USE OF WHOLE-FIELD DISPLACEMENT MEASUREMENTS FOR MODEL UPDATING OF BLADES

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ABSTRACT

Non-contacting measurement techniques such as modal holographic interferometry look very promising for finite element (F.E.) model updating or error localization of plate-like structures in the field of structural dynamics. The purpose of this work is to investigate a way to better exploit the high spatial resolution of optical techniques in order to correct FE meshing discretization errors and/or model parameter errors. The key idea developed in this paper is to calculate successively two error estimators using only measurement data. The experimental field is first used for the detection of singular regions corresponding to high deformation (or stress) gradients. This estimator indicates the regions where a mesh refinement is required. Thus a second estimator is calculated and used for parameter error detection.

1. INTRODUCTION

The design of compressor blades of turbojet engines requires accurate predictions of their dynamic behavior in terms of modal parameters (eigenfrequencies and mode-shapes). To this end, finite element (F.E.) models are usually built and dynamic testing is performed. Compressor blades are excited by pressure fluctuations induced in the gas flow which results in excitation frequencies corresponding to high order harmonics of the rotation speed of the engine. For this reason, modal testing of compressor blades has to be performed on a wide frequency range (typically up to 20 kHz) and necessitates the identification of a high number of modes (typically up to 20 modes). Due to the relative small size and the lightness of compressor blades, the use of distributed sensors (e.g. accelerometers or strain gauges) is not recommended and non intrusive measurement techniques are preferred. In this perspective, interferometry techniques such as Electronic (or Digital) Speckle Pattern Interferometry (ESPI or DSPI), holographic interferometry or velocimetry are very promising techniques for experimental modal analysis of turbomachine blades and more generally of plate-like structures [1-7]. Such techniques enable to extract the 3-D displacement fields at each point of the measured surface.

Updating methods may then be used to adjust finite element models to test results. The field of application of these methods includes :

- the correction of *approximation errors* ; this type of error is related to assumptions regarding the physics of the model as for example, the linear behavior of the structure, the physical behavior laws (model of elasticity, etc.), in modeling connections and boundary conditions, the limitations of the mathematical formulations used for deriving particular finite elements and the representation of non-accessible structural data (dissipative phenomena) ;
- the correction of *discretization errors* related to errors arising from a model that is too coarse for capturing some of the significant dynamics of the system ; this type of error is related to the optimization and automation of finite element meshing for dynamic computations ;
- the correction of *parametric errors* caused by the differences in measured material properties (elasticity modulus, density, section and thickness, etc.), especially for complex new material systems.

2. EXPLOITING WHOLE-FIELD MEASUREMENT DATA

Since the interferometry based measurement techniques allow to obtain a 2-D grid of points at a high resolution in comparison with the 3-D finite element analysis results, the major difficulty is no more the lack of experimental information but the way to exploit the measured field at its best. The main challenge becomes the transformation of optical measurement data into values that are meaningful from a F.E. point of view. For example, in the F.E. method, the approximation of the displacement field is of polynomial type ; moreover, as the displacement field is kinematically admissible, the F.E. stress field is discontinuous.

The methodology considered here for model error detection based on experimental data is established by analogy with adaptive finite element refinement techniques. The key idea is to introduce the experimental field as the reference field in the mesh discretization error calculation process in order to detect different types of errors.

In the following, the formulation will follow closely the one used in reference [8].

2.1. Representation of the experimental field

When the measured and calculated values are of the same nature and associated to the same topology, the discrepancy between the F.E. results and the experimental data is easy to determine. At this stage, one would like to be able to compare the measured value at each pixel of the holographic image with the corresponding value of the discretized F.E. solution. However, the transformation of the experimental data (defined in the absolute reference frame) into the intrinsic coordinate system of a single finite element reveals itself as being a difficult inverse problem. Moreover, the continuity of the stress field is not achieved in a kinematically admissible F.E. approach. For these reasons, it appears necessary to fit the experimental displacement field using the same polynomials as the ones used in the finite element formulation.

In the following, the notation φ_r will be used to represent a component of the recovered (displacement or stress) fields i.e. the continuous field resulting from the fitting of the experimental displacement field. The recovered field takes the form of a polynomial defined on a patch of elements corresponding to an area Ω_r of the measured surface.

$$\varphi_r = \mathbf{P}_n \ \mathbf{a} \tag{1}$$

or

$$\varphi_r = \sum_{i=1}^{T_n} \left(x^j \ y^k \ z^\ell\right)_i \ a_i \quad \text{with} \quad j+k+\ell \le n \tag{2}$$

where the number of terms is equal to :

$$T_n = \frac{(n+1)(n+2)(n+3)}{6}$$

In equation (1), n represents the maximum degree of the polynomial. In the case of whole-field measurements (such as interferometry techniques), the number of measured coordinates is generally much higher than the number of terms used to interpolate the displacement field in the F.E. model.

2.2. Definition of the patch area

The patches define the areas drawn by the projection of several finite elements on the measurement grid and in general, they are partially overlapping. Usually, the patch area associated to node j is made up of the set of elements that are connected to this node. This method is known as the "patch recovery" method [9].

$$\Omega_{rj} = \bigcup_{i=1}^{m_j} \Omega_i \tag{3}$$

where Ω_i represents the element n° i and m_j the number of elements connected to node j.

The nodes taken into account to build the patches are only those defining the surface of an element which contains measured points. When dealing with holographic measurement techniques, a patch corresponds to a 3-D surface and not to a volume.

2.3. Construction of the field φ_r

The vector of unknown parameters $\mathbf{a}$ in equation (1) are obtained by minimization of the difference between the experimental results and the corresponding fitted values at the points of measurement in the patch. This results in minimizing the function :

$$\omega(x_i, y_i, z_i)\left(\varphi_r(x_i, y_i, z_i) - \varphi_{\mathrm{EXP}}(x_i, y_i, z_i)\right) \tag{4}$$

where $\omega(x_i,\ y_i,\ z_i)$ is a weighting function that can be used to balance the influence of the point of coordinates $(x_i,\ y_i,\ z_i)$ according to its distance to the central node of the patch.

Taking into account equation (1), the function to be minimized takes the form

$$\underset{\mathbf{a}}{Min}\ R_i(\mathbf{a}) \overset{\Delta}{=} \omega(x_i, y_i, z_i)\left(\mathbf{P}_n(x_i, y_i, z_i)\,\mathbf{a} - \varphi_{\mathrm{EXP}}(x_i, y_i, z_i)\right) \tag{5}$$

Due to the high number of experimental data, the problem defined by equation (5) is over-determined and its solution can be estimated using the least square method. For this purpose, let us define :

$$F(\mathbf{a}) = \sum_{i=1}^{m_i} R_i^2 \tag{6}$$

where m_i is the total number of measured points in the patch.

The minimization procedure leads to the system of equations

$$\sum_{i=1}^{m_i} \omega^2(x_i, y_i, z_i)\,\mathbf{P}_n^T(x_i, y_i, z_i)\,\mathbf{P}_n(x_i, y_i, z_i)\,\mathbf{a}$$

$$= \sum_{i=1}^{m_i} \omega^2(x_i, y_i, z_i)\,\mathbf{P}_n^T(x_i, y_i, z_i)\,\varphi_{\mathrm{EXP}}(x_i, y_i, z_i)$$

which can be put in the matrix form :

$$\mathbf{a} = \mathbf{A}^{-1}\,\mathbf{b} \tag{7}$$

Note that the number of equations to be solved in each patch is low. This makes the method less expensive than a method that would use a global projection. It should be noted that this construction process leads to a smoothed experimental field. The difference between the fitted field and the experimental one has to be quantified at a previous stage when the F.E. mesh quality is checked. If the F.E. model discretization of the structure is well adapted, the difference between the two fields is negligible.

2.4. Recovery of the whole-field

Once the field φ_r has been built on the patch areas, it becomes possible to evaluate the value of the continuous experimental field at any point of the structure in the global coordinates. Let us consider a point located on the surface of the structure : one keeps first the values of φ_r on all the areas of the patch to which the point belongs. Thus one calculates the weighted average using the following equation

$$\tilde{\varphi}_{\mathrm{EXP}}\left(x_j, y_j, z_j\right) = \sum_{r=1}^{m_j} \frac{\varphi_r\left(x_j, y_j, z_j\right) / C_j^r}{\sum_{r=1}^{m_j} \frac{1}{C_j^r}} \tag{8}$$

where m_j is the number of areas including the point j ; C_j^r is a weighting factor corresponding to the distance between the point and the node which is associated to area r.

The value of the fitted experimental field is then available at any points and more particularly, at the Gauss points of a given finite element for instance.

2.5. Accuracy of the fitting process

The first step in the exploitation of the experimental data is now to quantify the accuracy of the fitting process. To this end, the discrepancy between each measured value and the fitted one is checked at the element level by verifying that

$$\int_{\Omega_e} \tilde{\varphi}^N_{\mathrm{exp}}(x_i, y_i, z_i) - \varphi_{\mathrm{exp}}(x_i, y_i, z_i) \le \varepsilon \tag{9}$$

where N symbolizes the degree of the fitting.

If this error is above a permissible value ε, it indicates discretization problems in the F.E. model. Otherwise the fitted experimental field may be considered as an acceptable representation of the measurement data and can be used as reference for error estimations.

3. ERROR ESTIMATES

Once the experimental displacement field has been fitted using a recovery technique, it can be used to compute error estimators. The error is defined as the difference between the fitted experimental field (considered as the "exact" solution) and the approximate one (i.e. the finite element solution). The estimators developed in this work are especially intended to the study of mid-size blades. These blades are usually modeled by two layers of solid elements (parallelepiped or prism) in which the displacement fields are interpolated by polynomials of degree two. It should be noticed that only the points on the external surface of the blade are measured.

The error estimator at a given point i is defined as

$$E(i) = \tilde{\varphi}_{\mathrm{EXP}}(i) - \varphi(i) \tag{10}$$

where φ_{EXP} is the fitted experimental field and φ may be either the unfitted experimental field or the finite element one. When the unfitted experimental field is considered, the estimator includes the discretization errors.

At an element level, the energy norm error estimator takes the form :

$$\left(\int_{\Omega_e} (\tilde{\boldsymbol{\sigma}} - \boldsymbol{\sigma})^T\ \mathbf{D}^{-1}\ (\tilde{\boldsymbol{\sigma}} - \boldsymbol{\sigma})\, d\Omega_e \right)^{1/2} \tag{11}$$

where $\mathbf{D}$ is the elasticity matrix and $\boldsymbol{\sigma}$ represents the stress tensor.

3.1. Discretization errors

3.1.1. Error estimation using the shape functions

Knowing the value of the fitted experimental field ($\tilde{\mathrm{q}}_{\mathrm{e}}$) at the nodes and at the Gauss points of the external surface of element e, the following error may be calculated :

$$E_N = \left(\sum_{i}^{np} \mathbf{N}_e(\chi_i)\, \tilde{\mathbf{q}}_e \quad - \quad \tilde{\varphi}_{\mathrm{EXP}}\ (\chi_i) \right) \frac{1}{np} \tag{12}$$

where np is the number of Gauss points inside element e, $\mathbf{N}_e$ is the shape function matrix of element e and χ_i are the normalized coordinates of the Gauss points.

This is an indicator of the ability of the shape functions to well represent the experimental field on an element.

3.1.2. Error estimation using the stresses

In order to evaluate the accuracy of the fitting of the stress field at the element level, the averaged stress value over an element *E* is compared with the averaged stress value of a smaller element (or sub-element) *e* built inside. The size of the element is maintained constant along the thickness of the blade. The error is defined as

$$E_\sigma = \frac{1}{np} \sum_{i}^{np} \mathbf{D}\left(\nabla\ \mathbf{N}_E\left(\chi_i^e\right)\right)\tilde{\mathbf{q}}_e - \mathbf{D}\left(\nabla\ \mathbf{N}_E\left(\chi_i^e\right)\right)(\mathbf{N}_e(\chi_i^E)\ \tilde{\mathbf{q}}_E) \tag{13}$$

The first term in equation (13) represents the averaged stress value of sub-element e calculated from the measured displacement field at the nodes of sub-element e. The second term is also an estimate of the averaged stress value of sub-element e but it is interpolated from the values of the measured displacement field at the nodes of element E. This error estimator enables the detection of finite element mesh discretization errors.

3.2. Parameterization error

Once the mesh refinement of the F.E. model has been checked and that the discretization errors have been evaluated, the fitted experimental field may be used to estimate possible errors in the parameters of the F.E. model (such as Young modulus, thickness, density, …). The basic equation for the calculation of the discrepancy between the fitted experimental field and the corresponding F.E. field is :

$$E_P(i) = \tilde{\varphi}_{\text{EXP}}(i) - \varphi_{EF}(i) \qquad (14)$$

where φ designates a displacement, a strain or a stress field and i is a node or any point inside an element.

4. APPLICATION EXAMPLE : THE CANTILEVER PLATE

The process of detection of the different types of errors in a F.E. model has been validated on the example of a cantilever plate structure of dimensions (60 x 90 x 3 mm). The plate is made of steel (Young's modulus = 2.1 10^{11} N/m^2). The finite element model considered to simulate the measured displacement field totalizes 9,256 elements and 154,860 degrees of freedom. For sake of conciseness, one single high frequency mode is considered here (figure 1) but a complete study would include the whole set of modes over the frequency band of interest. The considered mode was corrugated by noise to simulate experimental data. The maximum amplitude of noise was fixed to 2% of the maximum displacement amplitude. A colored noise (composed of 1% of white noise and 1% of a combination of the two nearest modes) was considered. To illustrate the different error estimation indicators, a simulated defect was also introduced in the structure in terms of a stiffness reduction located at point ***D*** (at coordinates 60 x 55 mm) (figure 1). The defect corresponds to a 8 x 4 x 3 mm area in which the elasticity modulus was reduced by 30 %.

4.1. Initial finite element model

An initial finite element model made of 400 elements (7,080 degrees of freedom) was first considered. For a fitting with polynomial function of degree 4, the maximum of the error on the fitting is about 0.037% (equation (10)).
The correlation in terms of Modal Assurance Criterion (MAC) between the fitted experimental field and the measured one for mode n° 11 and for the three measurement directions *x*, *y* and *z* is excellent (MAC values are respectively equal to 0.99981, 0.99909, 0.99994). Such values would cause the end of the optimization process of any updating programs based on the MAC values.

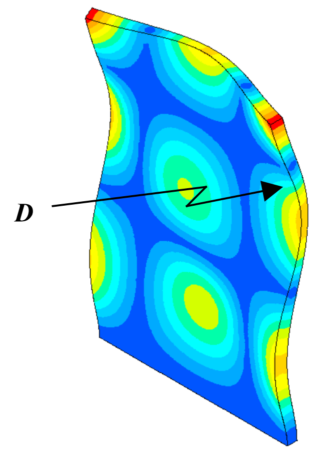

Figure 1.- Simulated optical measurements for mode n°11 at 12,020 Hz (no noise). Location of the defect.

4.1.1. F.E. mesh discretization errors

Results of the procedure for the discretization error localization are shown in figure 2.

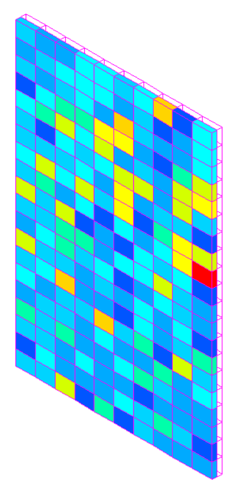

Figure 2.- Error estimation using equation (13)

It can be observed that the global mode is not correctly represented and that the mesh is not sufficiently fine near the clamped edge. Accordingly, the mesh should be completely refined.

4.1.2. F.E. parameterization errors

Despite the presence of discretization errors, the detection of the errors on the parameters clearly indicates (figures 3 and 4) the presence of an error near the point of coordinates (60 x 55 mm) but also errors at the clamped edge.

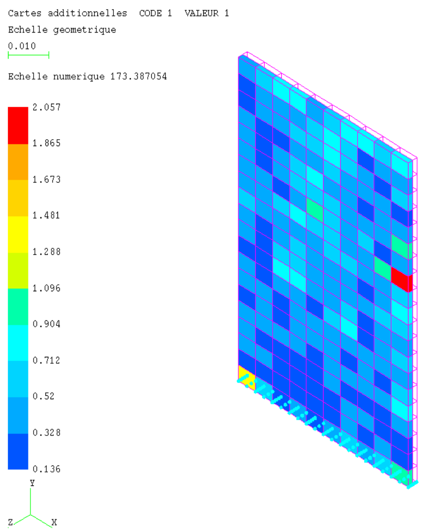

Figure 3.- Error estimation using equation (14) in terms of stresses

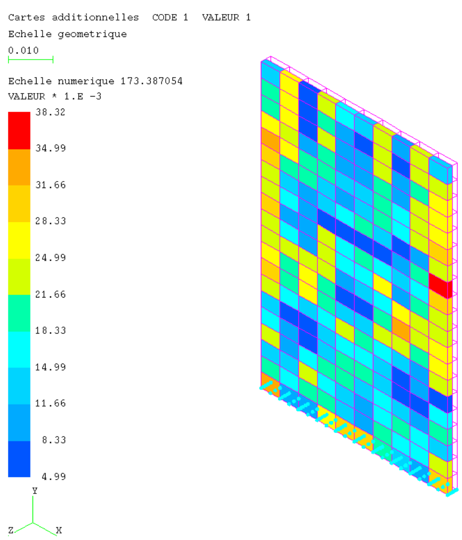

Figure 4.- Error estimation using equation (14) in terms of displacement gradients

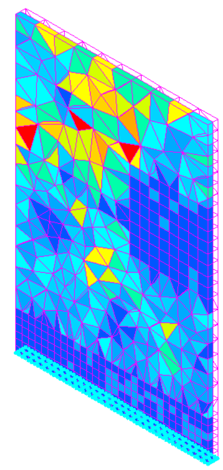

Figure 5.- Error estimation on the discretization using equation (12)

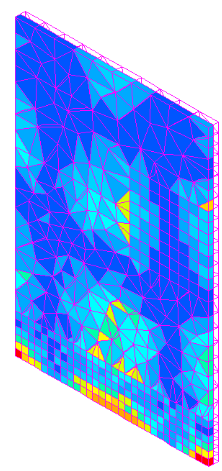

Figure 6.- Error estimation on the discretization using equation (13)

4.2. Refined F.E. model

A very basic mesh refinement was realized on the areas where problems were outlined by the first model (near the point of coordinates 60 x 55 mm and close to the clamping). The refined model totalizes 1,200 elements and 15,912 degrees of freedom.

4.2.1. F.E. mesh discretization errors

A discretization error is still visible (figures 5 and 6) near the clamped end but it is several orders of magnitude lower than the one revealed by the first model. Near the defect, the mesh is good enough and the global mode is now correctly represented.

4.2.2. FE parameterization errors

The error on the elements that have been localized is about 30 % of the absolute value of the three principal components of the stress calculated at the super convergence points of the F.E. elements (figures 7 and 8).

5. CONCLUSIONS

The results of field measurement techniques have been exploited for the detection and the localization of errors in a FE model. The proposed method has been tested using simulated data with noise and has shown its efficiency and its performance. The next step will be to validate the method on a compressor blade using actual optical data.

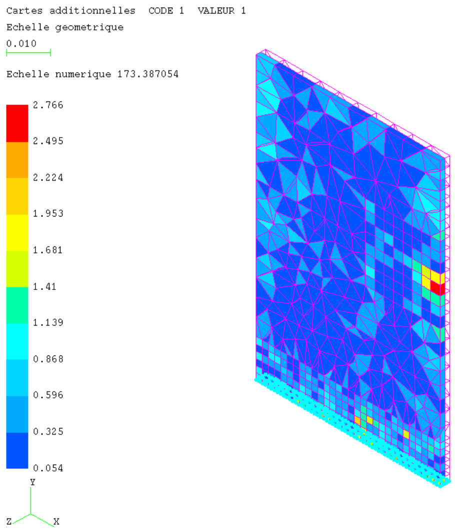

Figure 7.- Error estimation using equation (14) in terms of stresses

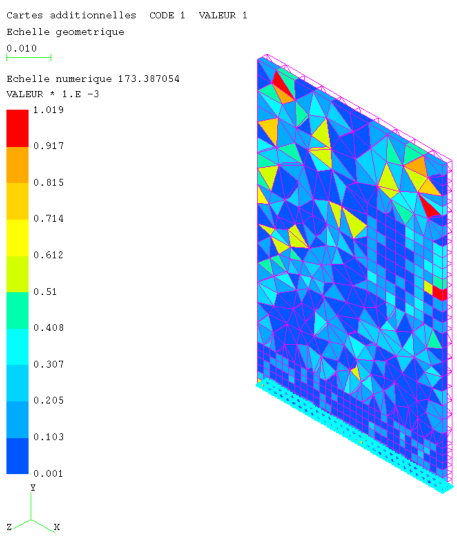

Figure 8.- Error estimation using equation (14) in terms of displacement gradients

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