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A Fuzzy Approach to the Definition of Standardized Visibility in Fog*

Jean-Jacques Boreux

Surveillance de l'Environnement
Fondation Universitaire Luxembourgeoise
185 avenue de Longwy
B-6700 Arlon, Belgium

and

Lucien Duckstein

Systems and Industrial Engineering Department
University of Arizona
Tucson, Arizona 85721

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ABSTRACT

The problem of defining the physical concept of standardized visibility is addressed by means of a fuzzy logic approach. Assuming that several hypotheses hold, it is possible to evaluate a standardized visibility V_k (or meteorological visual range) as the ratio of a psychometric coefficient k and the extinction coefficient σ of the aerosol. The main disadvantage of this relationship due to Koschmieder [7] stems from the fact that the meteorological visual range can be very different from visibility estimated by a human observer with normal sight. This is particularly true during fog occurrences. A fuzzy approach is used to evaluate a relationship between the fog extinction coefficient supplied by a transmissometer and the corresponding visibility evaluated by an expert. Calculations are performed according to two different approaches. First, a so-called fuzzy hyperbolic model based on a distance defined by Bardossy et al. [2] is presented, yielding a crisp (nonfuzzy) value of the coefficient k. Next, we use the multiplication of two fuzzy numbers, taken as triangular, to build an average fuzzy number $\tilde{k}$ that

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accounts for the different values found in the literature. The defuzzification of $\bar{k}$ confirms the result obtained by the first approach.

1. INTRODUCTION

The main purpose of this paper is to define standard visibility by fuzzifying the relationship between the observed visibility in fog and the corresponding fog extinction coefficient given by a transmissometer.

Highway traffic has greatly benefited in the last half-century from investments by government agencies (industrialized countries). Traffic flow is increasing, making road safety problems much more complex, especially when visibility is poor. In short, visibility can be defined as the greatest distance from which an observer can clearly see a desired object [9]. This psychophysical definition can be understood introducing the notion of contrast between an object and its background. The visibility is then the maximum distance at which the outline of an object can be recognized against the horizon as background [5]. For instance, when the weather is foggy, the speed limit may be set according to the horizontal visibility, which is supplied by various measuring instruments. In fact, all visibility measurement devices and techniques require that certain parameters be estimated, such as the threshold contrast ε of the eye, inherent contrast of target C_0 rate ϑ between the luminance of the background at zero distance and L distance from the object and distribution of particles in the portion of atmosphere considered σ. The visibility may be given as:

$$V_k = \frac{k(\varepsilon, C_0, \vartheta)}{\sigma} \tag{1}$$

where the coefficient k depends upon the three parameters ε, C_0, and ϑ.

In estimating the various parameters, several assumptions must be made, for example, homogeneity of fog. Poor estimations of parameter and violation of assumptions often cause disagreements between the visibility perceived by most motorists and that measured for setting the speed limit. This problem stems from the fact that the human eye is a very complex sensoring device. It should be noted that visibility is, in fact, a psychophysical concept and thus instrumental methods for determining horizontal visibility are based on an imprecise definition of the concept.

Still a suitable estimation of visibility remains of prime importance for road traffic, especially in foggy weather; it thus appears to be necessary to define a standardized visibility compatible with the one observed by a majority of the motorists.

Since visibility is a psychophysical concept, a fuzzy approach may be used. A fuzzy set is, in general, a set whose boundaries are not defined precisely, such as the visibility estimated by motorists. In fact, most motorists describe the visual range using intervals of visibility, for instance in these terms: I believe the visibility is about 200 m, certainly more than 150 m and less than 300 m. If a series of such intervals can be provided for "degree of credibility" between 0 and 1, then a so-called membership function, whose value is between 0 and 1, can be defined [6, 11]. This approach is used to estimate a suitable relationship between the horizontal visibility (estimated by an expert) and the extinction coefficient of fog, taking into account the vagueness of the data and psychophysical processes.

This paper is organized as follows. The second section summarizes visibility theory. The third section presents a "hyperbolic" fuzzy least squares model defining a crisp value of k to be used in (1) from N pairs of triangular fuzzy numbers: $\tilde{A}_i = \tilde{\sigma}_i$, $\tilde{B}_i = \tilde{V}_i$, $1 \leqslant i \leqslant N$. The most credible value of the independent variable $\tilde{\sigma}_i$ is measured using a transmissometer and the dependent variable $\tilde{V}$ is the visibility estimated by an expert. In the fourth section, the robustness of the fuzzy approach is investigated by taking k itself as a fuzzy number $\bar{k}$, calculated as the average of the product of two triangular fuzzy numbers (i.e., $\tilde{k}_i = \tilde{A}_i \otimes \tilde{B}_i$). The resulting fuzzy member $\bar{k}$ is then defuzzified by the barycenter, giving a second value of k, which can be compared with the first. In the last two sections, we present results.

2. VISIBILITY THEORY

The atmosphere is an aerosol or a colloidal system where the gaseous phase is the dispersion medium and the suspended particles constitute the dispersed phase in which transmission of light dims with the distance. Figure 1, drawn after Tombach [9], summarizes the basic elements of visibility. A human observer looks horizontally at an object through a homogeneous atmosphere in the daylight. In that figure, B_0 and B_{0b} are, respectively, the luminances of the object and the background when observed at close range (the subscript b denotes background). Similarly, B_L and B_{Lb} are the luminances of the same targets when observed at the distance L. The quantity B^* is the luminance of the path between the object and the receiver due to scattered sunlight.

We define the intrinsic contrast of the object against its background as

$$C_0 = \frac{B_0 - B_{0b}}{B_{0b}}. \tag{2}$$

For a perfect black body, $B_0 = 0$ and $C_0 = -1$, otherwise $C_0 > -1$; the range of C_0 is thus $(-1, +\infty)$, where negative values correspond to an object

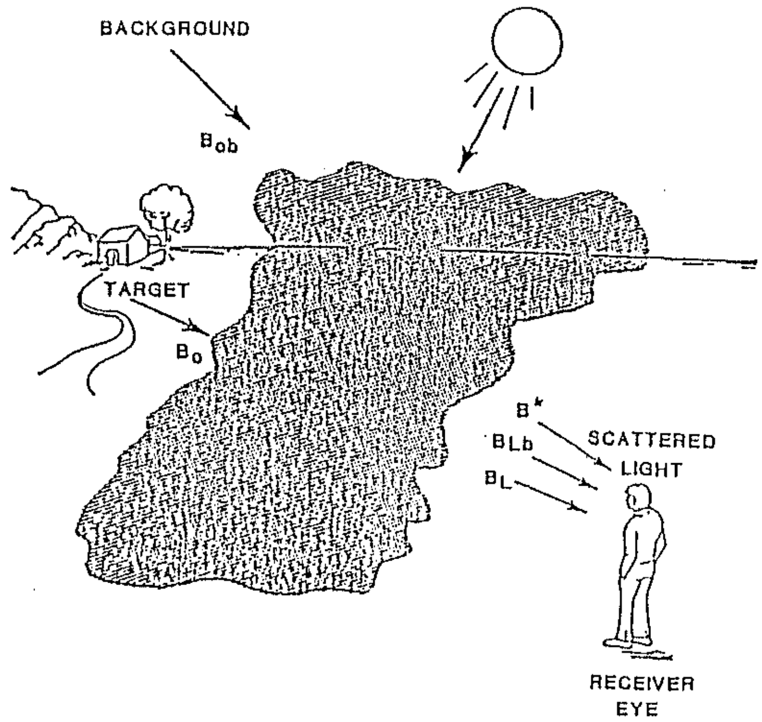

FIG. 1. Basic elements of visibility. B denotes luminance, B_0 is the intrinsic luminance of the targets (object and background), B_L is the luminance at the receiver, and B^* is the path luminance due to scattered sunlight. The subscript "b" denotes the background.

darker than the background and positive values are obtained for an object brighter than the background.

Similarly, the apparent contrast detected by the observer located L meters from the object is given as

$$C_L = \frac{B_L - B_{Lb}}{B_{Lb}} \tag{3}$$

In fact, the observed luminance B_L is the sum of the inherent luminance B_0, as diminished by the transmittance T of the atmosphere and the path luminance B^*

$$B_L = B_0 T + B^* \tag{4}$$

where the transmittance T ranges from 0 for a perfectly opaque medium to 1 for a perfectly transparent medium. Similarly,

$$B_{Lb} = B_{0b} T + B^*. \tag{5}$$

Combining the four previous equations, we obtain

$$C_L = C_0 T \frac{B_{0b}}{B_{Lb}}. \tag{6}$$

Concerning the transmittance T of the atmosphere, it has been established theoretically and experimentally that a thin section of an aerosol, here denoted dL, will both scatter and absorb light in an amount proportional to the luminous flux F (lumen) entering the section:

$$dF = -\sigma F \, dL \tag{7}$$

where σ is a factor of proportionality called aerosol extinction coefficient. Integration of (7) gives what is known as Bouguer's law:

$$F_L = F_0 \exp(-\sigma L). \tag{8}$$

The transmittance T is then given as

$$T = \frac{F_L}{F_0} = \exp(-\sigma L), \tag{9}$$

and can be determined using a transmissometer over a path length L, defined as the instrument "baseline."

Combining (6) and (9) yields the following relationship:

$$C_L = C_0 \frac{B_{0b}}{B_{Lb}} \exp(-\sigma L). \tag{10}$$

If we know the intrinsic contrast C_0 of an object, the minimum level of contrast that a normal human eye can distinguish, denoted ε, often called threshold of brightness contrast, and the extinction coefficient σ of the aerosol, then the visibility V_k is defined as the distance L at which C_L is just equal to ε. From (10), in which we replace C_L by ε and L by V_k, the standardized visibility is given as

$$V_k = \frac{-1}{\sigma} \ln \left| \frac{\varepsilon}{C_0} \cdot \frac{B_{Lb}}{B_{0b}} \right|. \qquad (11)$$

Setting $\vartheta = B_{Lb}/B_{0b}$, the combination of (1) and (11) yields

$$V_k = \frac{1}{\sigma}(\ln|C_0| - \ln|\varepsilon| - \ln|\vartheta|) = \frac{k}{\sigma}. \qquad (12)$$

The subscript k indicates that the visibility defined in this way depends strongly on the hypotheses about ε, C_0, and ϑ, without taking into account any hypothesis on σ.

It is generally assumed that, for targets located horizontally at less than 15 km from the observer, $\vartheta \cong 1$ [8]. Taking $\varepsilon = 0.02$ (perfect observer) and $C_0 = -1$ (perfect black target against the daytime horizon), we obtain Koschmieder's relationship [7]:

$$V_K = \frac{3.9}{\sigma}. \qquad (13)$$

Here the subscript K refers to Koschmieder. It has to be pointed out that this relationship is obtained making strong assumptions (beyond having perfect observer under daylight conditions and a perfect black target): homogeneous atmosphere (i.e., σ = constant along the sight path); horizontal viewing, earth without curvature, cloudless sky (i.e., $\vartheta = 1$); and aerosol particles scattering light individually (i.e., multiple scattering is neglected).

Deviations from these assumptions have a substantial effect on the resulting value of k [1, 5, 10]. The numerator of (1) thus varies according to the investigation (quoted from Weintraub and Saxena [10]): 1.8 (Dzubay et al., 1982); 2.9 (U.S. EPA, 1980); 3.35 (Hovarth and Noll, 1969); 1.9 ± 0.4 (Griffing, 1980), and 3.0 (Tombach and Allard, 1983). This range of values of k must be extended because these researchers have considered idealized cases (in particular homogeneity of aerosol). Either one of the fuzzy models presented next appears to account for a suitable range of values of k.

3. A FUZZY HYPERBOLIC MODEL FOR KOSCHMIEDER'S RELATIONSHIP (MODEL II)

The simplest fuzzy numbers are so-called triangular fuzzy numbers or TFN [4]. The fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 \leqslant a_2 \leqslant a_3$ is a TFN if its membership function $\mu_{\tilde{A}}$ can be written as in (14) and Figure 2.

$$\mu_{\tilde{A}}(\xi) = \begin{cases} 0 \Leftarrow \xi \leqslant a_1 \\ \dfrac{\xi - a_1}{a_2 - a_1} \Leftarrow a_1 < \xi \leqslant a_2 \\ \dfrac{\xi - a_3}{a_2 - a_3} \Leftarrow a_2 < \xi \leqslant a_3 \\ 0 \Leftarrow a_3 < \xi \end{cases} \qquad (14)$$

Consider two TFN: $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$.

Using the following notations:

$$\delta_a \equiv a_2 - a_1; \quad a \equiv a_2; \quad \eta_a \equiv a_3 - a_2,$$
$$\delta \equiv b_2 - b_1; \quad b \equiv b_2; \quad \eta_b \equiv b_3 - b_2,$$

we define a distance between the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ as [2]

$$D^2(\tilde{A}, \tilde{B}, h) \equiv \int_0^1 \{[a - b - (1 - h)(\delta_a - \delta_b)]^2 + [a - b + (1 - h)(\eta_a - \eta_b)]^2\} h \, dh. \qquad (15)$$

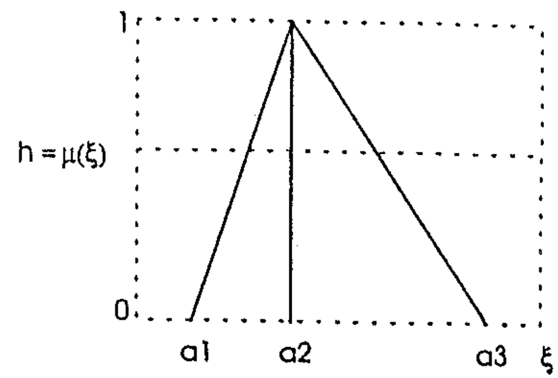

FIG. 2. Sketch of the membership function of the triangular fuzzy number (a_1, a_2, a_3).

Now, let us consider N pairs of TFNs $\{\tilde{A}_i, \tilde{B}_i\}$, $1 \leq i \leq N$. If the relationship between the independent variable (say $\tilde{A}$) and the dependent variable $\tilde{B}$ is seen as hyperbolic, then a fuzzy hyperbolic model may be estimated by minimizing the sum of N distances $D^2(\tilde{A}_i, \tilde{B}_i, h)$:

$$0 = \frac{\partial}{\partial k} \sum_{i=1}^{N} D^2\left(\frac{k}{\tilde{A}_i}, \tilde{B}_i, h\right) \tag{16}$$

where k is the coefficient sought (assumed to be crisp in this case).

In our case, referring to (1), the peak of $\mu(\tilde{B}_i)$ is taken as the visibility V_i observed by a well-trained individual, while that of $\tilde{A}_i$ is the corresponding extinction coefficient $\tilde{\sigma}_i$ measured by means of a transmissometer.

Using the notations,

$$\tilde{V}_i \equiv (v_2 - v_1, v_2, v_3 - v_2)_i \equiv (\delta_i, v_i, \eta_i),$$

$$\tilde{\sigma}_i \equiv (s_2 - s_1, s_2, s_3 - s_2)_i \equiv (\varphi_i, s_i, \psi_i),$$

$$I_1 = I_1(x, y) = \left(\frac{x-y}{x} + \ln\frac{x}{x-y} - 1\right)\frac{1}{y^2},$$

$$I_2 = I_2(x, y) = \frac{1}{y} - \frac{x-y}{y^2}\ln\frac{x}{x-y},$$

$$I_3 = I_3(x, y) = \frac{1}{2y} - \frac{x}{y^2}\left[1 + (x-y)\ln\frac{x}{x-y}\right],$$

and

$$\begin{cases} I_1^* = I_1(s_i, \varphi_i); & I_1^{**} = I_1(s_i, -\psi_i), \\ I_2^* = I_2(s_i, \varphi_i); & I_2^{**} = I_2(s_i, -\psi_i), \\ I_3^* = I_3(s_i, \varphi_i); & I_3^{**} = I_3(s_i, -\psi_i), \end{cases} \tag{17}$$

the constant k to be used in (1) is calculated to be

$$k = \frac{\sum_{i=1}^{N} v_i(I_2^* + I_2^{**}) - \sum_{i=1}^{N} \eta_i I_3^* + \sum_{i=1}^{N} \delta_i I_3^{**}}{\sum_{i=1}^{N} (I_1^* + I_1^{**})}. \tag{18}$$

4. TAKING THE "CONSTANT" k AS A FUZZY NUMBER (MODEL P)

For each one of the N observations, $1 \leq i \leq N$, (1) can be rewritten as

$$\tilde{k}_i = \tilde{V}_i \otimes \tilde{\sigma}_i = (\delta_i, v_i, \eta_i) \otimes (\varphi_i, s_i, \psi_i) \tag{19}$$

where $\otimes$ represents the fuzzy multiplications shown in Figure 3. As the product of two TFN (solid line) is not a TFN (dotted line), the N fuzzy numbers $\tilde{k}_i$ must be calculated using the inverse function $\mu^{-1}(h, i)$ of the membership function $\mu(\xi, i)$. At each level h, $0 \leq h \leq 1$, and for each observation i, $\leq i \leq N$, we have

$$\begin{cases} \mu_L^{-1}(h, i) = \left[v_1 s_1 + (v_1\varphi + s_1\delta)\cdot h + \delta\varphi\cdot h^2\right]_i & \text{(Left)} \\ \mu_R^{-1}(h, i) = \left[v_3 s_3 + (v_3\psi + s_3\eta)\cdot h + \eta\psi\cdot h^2\right]_i & \text{(Right)}. \end{cases} \tag{20}$$

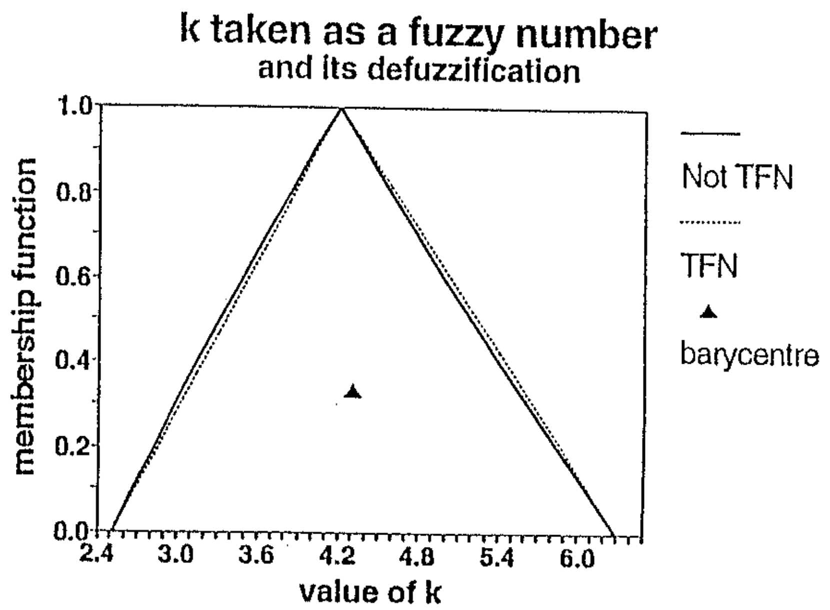

FIG. 3. The numerator k of the visibility relationship taken as a fuzzy number. The thick line shows the fuzzy number $\tilde{k}$ (not a TFN), calculated as the average of $N = 25$ fuzzy numbers, each of which being the product of two TFN $\tilde{k}_i = \tilde{\sigma}_i \otimes \tilde{V}_i$. The defuzzification consists in calculating the barycenter $\bar{k}$ of this function. Note that in our case, the triangular approximation would be sufficient.

The average fuzzy number $\tilde{k}$ is obtained by computing the mean of N relations (20) at each level h (Figure 3). The defuzzification consists in computing the barycenter $\bar{k}$ of the membership function μ_ξ sketched in Figure 3 using the following definition:

$$\bar{k} = \frac{\int_{-\infty}^{\infty} \xi \cdot \mu_\xi(\xi)\, d\xi}{\int_{-\infty}^{\infty} \mu_\xi(\xi)\, d\xi} \tag{21}$$

where $\mu_\xi(\xi)$ refers to the ordinate of the membership function of the fuzzy number $\tilde{k}$.

5. RESULTS

We have used $N = 25$ observations taken during fog events at the Lille airport [3]. The independent variable is the extinction coefficient σ supplied by a transmissometer giving the transmittance T of the fog over a path length L, defined as the instrument "baseline," equal here to 30 meters:

$$\sigma = \frac{-\ln T}{L}, \tag{22}$$

while the dependent variable is the observed visibility V given by the airport watchman.

As both variables are in fact fuzzy, the errors on the mode of $\tilde{\sigma}$ are assumed to be symmetric and equal to about 20% of the measured values. Similarly, the errors on the mode of $\tilde{V}$ are assumed to be equal to $\pm 25\%$ of its estimated value. Furthermore, $\tilde{\sigma}$ and $\tilde{V}$ are assumed to be symmetric triangular fuzzy numbers with support equal to the estimated error interval. These error estimations are based on various theoretical considerations, and, in all cases, must be considered as maximas.

Equation (18) has first been applied (model H), yielding $k = k_H = 4.19$. Next, using (19), (20), and (21) (model P), one finds $\bar{k} = k_P = 4.30$. The second value k_P is quite close to the first one k_H. Therefore, the two models seem to be consistent and the k value to be used in (1) can be taken as, say, $k = 4.3$, because the second type of analysis uses defuzzification at the end of the calculation, rather than at the beginning.

6. DISCUSSION AND CONCLUSIONS

First, the variability or sensitivity of these two models H and P with regard to the vagueness of the variables is investigated. Varying the left and right sides of our variables (TFNs) step by step from 5% to 25% of their central value (i.e., the value corresponding to $\mu = 1$), we have compared 625 pairs $(k_H, k_P)_i$, $1 \leqslant i \leqslant 625$. For each, we have calculated

$$\Delta k_i(\%) = 200 \frac{k_{Pi} - k_{Hi}}{k_{Pi} + k_{Hi}}. \tag{23}$$

As sketched in Figure 4, the distribution of Δk is approximately normal ($\overline{\Delta k} \cong 1.3\%$; $S_{\Delta k} \cong 2.4\%$). In other words, the probability that the difference between k_H and k_P exceeds 5% of their average value is less than 0.06, while

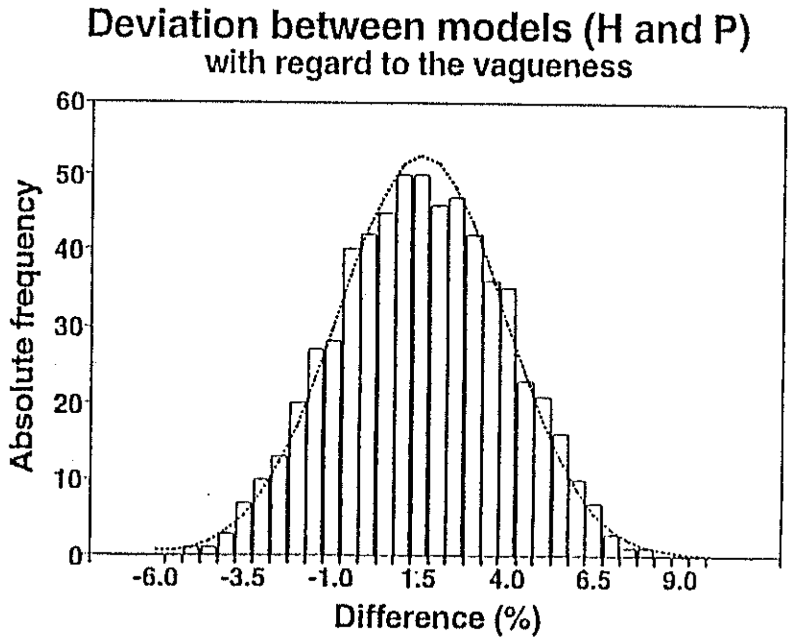

FIG. 4. Comparison between model H and model P with regard to the vagueness of both variables, fog extinction coefficient σ (transmissometer) and observed visibility V (expert). The distribution of $\Delta k(\%)$ is nearly normal and the probability that k_P (model P) and k_H (model H) differ by more than 5% from their average value is less than 0.06.

the probability that it exceeds 2.5% is less than 0.35. Therefore, both models appear to produce similar results when the vagueness of variables is varied, and we may consider that these fuzzy models are coherent.

Second, the value of k in either model P or model H depends strongly on the shapes of the input TFNs. For instance, if all vagueness occurs on the left side of the variables (i.e., $\tilde{\sigma}_i = (0.25s_i, s_i, 1.05s_i)$ and $\tilde{V}_i = (0.25v_i, v_i, 1.05v_i)$; $1 \leqslant i \leqslant N$), we find smaller k values. In the opposite case, we find larger values of k if all vagueness is concentrated on the right band of the TFN. However, we cannot define the TFN $\tilde{\sigma}$ and $\tilde{V}$ as we wish. In the case considered here, we have selected the maximum uncertainties on both the fog extinction coefficient $\tilde{\sigma}$ and the estimated visibility $\tilde{V}$.

Third, k taken as fuzzy number $\tilde{k}$ (model P) includes most values proposed by researchers. Referring to (12), assuming $\vartheta = 1$ [8] and $\varepsilon = 0.02$ [7], we can calculate the inherent contrast C_0 of an object in fog (for instance a tree):

$$|C_0| = |\varepsilon| e^k. \tag{24}$$

In other words, the absolute value of inherent contrast C_0 of a tree in fog should be about 1.32 ($k_P = 4.19$) or 1.47 ($k_H = 4.30$). As the infimum of inherent contrasts is -1 (perfect black body against a white background), C_0 from (24) is necessarily a positive quantity, that is to say an object brighter than fog. This result can be true only for a target brighter than the background [5]. In the daytime, a majority of the trees are darker than the fog, meaning that these hypotheses are not valid. We thus think that several of the usual assumptions made on parameters in conjunction with (12) may be very risky and can be avoided using fuzzy logic.

The following concluding points have been reached:

1. A proper definition of standardized visibility is necessary for decision making under conditions of fog, in particular, for setting the speed limit.
2. Physical considerations provide the relationship $k = \sigma \cdot V$. Assumptions must be made about k as a function of ε, C_0 and ϑ. Furthermore, observations on V, σ, and $k(\varepsilon, C_0, \vartheta)$ are imprecise and often not available.
3. Depending on the assumptions and experimental data available, various researchers have found the value of k to vary from 1.8 to 3.9 although they had considered only idealized cases.
4. A fuzzy approach can account for a realistic range of values of k; here two fuzzy models, H and P, have been developed.
5. The first approach (model H), a fuzzy hyperbolic model, minimizes the distance between the estimated visibility and the corresponding measured fog extinction coefficient. This model yields a crisp value of k.

6. The second (model P) calculates a fuzzy number $\tilde{k}$ as the product of TFN's: $\tilde{k} = \tilde{\sigma} \cdot \tilde{V}$, and then a point value is calculated by defuzzification.
7. The two values $k_P = 4.19$ and $k_H = 4.30$ are quite close so that the value $k = 4.30$ may be selected to determine a standardized visibility.

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