the pole–luminaire system. While fluctuating forces due to wind turbulence seem to be predictable with reasonable accuracy and simplicity, the understanding of the physical mechanism of wake excitation has proved to be much more complex and the application of analytical methods to predict the related wind loading is still fairly uncertain. The theory first developed by *Vickery* simulated Von–Karman vortices by a *PSD* of a Gaussian function $\Phi_{f_i f_j}(\omega)$ [10]. In order to improve the accuracy of function f_i in the low and high frequency ranges, the function used in this study is taken from reference [8] :

$$f_i = \frac{\frac{\beta_i\,\left(1-0.64\,\beta_i^2\right)\,\omega}{(0.964-0.353\,\beta_i)\,\omega_{si}^2}}{\left[1-\left(1-0.64\,\beta_i^2\right)\,\frac{\omega^2}{\omega_{si}^2}\right]^2 + 2.56\,\beta_i^2\,\left(1-0.64\,\beta_i^2\right)\,\frac{\omega^2}{\omega_{si}^2}} \tag{2.5}$$

where β_i is a bandwidth parameter function of longitudinal turbulence intensity and ω_{si} is the shedding frequency at height z_i [8].

Finally, the across–wind *PSD* matrix of forces $\mathbf{\Phi}_{\mathbf{F_i F_j}}(\omega)$ is obtained by introducing the profile aerodynamic properties ν_i^{vk} [8] :

$$\mathbf{\Phi}_{\mathbf{F_i F_j}}(\omega) = \nu_i^{vk}\, \nu_j^{vkT}\, \Phi_{f_i f_j}(\omega) \tag{2.6}$$

$$\begin{cases} \nu_i^{vk} = \frac{1}{2}\,\rho\, u_{ni}^2\, (\tilde{S}_i\, \tilde{C}_{F_i})_{(\alpha_1,\alpha_2)} & \text{force coefficient} \\ \nu_i^{vk} = \frac{1}{2}\,\rho\, u_{ni}^2\, (\tilde{V}_i\, \tilde{C}_{T_i})_{(\alpha_1,\alpha_2)} & \text{torque coefficient} \end{cases}$$

where u_{ni} is the mean wind velocity normal to the structure at i, $\tilde{S}_i$ and $\tilde{V}_i$ are the area and volume associated with point i, $\tilde{C}_{F_i}$ and $\tilde{C}_{T_i}$ are the aerodynamic force and torque coefficients at i for Von–Karman vortices.

3 Accelerated fatigue testing methodology

3.1 Equivalence criteria

Once the response of the pole–luminaire system is obtained, it can be used to compute the severity of the luminaire vibration environment during all its lifetime. Three basic criteria are available in the literature [1], [5] to quantify the severity of a vibration environment :

- the maximax response spectrum, associated with the maximum displacement representative of the maximum stress in the equipment;
- the fatigue damage spectrum, which is related to the deterioration of the material when submitted to repeated stresses;
- the dissipative damage spectrum, based on the assumption that the energy dissipated by the equipment is correlated with the severity of the vibration environment.

Since fatigue effects of long time ambient vibration are the leading cause of structural failures in outdoor pole mounted luminaires, the most representative criterion is the *fatigue damage spectrum.*

In order to simulate in laboratory the vibration environment of the luminaire, accelerated fatigue testing has to be defined. The problem of equivalence is therefore of prime importance. The definition of equivalence criteria is based on the application of the luminaire vibration excitation to the base of a reference one–degree–of–freedom system (Fig.3). The equation of movement of such a system is given by the relation (3.7)

$$\ddot{z} + 2\,\varepsilon\,\omega_0\,\dot{z} + \omega_0^2\,z = -\ddot{x} \qquad (3.7)$$

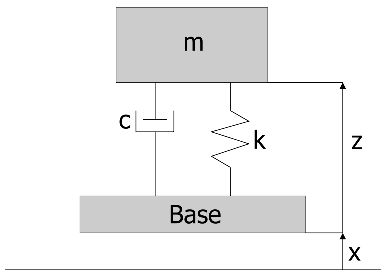

Figure 3: Reference 1–dof mass–spring–damper system.

where $x(t)$ describes the motion imposed to the base, $z(t)$ the relative position of the mass, ε the damping ratio and ω_0 the natural pulsation of the one–dof system. The mass m, stiffness k and damping c are the parameters of a modal model of the luminaire obtained either by finite element analysis or by experimental modal identification.

For a given vibration environment $x(t)$, the maximal response Z_{max} of the relative displacement is a function of the natural frequency f_0 of the system and of the damping ratio ε. For a stress level σ_i, the corresponding damage d_i is defined by $d_i = n_i/N_i$, n_i being the number of cycles of amplitude σ_i and N_i the maximum number of cycles before deterioration at the same stress level. N_i is given by the classical Wöhler curves of the material, which in their central part, can be approximated by *Basquin*'s relationship :

$$N_i \, \sigma_{i,max}^b = A^b \tag{3.8}$$

where b and A are two material dependent parameters. If one assumes a linear behaviour of the material, stress and displacement may be related by a constant K :

$$\sigma_{i,max} = K \, Z_{i,max} \tag{3.9}$$

According to *Miner*'s linear cumulative damage law and using equations (3.8–3.9), one obtains the total damage D corresponding to the reference 1–dof system in the form :

$$D = \left(\frac{K}{A}\right)^b \sum_i n_i \, Z_{i,max}^b \, (f_0, \varepsilon) \tag{3.10}$$

The fatigue damage spectrum is the curve which represents the total damage D as a function of the natural frequency f_0, for a given ε. In the case of a random Gaussian excitation, the expected value of $Z_{i,max}$ may be calculated by the theory of random vibration [5], [6]. The localisation in the structure of the maximal stress needs the use of a more elaborate finite element model or of an experimental model with strain gauges.

3.2 Optimisation process

To ensure a reliable design, fatigue vibration testing has to be carried out in laboratory in order to reproduce the real environment severity as accurately as possible. The objectives of the methodology are twofold :

- to define the vibration test specifications;
- to reduce the test duration.

The use of the equivalence spectra, developed for a 1-dof system, allows to compare the severity of two different vibration environments. For a vibration defined in a given frequency range $[f_1, f_2]$, the methodology, consists in :

1. computing the reference spectrum in a frequency range $[f_{01}, f_{02}]$ large enough to cover the most important natural frequencies of the equipment;
2. searching the equivalent test specifications so that the equivalence between spectra is obtained for each frequency in the range $[f_{01}, f_{02}]$.

The research of the equivalent test can be performed using analytical expressions of criteria or using an iterative process based on an optimisation process. The advantages of the last method are multiple :

- there is no restriction on the value of the natural frequency f_0 which may belong or not to $[f_1, f_2]$;
- the equivalent test is obtained independently of the natural frequency of the structure;
- the method is efficient even for complex problems (complex excitations, multi–degree–of–freedom systems).

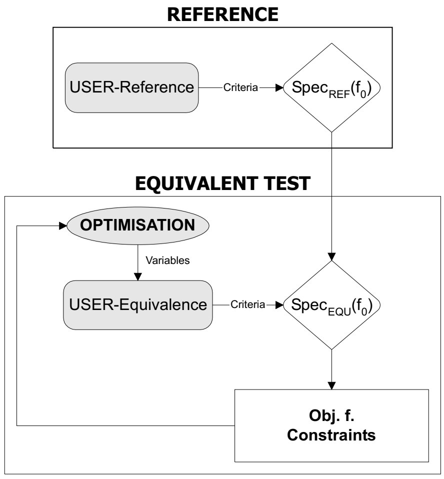

Figure 4: Optimisation process in *Boss/QUATTRO*.

This methodology has been developed in the '*Boss/QUATTRO*' software [7]. The successive tasks are organised as shown in the flowchart of Fig.4 :

1. the script *USER-Reference* runs the computation of the comparison criterion for the reference test;
2. from the initial values of the design variables, the script *USER-Equivalence* computes the same criterion for the equivalent test;
3. using results obtained in steps 1 and 2, the objective function to minimise and the constraints are generated;
4. the sensitivities of the different functions with respect to each design variable are computed and the parameters up-dated;
5. steps 2 to 4 are repeated until an optimum is reached.

4 Application example

The accelerated fatigue testing methodology has been applied to a common pole–luminaire system like the one shown in Fig.1. The parameters of the model are summarised in Tab.1.

Site roughness	II
Reference speed u_{10} $[m/s]$	10
Azimuth α_1 $[deg]$	90
Elevation α_2 $[deg]$	0
Damping ratio ε $[\%]$	0.5
Luminaire mass $[kg]$	20
Pole height $[m]$	10

Table 1: Parameters of the model.

4.1 Luminaire excitation : computations and validation

Fig.5 represents the acceleration *PSD* in x–direction computed at the fixation point of the luminaire on the pole (responses in y and z–directions may be obtained with a similar analysis).

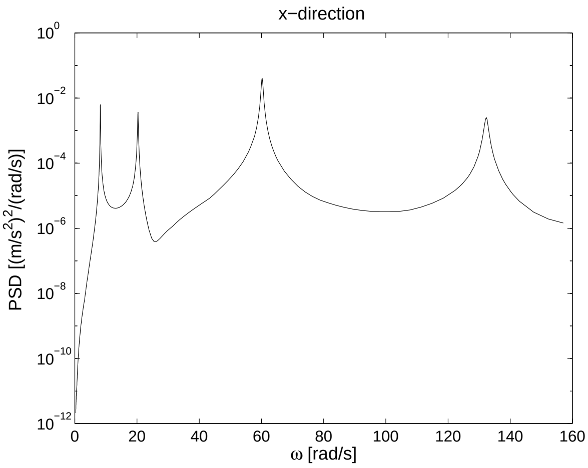

Figure 5: Response *PSD* $[(m/s^2)/(rad/s)]$ at the fixation point (x–direction).

To validate the spectral analysis, these results are compared to those obtained by *Van Dusen* on street lighting pole vibration [9]. The approach followed by *Van Dusen* consisted in determining experimentally envelopes of vibration amplitudes and frequencies corresponding to the limits of the vibration intensities normally occuring in street–lighting service. A great number of different pole–luminaire systems were observed for a mean wind velocity of $[2, 18]\,m/s$ and a damping ratio in the range $[0.05, 0.5]\,\%$.

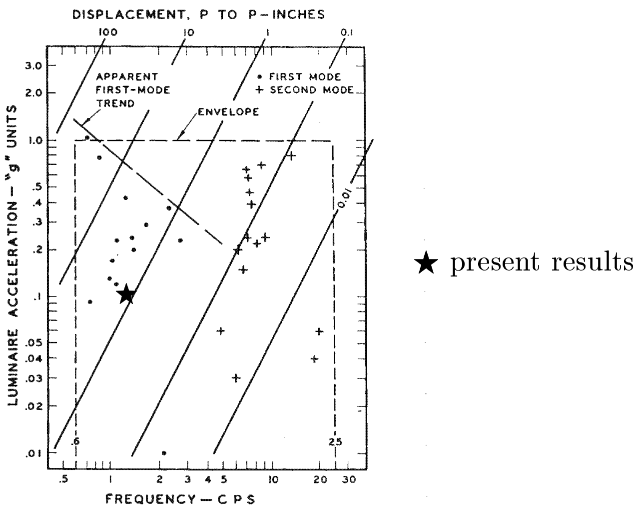

Figure 6: Comparison with Van Dusen's observations [9].

Fig.6 shows that the maximal amplitude observed by *Van Dusen* during its experiments was $1g$ for a frequency range of $[0.6, 25]\,Hz$. The black star locates the present computed results in *Van Dusen*'s envelope.

4.2 Definition of equivalent tests

The results of the spectral analysis are then used to define reference tests which provide, for each structural direction, one reference fatigue damage spectrum. The duration of the 'reference test' corresponding to the lifetime of the luminaire may be estimated to 20 years. For illustration, the values of the parameters used to compute the fatigue damage spectrum are $K = 1\,N/m^3$, $A = 1\,N/m^2$, $b = 10$ and $\varepsilon = 0.5\,\%$. In practice, they should be accurately determined.

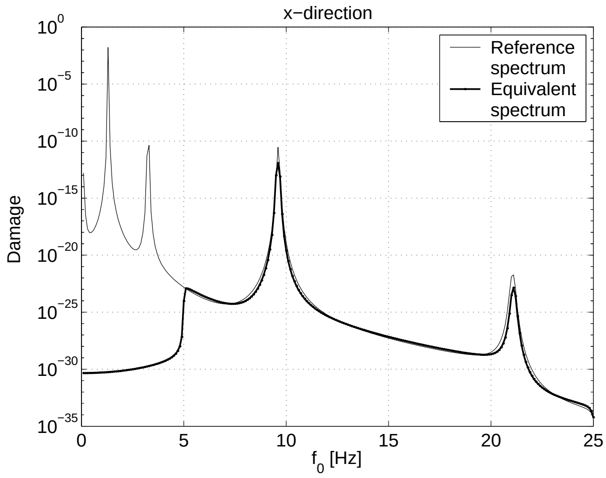

Figure 7: Fatigue damage spectrum of the reference and equivalent tests (x–direction).

The equivalent fatigue test for each structural direction corresponds to a random testing in the frequency range of $[5, 25]\,Hz$ divided into 5 intervals where the PSD remains constant. The allowable minimal frequency is imposed by the technical capabilities of electro–dynamic shakers. This restriction does not question the efficiency of the test because the first natural frequency of the luminaire is above this minimal value. The test levels and frequencies defining the intervals between $[5, 25]\,Hz$ have been chosen as the parameters of the test for a duration of one hour per axis.

	Excit.	Freq. range $[Hz]$	Amp. $[(m/s^2)/Hz]$	Duration $[s]$
Equiv.	random $[5, 25]Hz$	$[5.0 - 9.5]$	9.25e-4	3600
		$[9.5 - 9.7]$	1.20e+0	
		$[9.7 - 21.0]$	2.40e-3	
		$[21.0 - 21.2]$	1.03e-1	
		$[21.2 - 25.0]$	5.41e-4	

Table 2: Specifications of the equivalent test (x–direction).

The solution given by the optimisation process is shown in Fig.7 and the specifications of the equivalent test for x–direction are summarised in Tab.2. It results that :

- the equivalent spectrum fits very well with the reference one in the excitation frequency range;
- the natural frequency of the luminaire must be higher than 5 Hz and not coincide with the peaks of the pole-luminaire system.

5 Conclusion

The developed methodology enables the manufacturer to test luminaires in laboratory according to their long time vibration environment. The spectral response of the pole-luminaire system subject to wind excitation is computed in order to know the vibration spectra at the interface between pole and luminaire.

The definition of equivalence criteria (maximax response spectrum, fatigue damage spectrum, dissipative damage spectrum) allows the research of an optimised equivalent test.

Equivalence can generally not be defined by respecting simultaneously each of the equivalence criteria. To choose the most appropriate criterion, the identification and knowledge of failure processes are of prime importance.

The random behaviour of the wind is well reproduced by an accelerated fatigue random test. The use of a sine excitation at the natural frequency of the structure, as it is described in some standards, would not be as representative as a wide band random test : most of the time, the natural frequency of the luminaire varies during the test because of its non–linear behaviour.

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