A Fuzzy Approach to the Definition of Standardized Visibility in Fog

Jean-Jacques Illoereux
Surveillance de l'Environnement
Fondation Universitaire Luxembourgeoise
185 avenue de Luxembourg
B-6700 Arlon, Belgium

and

Lucien Duckstein
Systems and Industrial Engineering Department
University of Arizona
Tucson, Arizona 85721

Transmitted by John Caeti

ABSTRACT

The problem of defining the physical concept of standardized visibility is addressed by means of a fuzzy logic approach. Assuming that several hypotheses hold, it is possible to evaluate a standardized visibility $V_s$ for meteorological visual range as the ratio of a psychometric coefficient $k$ and the extinction coefficient $\sigma$ of the aerosol. The main disadvantage of this relationship due to Koschmieder [1] stems from the fact that the meteorological visual range can be very different from visibility estimated by a human observer with normal sight. This is particularly true during fog occurrences. A fuzzy approach is used to evaluate a relationship between the fog extinction coefficient applied by a transmittometer and the corresponding visibility evaluated by an expert. Calculations are performed according to two different approaches. First, a so-called fuzzy hyperbolic model based on a distance defined by Bardossy et al. [2] is presented, yielding a crisp (nonfuzzy) value of the coefficient $k$. Next, we use the multiplication of two fuzzy numbers, taken as triangular, to build an average fuzzy number $k$ that

* This research has been supported in part by grants from the U.S. National Science Foundation. The help and encouragement that A. Bardossy has provided us, in particular during the NATO Advanced Study Institute, "Engineering Risk and Reliability in a Changing Physical Environment," held 24 May-4 June 1993 in Deauville, France, is gratefully acknowledged.

© Elsevier Science Inc., 1994
655 Avenue of the Americas, New York, NY 10010 0000-3003/94/$7.00
Visibility in Fog

Since visibility is a psychophysical concept, a fuzzy approach may be used. A fuzzy set is, in general, a set whose boundaries are not defined precisely, such as the visibility estimated by motorists. In fact, most motorists describe the visual range using intervals of visibility, for instance in these terms: I believe the visibility is about 200 m, certainly more than 150 m and less than 300 m. If a series of such intervals can be provided for “degree of credibility” between 0 and 1, then a so-called membership function, whose value is between 0 and 1, can be defined [6, 11]. This approach is used to estimate a suitable relationship between the horizontal visibility (estimated by an expert) and the extinction coefficient of fog, taking into account the vagueness of the data and psychophysical processes.

This paper is organized as follows. The second section summarizes visibility theory. The third section presents a “hyperbolic” fuzzy least squares model defining a crisp value of $k$ to be used in (1) from $N$ pairs of triangular fuzzy numbers: $A_i = \tilde{A}_i$, $B_i = \tilde{B}_i$, $1 \leq i \leq N$. The most credible value of the independent variable $\tilde{A}_i$ is measured using a transmissometer and the dependent variable $\tilde{V}$ is the visibility estimated by an expert. In the fourth section, the robustness of the fuzzy approach is investigated by taking $k$ itself as a fuzzy number $\tilde{k}$, calculated as the average of the product of two triangular fuzzy numbers (i.e., $\tilde{k} = A \otimes B$). The resulting fuzzy member $\tilde{k}$ is then defuzzified by the barycenter, giving a second value of $k$, which can be compared with the first. In the last two sections, we present results.

2. VISIBILITY THEORY

The atmosphere is an aerosol or a colloidal system where the gaseous phase is the dispersion medium and the suspended particles constitute the dispersed phase in which transmission of light dims with the distance. Figure 1, drawn after Tombach [9], summarizes the basic elements of visibility. A human observer looks horizontally at an object through a homogeneous atmosphere in the daylight. In that figure, $B_0$ and $B_{ob}$ are, respectively, the luminances of the object and the background when observed at close range (the subscript $b$ denotes background). Similarly, $B_L$ and $B_{lb}$ are the luminances of the same targets when observed at the distance $L$. The quantity $B^*$ is the luminance of the path between the object and the receiver due to scattered sunlight.

We define the intrinsic contrast of the object against its background as

$$ C_0 = \frac{B_0 - B_{ob}}{B_{ob}}. $$

For a perfect black body, $B_0 = 0$ and $C_0 = -1$, otherwise $C_0 > -1$; the range of $C_0$ is thus $(-1, +\infty)$, where negative values correspond to an object
Visibility in Fog

where the transmittance $T$ ranges from 0 for a perfectly opaque medium to 1 for a perfectly transparent medium. Similarly,

$$B_{L,b} = B_{a,b}T + B^*.$$  \hfill (5)

Combining the four previous equations, we obtain

$$C_L = C_0 T \frac{B_{a,b}}{B_{L,b}}.$$  \hfill (6)

Concerning the transmittance $T$ of the atmosphere, it has been established theoretically and experimentally that a thin section of an aerosol, here denoted $dL$, will both scatter and absorb light in an amount proportional to the luminous flux $F$ (lumen) entering the section:

$$dF = -\sigma F dL.$$  \hfill (7)

where $\sigma$ is a factor of proportionality called aerosol extinction coefficient. Integration of (7) gives what is known as Bouguer's law:

$$F_L = F_0 \exp(-\sigma L).$$  \hfill (8)

The transmittance $T$ is then given as

$$T = \frac{F_L}{F_0} = \exp(-\sigma L),$$  \hfill (9)

and can be determined using a transmissometer over a path length $L$, defined as the instrument "baseline."

Combining (6) and (9) yields the following relationship:

$$C_L = C_0 \frac{B_{a,b}}{B_{L,b}} \exp(-\sigma L).$$  \hfill (10)

If we know the intrinsic contrast $C_L$ of an object, the minimum level of contrast that a normal human eye can distinguish, denoted $\varepsilon$, often called threshold of brightness contrast, and the extinction coefficient $\sigma$ of the aerosol, then the visibility $V_k$ is defined as the distance $L$ at which $C_L$ is just
equal to \( \sigma \). From (10), in which we replace \( C_k \) by \( \varepsilon \) and \( I \) by \( V_k \), the standardized visibility is given as

\[
V_k = \frac{-1}{\sigma} \ln \left( \frac{\varepsilon}{C_0} \cdot \frac{B_{1k}}{B_{2k}} \right)
\]  

(11)

Setting \( \varepsilon = B_{1k}/B_{2k} \), the combination of (1) and (11) yields

\[
V_k = \frac{1}{\sigma} \left( \ln|C_0| - \ln|\varepsilon| - \ln|\varepsilon| \right) = \frac{k}{\sigma}.
\]  

(12)

The subscript \( k \) indicates that the visibility defined in this way depends strongly on the hypotheses about \( \varepsilon \), \( C_0 \), and \( \varepsilon \), without taking into account any hypothesis on \( \sigma \).

It is generally assumed that, for targets located horizontally at less than 15 km from the observer, \( \varepsilon = 1 \) [8]. Taking \( \varepsilon = 0.02 \) (perfect observer) and \( C_0 = -1 \) (perfect black target against the daytime horizon), we obtain Koschmieder's relationship [7]:

\[
V_k = \frac{3.9}{\sigma}.
\]  

(13)

Here the subscript \( K \) refers to Koschmieder. It has to be pointed out that this relationship is obtained making strong assumptions (beyond having perfect observer under daylight conditions and a perfect black target; homogeneous atmosphere (i.e., \( \sigma \) = constant along the sight path), horizontal viewing, earth without curvature, cloudless sky (i.e., \( \varepsilon = 1 \)), and aerosol particles scattering light individually (i.e., multiple scattering is neglected).

Deviations from these assumptions have a substantial effect on the resulting value of \( k \) [1, 5, 10]. The numerator of (1) thus varies according to the investigation (quoted from Weintraub and Saxena [10]): 1.6 (Deehay et al., 1982), 2.9 (U.S. EPA, 1980), 3.35 (Hovarth and Null, 1980); 1.9 ± 0.4 (Griffing, 1980), and 3.0 (Tombach and Allard, 1985). This range of values of \( k \) must be extended because these researchers have considered idealized cases (in particular homogeneity of aerosol). Either one of the fuzzy models presented next appears to account for a suitable range of values of \( k \).
Now, let us consider $N$ pairs of TFNs $\{\tilde{A}_i, \tilde{B}_i\}$, $1 \leq i \leq N$. If the relationship between the independent variable (say $A$) and the dependent variable $B$ is seen as hyperbolic, then a fuzzy hyperbolic model may be estimated by minimizing the sum of $N$ distances $D^2(\tilde{A}_i, \tilde{B}_i, h)$:

$$0 = \frac{\partial}{\partial k} \sum_{i=1}^{N} D^2 \left( k \frac{x}{A_i}, \tilde{B}_i, h \right)$$

where $k$ is the coefficient sought (assumed to be crisp in this case).

In our case, referring to (1), the peak of $\mu(A_i)$ is taken as the visibility $V_i$ observed by a well-trained individual, while that of $\tilde{A}_i$ is the corresponding extinction coefficient $\tilde{\sigma}_i$ measured by means of a transmissometer.

Using the notations,

$$\tilde{V}_i = (v_0 - v_1, v_2, v_3 - v_2), = (\delta_i, \varphi_i, \eta_i),$$

$$\tilde{\sigma}_i = (\varphi_i - \varphi_1, \varphi_2, \varphi_3 - \varphi_2), = (\varphi_i, s_i, \psi_i),$$

$$I_i = I_i(x, y) = \left( \frac{x - y}{x} + \ln \frac{x}{x - y} - 1 \right) \frac{1}{y^2},$$

$$I_2 = I_2(x, y) = \frac{1}{y} - \frac{x - y}{y^2} \ln \frac{x}{x - y},$$

$$I_2 = I_3(x, y) = \frac{1}{2y} - \frac{x}{y^2} \left[ 1 + (x - y) \ln \frac{x}{x - y} \right],$$

and

$$I_i^* = I_i(x_i, \varphi_i); \quad I_i^{**} = I_i(x_i, -\psi_i),$$

$$I_i^{**} = I_i(x_i, \varphi_i); \quad I_i^{**} = I_i(x_i, -\psi_i),$$

$$I_i^{**} = I_i(x_i, \varphi_i); \quad I_i^{**} = I_i(x_i, -\psi_i)$$

the constant $k$ to be used in (1) is calculated to be

$$k = \frac{\sum_{i=1}^{N} v_i (I_i^* + I_i^{**}) - \sum_{i=1}^{N} \eta_i I_i^* + \sum_{i=1}^{N} \delta_i I_i^{**}}{\sum_{i=1}^{N} (I_i^* + I_i^{**})}$$

4. **TAKING THE "CONSTANT" $k$ AS A FUZZY NUMBER** (MODEL P)

For each one of the $N$ observations, $1 \leq i \leq N$, (1) can be rewritten as

$$\tilde{k}_i - \tilde{V}_i \otimes \tilde{\sigma}_i = (\delta_i, \varphi_i, \eta_i) \otimes (\varphi_i, s_i, \psi_i)$$

where $\otimes$ represents the fuzzy multiplications shown in Figure 3. As the product of two TFNs (solid line) is not a TFN (dotted line), the $N$ fuzzy numbers $\tilde{k}_i$ must be calculated using the inverse function $\mu^{-1}((h, i))$ of the membership function $\mu(\tilde{A}_i, i)$. At each level $h, 0 \leq h \leq 1$, and for each observation $1 \leq i \leq N$, we have

$$\mu^{-1}_i((h, i)) = \left[ \frac{h}{v_1 s_i + (v_2 \varphi + s_2 \delta \cdot h + \delta \varphi \cdot h^2)} \right]$$

where

$$\mu^{-1}_i((h, i)) = \left[ h \frac{v_1 s_i + (v_2 \varphi + s_2 \delta \cdot h \cdot \eta \varphi \cdot h^2)}{v_1 s_i + (v_2 \varphi + s_2 \delta \cdot h \cdot \eta \varphi \cdot h^2)} \right]$$

**k taken as a fuzzy number and its defuzzification**

![Diagram showing fuzzy number and its defuzzification](image)

**Fig. 3.** The numerator $k$ of the visibility relationship taken as a fuzzy number. The thick line shows the fuzzy number $\tilde{k}$ (not a TFN), calculated as the average of $N = 25$ fuzzy numbers, each of which being the product of two TFNs $\tilde{k}_i = \tilde{V}_i \otimes \tilde{\sigma}_i$. The defuzzification consists in calculating the barycenter $\tilde{k}$ of this function. Note that in our case, the triangular approximation would be sufficient.
The average fuzzy number \( \bar{k} \) is obtained by computing the mean of \( N \) relations (20) at each level \( h \) (Figure 3). The defuzzification consists in computing the barycenter \( \bar{k} \) of the membership function \( \mu_k \) sketched in Figure 3 using the following definition:

\[
\bar{k} = \frac{\int_{-\infty}^{\infty} \xi \mu_k(\xi) d\xi}{\int_{-\infty}^{\infty} \mu_k(\xi) d\xi}
\]  

(21)

where \( \mu_k(\xi) \) refers to the ordinate of the membership function of the fuzzy number \( k \).

5. RESULTS

We have used \( N = 25 \) observations taken during fog events at the Lille airport [3]. The independent variable is the extinction coefficient \( \sigma \) supplied by a transmissometer giving the transmittance \( T \) of the fog over a path length \( L \), defined as the instrument “baseline,” equal here to 30 meters:

\[
\sigma = -\frac{\ln T}{L}
\]

(22)

while the dependent variable is the observed visibility \( V \) given by the airport watchman.

As both variables are in fact fuzzy, the errors on the mode of \( \sigma \) are assumed to be symmetric and equal to about 20% of the measured values. Similarly, the errors on the mode of \( V \) are assumed to be equal to \( \pm 25\% \) of its estimated value. Furthermore, \( \sigma \) and \( V \) are assumed to be symmetric triangular fuzzy numbers with support equal to the estimated error interval. These error estimations are based on various theoretical considerations, and, in all cases, must be considered as maximums.

Equation (18) has first been applied (model H), yielding \( k = k_H = 4.19 \). Next, using (19), (20), and (21) (model P), one finds \( k = k_P = 4.30 \). The second value \( k_P \) is quite close to the first one \( k_H \). Therefore, the two models seem to be consistent and the \( k \) value to be used in (1) can be taken as, say, \( k = 4.3 \), because the second type of analysis uses defuzzification at the end of the calculation, rather than at the beginning.

6. DISCUSSION AND CONCLUSIONS

First, the variability or sensitivity of these two models \( H \) and \( P \) with regard to the vagueness of the variables is investigated. Varying the left and right sides of our variables (TFNs) step by step from 5% to 25% of their central value (i.e., the value corresponding to \( \mu = 1 \)), we have compared 625 pairs \((k_H, k_P)\), \(1 \leq t \leq 625\). For each, we have calculated

\[
\Delta k_t(\%) = \frac{k_{Ht} - k_{Pt}}{k_{Ht} + k_{Pt}}
\]

(23)

As sketched in Figure 4, the distribution of \( \Delta k \) is approximately normal (\( \Delta k = 1.3\% \); \( \sigma_{\Delta k} = 2.4\% \)). In other words, the probability that the difference between \( k_H \) and \( k_P \) exceeds 5% of their average value is less than 0.06, while

![Deviation between models (H and P) with regard to the vagueness](image-url)
the probability that it exceeds 2.5% is less than 0.35. Therefore, both models appear to produce similar results when the vagueness of variables is varied, and we may consider that these fuzzy models are coherent.

Second, the value of $k$ in either model $P$ or model $H$ depends strongly on the shapes of the input TPNs. For instance, if all vagueness occurs on the left side of the variables (i.e., $\vec{x}_i = 0.25x_i$, $\vec{y}_i = 1.05y_i$, and $\vec{V}_i = 0.25V_i$, $\vec{V}_i = 1.05V_i$; $1 \leq i \leq N$), we find smaller $k$ values. In the opposite case, we find larger values of $k$ if all vagueness is concentrated on the right hand of the TPN. However, we cannot define the TPN $\vec{\sigma}$ and $\vec{V}$ as we wish. In the case considered here, we have selected the maximum uncertainties on both the fog extinction coefficient $\vec{\sigma}$ and the estimated visibility $\vec{V}$.

Third, $k$ taken as fuzzy number $\tilde{k}$ (model $P$) includes most values proposed by researchers. Referring to (12), assuming $\vec{\sigma} = 1$ [6] and $\alpha = 0.02$ [7], we can calculate the inherent contrast $C_0$ of an object in fog (for instance: a tree):

$$|C_0| = |\varepsilon|^{4.2}. \quad (24)$$

In other words, the absolute value of inherent contrast $C_0$ of an object in fog should be about 1.32 ($k_P = 4.19$) or 1.47 ($k_H = 4.30$). As the inherent contrasts is $\varepsilon = 1$ (perfect black body against a white background), $C_0$ from (24) is necessarily a positive quantity, that is to say an object brighter than fog. This result can be true only for a target brighter than the background [5]. In the daytime, a majority of the trees are darker than the fog, meaning that these hypotheses are not valid. We thus think that several of the usual assumptions made on parameters in conjunction with (12) may be very risky and can be avoided using fuzzy logic.

The following concluding points have been reached:

1. A proper definition of standardized visibility is necessary for decision making under conditions of fog, in particular, for setting the speed limit.
2. Physical considerations provide the relationship $k = \varepsilon \cdot V$. Assumptions must be made about $k$ as a function of $\varepsilon$, $C_0$, and $\vec{\sigma}$. Furthermore, observations on $V$, $\sigma$, and $k(\varepsilon, C_0, \sigma)$ are imprecise and often not available.
3. Depending on the assumptions and experimental data available, various researchers have found the value of $k$ to vary from 1.8 to 3.9 although they had considered only idealized cases.
4. A fuzzy approach can account for a realistic range of values of $k$; here two fuzzy models, $H$ and $P$, have been developed.
5. The first approach (model $H$), a fuzzy hyperbolic model, minimizes the distance between the estimated visibility and the corresponding measured fog extinction coefficient. This model yields a crisp value of $k$.

6. The second (model $P$) calculates a fuzzy number $\tilde{k}$ as the product of TPN's $\vec{k} = \vec{\sigma} \cdot \vec{V}$, and then a point value is calculated by defuzzification.
7. The two values $k_P = 4.19$ and $k_H = 4.30$ are quite close so that the value $k = 4.30$ may be selected to determine a standardized visibility.

REFERENCES