MODAL IDENTIFICATION OF THE "STEEL-QUAKE" STRUCTURE USING THE DATA-DRIVEN STOCHASTIC SUBSPACE AND ARMAV METHODS

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SUMMARY: In this paper, two techniques used for modal identification of output-only systems are presented and compared. The first technique is based on an ARMAV model. The method is known as the prediction error method (PEM) and requires a non-linear iterative optimisation procedure. The second technique is a stochastic subspace method that estimates the system matrices of a stochastic state space model by a data-driven algorithm and by using numerical techniques such as singular value and QR decompositions. The comparison between both techniques is performed over the "Steel-Quake" benchmark proposed in the framework of COST Action F3 "Structural Dynamics". The results show that the investigated techniques give good results in term of estimated modal parameters. Especially, it is found that the stochastic subspace technique is much faster than the PEM.

KEYWORDS: stochastic process, optimal prediction, prediction error method, stochastic subspace algorithm, singular value and QR decompositions, modal parameters uncertainties.

INTRODUCTION

In modal analysis of civil engineering structures, the artificial controlled excitations are often not practical and too expensive. On the other hand, ambient excitation sources such as traffic and wind are natural excitations. To identify structures excited by unknown input, different methods based on output-only measurements have been developed. In this paper, two system identification techniques are presented: the prediction error method applied to ARMAV (Auto Regressive Moving Average Vector) models and the data-driven stochastic subspace method. The first method allows to identify the model parameters in a non-linear iterative optimisation way [1] while the second technique finds the system matrices of a stochastic state space model without any non-linear calculation [2]. Practical aspects such as the model order selection and the distinction between the physical modes and the spurious modes are outlined. The estimation of the modal parameter uncertainties are also investigated for both methods. The quantification of these uncertainties is very relevant for structural monitoring based on vibration measurements. In this way, if the uncertainties of the estimated modal parameters can be computed, it becomes possible to establish a probabilistic confidence in the existence of a damage [3].

ARMAV (AUTO REGRESSIVE MOVING AVERAGE VECTOR) TECHNIQUE

The ARMAV technique works directly with the recorded time signals. The identification method known as the *Prediction Error Method* (PEM) allows to find the ARMAV model parameters in a non-linear, iterative way. A detailed description of the PEM is provided in [1]. A description of the ARMAV technique can also be found in a recent paper by the present authors [3]. Here only the main ideas behind the method are recalled.

Let us assume that the structure behaves linearly and is time-invariant and that the unknown input force can be modelled by a white noise. Given a *m*-dimensional time series $\mathbf{y}[n]=\mathbf{y}(n\Delta t)$, where Δt is the sampling period, the parametric ARMAV(*p*,*q*) model is described as:

$$\mathbf{y}[n] = \sum_{k=1}^{p} \mathbf{a_k}\ \mathbf{y}[n-k] + \mathbf{u}[n] + \sum_{k=1}^{q} \mathbf{b_k}\ \ \mathbf{u}[n-k] \tag{1}$$

where $\mathbf{u}[n]$ is a stationary zero-mean Gaussian white noise process. $\mathbf{a_k}$ and $\mathbf{b_k}$ are (*m*,*m*) matrices of AR (Auto Regressive) and MA (Moving Average) coefficients. The AR part of order *p* describes the system dynamics while the MA part of order *q* is related to the external noise as well as to the white noise excitation. In these linear parametric models, the system output $\mathbf{y}[n]$ is supposed to be produced by a stationary Gaussian white noise input $\mathbf{u}[n]$. By using this approach, one may analyse linear systems where only the system output is measured, while the input is unknown but produced by uncorrelated random signals. These models can also be used directly to analyse data obtained from the free response of linear systems [4].

Let us note by $\mathbf{q}$ the set of model parameters to be determined. The model parameters are estimated so that the prediction error $\mathbf{e}[n \mid \mathbf{q}] = \mathbf{y}[n] - \hat{\mathbf{y}}[n \mid \mathbf{q}]$, i.e. the difference between the measured time signals and the predicted output of the ARMAV model, becomes as small as possible. For this purpose, a criterion function $V_N(\mathbf{q})$ is formed:

$$V_N(\mathbf{q}) = \det\left(\frac{1}{N} \sum_{n=1}^{N} \mathbf{e}[n \mid \mathbf{q}]\, \mathbf{e}[n \mid \mathbf{q}]^T \right) \tag{2}$$

The model parameter estimate based on N samples is then defined by minimisation of the criterion function (2). The predictor $\hat{\mathbf{y}}[n \mid \mathbf{q}]$ of the ARMAV model is non-linear. This implies that an iterative numerical minimisation of the criterion function has to be applied [1]. To start the iterative procedure, a first estimation of model parameters is needed. Due to the possible occurrence of undesired local minima in the criterion function, it is worthwhile to spend some efforts on producing good initial values for the iterative search procedure. Therefore, a *Multi-Stage Least-Squares Method* [5] is used before the optimisation procedure.

The model order is in general not known *a priori*, and several criteria have been proposed to find the best model order. Two of the most widely used techniques for selecting the order of a parametric model are the *Akaike's Final Prediction Error Criterion (FPE)* and *Akaike's Information Theoretic Criterion (AIC*) [1].

Estimation of ARMAV models using the prediction error method is known to provide asymptotically unbiased and efficient model parameter estimates for Gaussian distributed prediction errors. In this case, an estimate of the model parameter covariance matrix of the difference between the true parameter and estimated parameter as N tends to infinity can be performed [1] and it becomes possible to determine the modal parameter uncertainties [6].

STOCHASTIC SUBSPACE TECHNIQUE

State space modelling of ambient vibrating structures

Like the ARMAV method, the stochastic subspace method directly works with the recorded time signals. In the case of ambient vibration testing, the technique assumes that the dynamic behaviour of a structure can be described by a stochastic state space formulation of the form:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{A}\,\mathbf{x}[k] + \mathbf{w}[k] \\ \mathbf{y}[k] &= \mathbf{C}\,\mathbf{x}[k] + \mathbf{v}[k] \end{aligned} \tag{3}$$

The general assumptions are that the underlying physical system behaves linearly and is time-invariant and that the stochastic process is stationary with zero-mean. The matrices **A** and **C** are respectively the state space matrix and the output matrix; **x**[k] represents the state vector of dimension n and **y**[k] is the output vector of dimension m. **w**[k] and **v**[k] represent the process and the measurement noises respectively. The process noise is due to disturbances and modelling inaccuracy, whereas the measurement noise is due to sensor inaccuracy. The unmeasured inputs are implicitly modelled by the noise terms **w**[k] and **v**[k]. These unmeasurable noises are assumed to be zero-mean Gaussian white noise processes. The output covariance matrices $\mathbf{L}_0$, $\mathbf{L}_i$ and the next state-output covariance matrix **G** are defined as

$$\mathbf{L}_0 = \mathbf{E}\left[\mathbf{y}[k]\,\mathbf{y}[k]^{\mathrm{T}}\right] \quad , \qquad \mathbf{L}_i = \mathbf{E}\left[\mathbf{y}[k+i]\,\mathbf{y}[k]^{\mathrm{T}}\right] \quad , \qquad \mathbf{G} = \mathbf{E}\left[\mathbf{x}[k+1]\,\mathbf{y}[k]^{\mathrm{T}}\right] \tag{4}$$

The problem of noise contaminated systems is that it is only possible to predict the response. This prediction is accomplished by the construction of the associated Kalman filter. The aim of the Kalman filter is to produce an optimal prediction $\hat{\mathbf{x}}[k+1]$ for the state vector **x**[k+1] based on the system matrices of the stochastic state space model (3) and on available outputs up to time k. By constructing the Kalman filter, the model (3) can be converted into a so-called *innovation model* by solving a Riccati equation [2]:

$$\begin{aligned} \hat{\mathbf{x}}[k+1] &= \mathbf{A}\,\hat{\mathbf{x}}[k] + \mathbf{K}\,\mathbf{e}[k] \\ \mathbf{y}[k] &= \mathbf{C}\,\hat{\mathbf{x}}[k] + \mathbf{e}[k] \end{aligned} \tag{5}$$

where **K** is the Kalman gain and **e**[k] is the innovation which is a zero-mean Gaussian white noise process. The Kalman gain includes the description of the disturbance on the system as well the white noise excitation.

Data-driven stochastic subspace method

The data-driven stochastic subspace method relies on an output block Hankel matrix of the form [2]:

$$\mathbf{H}_{0,2i-1} = \frac{1}{\sqrt{\mathrm{j}}} \begin{bmatrix} \mathbf{y}[0] & \mathbf{y}[1] & \dots & \mathbf{y}[j-1] \\ \dots & \dots & \dots & \dots \\ \mathbf{y}[i-1] & \mathbf{y}[i] & \dots & \mathbf{y}[i+j-2] \\ \hline \mathbf{y}[i] & \mathbf{y}[i+1] & \dots & \mathbf{y}[i+j-1] \\ \dots & \dots & \dots & \dots \\ \mathbf{y}[2i-1] & \mathbf{y}[2i] & \dots & \mathbf{y}[2i+j-2] \end{bmatrix} \equiv \begin{pmatrix} \mathbf{H}_{0,i-1} \\ \hline \mathbf{H}_{i,2i-1} \end{pmatrix} \equiv \begin{pmatrix} \mathbf{Y}_p \\ \hline \mathbf{Y}_f \end{pmatrix} \begin{matrix} \text{"past"} \\ \text{"future"} \end{matrix} \tag{6}$$

where i is the user-defined number of block rows and j the number of columns which should be as large as possible. The Hankel matrix $\mathbf{H}_{0,2i-1} \in \Re^{2mi \times j}$ can be splitted into a past and a future part of i block rows. For all theoretical derivations, it is assumed that $j \to \infty$.

In order to obtain a data reduction, the first step of the procedure consists in the QR-factorisation of the block Hankel matrix (6):

$$\mathbf{H}_{0,2i-1} = \mathbf{R}\mathbf{Q}^{\mathrm{T}} \tag{7}$$

where $\mathbf{Q}^{\mathrm{T}} \in \Re^{2mi \times j}$ is an orthonormal matrix ($\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{I}_{2mi}$) and $\mathbf{R} \in \Re^{2mi \times 2mi}$ is a lower triangular matrix (since typically $2mi << j$, zeros in $\mathbf{R}$ and corresponding values of $\mathbf{Q}$ have been omitted, leading to a first data reduction). Due to the orthonormality of $\mathbf{Q}$-factors, it can be shown that the final calculations of the system matrices will be performed only from the matrix $\mathbf{R}$ [7]. Therefore, an important data reduction is achieved.

The second step of the procedure concerns the projections which are important in data-driven subspace methods. In order to retain all the information in the past that is useful to predict the future, an orthogonal projection of the row space of future outputs into the row space of past outputs is performed [2]:

$$\mathbf{P}_i = Proj(\mathbf{Y}_f, \mathbf{Y}_p) = \mathbf{Y}_f \mathbf{Y}_p^{\mathrm{T}} \left(\mathbf{Y}_p \mathbf{Y}_p^{\mathrm{T}}\right)^{\oplus} \mathbf{Y}_p \tag{8}$$

where superscript $^{\oplus}$ denotes the Moore-Penrose pseudo-inverse of the matrix. It should be noted that this projection can be easily implemented using the above QR-factorisation [7].

The main theorem of stochastic identification which allows to recover the Kalman filter state estimates $\hat{\mathbf{x}}[k]$ directly from the output data is now presented. For this purpose, let us define the Kalman filter state sequence as:

$$\hat{\mathbf{X}}_i \equiv \left\{ \hat{\mathbf{x}}[i] \;\; \hat{\mathbf{x}}[i+1] \;\; ... \;\; \hat{\mathbf{x}}[i+j-1] \right\} \tag{9}$$

This state sequence is generated by a bank of Kalman filter state estimates working in parallel on each of the columns of the past output block Hankel matrix $\mathbf{Y}_p$. Indeed, as explained in [2], the Kalman filter state estimate $\hat{\mathbf{x}}[k]$ can be written as a linear combination of the past outputs $\mathbf{y}[0]$, …, $\mathbf{y}[k\text{-}1]$.

The subspace identification algorithm makes extensive use of the observability matrix and of its structure. The extended ($i > n$) observability matrix is defined as:

$$\boldsymbol{O}_i = \left[\mathbf{C} \quad \mathbf{CA} \quad ... \quad \mathbf{CA}^{i-1} \right]^{\mathrm{T}} \in \Re^{mi \times n} \tag{10}$$

Note that, in order to use the stochastic subspace method, the matrix pair {$\mathbf{A}$, $\mathbf{C}$} is assumed to be observable, which implies that the rank of $\boldsymbol{O}_i$ is equal to n.

Main theorem of stochastic identification

The projection $\mathbf{P}_i$ defined in equation (8) is equal to the product of the extended observability matrix (10) and the Kalman filter state sequence (9) [2]:

$$\mathbf{P}_i = \boldsymbol{O}_i \, \hat{\mathbf{X}}_i \tag{11}$$

To obtain both factors $\boldsymbol{O}_i$ and $\hat{\mathbf{X}}_i$ of equation (11), the singular value decomposition is applied to the weighted projection matrix $\mathbf{W}_1 \, \mathbf{P}_i \, \mathbf{W}_2$:

$$\mathbf{W}_1 \, \mathbf{P}_i \, \mathbf{W}_2 = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 \end{bmatrix}^{\mathrm{T}} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^{\mathrm{T}} \tag{12}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ are two user-defined weighting matrices. $\mathbf{S}_1$ contains $n = 2N_m$ non-zero singular values in decreasing order, where N_m is the number of system modes; n is the order of the system (3) and $rank\left(\mathbf{W}_1 \mathbf{P}_i \mathbf{W}_2\right) = n$. Combining equations (11) and (12) gives

$$\boldsymbol{O}_i = \mathbf{W}_1^{-1} \mathbf{U}_1 \mathbf{S}_1^{1/2} \quad , \qquad \hat{\mathbf{X}}_i = \boldsymbol{O}_i^{\oplus} \, \mathbf{P}_i \tag{13}$$

The major observation in the main theorem of stochastic identification is that the system matrices do not have to be known to determine the Kalman filter state sequence $\hat{\mathbf{X}}_i$. It can be determined directly from output data through orthogonal projection. It explains why this algorithm is called *subspace* identification technique: the algorithm retrieves the system related matrices as subspaces of projected data matrices. Different choices of weighting matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ will lead to different stochastic subspace identification methods. Three particular choices for the weighting matrices give rise to the *unweighted principal component* (UPC), the *principal component* (PC) and the *canonical variate* algorithms (CVA) [2]. The specific technique used in this investigation is the CVA.

In theory, the system order can be determined as the number of non-zero singular values of the weighted projection matrix. In practice, the number of measurements is not infinite and noises such as modelling inaccuracies, measurement noise and computational noise are present. Therefore, the “higher” singular values are not exactly zeros. In this case, the order can be selected by inspection of the singular values. The singular value where a significant drop occurs yields the model order. However, in many practical cases, no significant drop in the singular values can be observed. In this case, the problem of order determination can be solved by constructing a stabilisation diagram for increasing model order.

Determination of the system matrices

The block Hankel matrix (6) can also be expressed by the following formulation:

$$\mathbf{H}_{0,2i-1} \equiv \left(\frac{\mathbf{H}_{0,i}}{\mathbf{H}_{i+1,2i-1}} \right) \equiv \left(\frac{\mathbf{Y}_p^+}{\mathbf{Y}_f^-} \right) \begin{matrix} \text{"past"} \\ \text{"future"} \end{matrix} \tag{14}$$

where $\mathbf{Y}_p^+$ is obtained by adding one block row to the past outputs and $\mathbf{Y}_f^-$ is obtained by omitting the first block row of the future outputs. Through a similar development as for the main theorem of subspace identification, it can be shown that

$$\mathbf{P}_{i-1} = Proj(\mathbf{Y}_f^-, \mathbf{Y}_p^+) = \boldsymbol{O}_{i-1} \, \hat{\mathbf{X}}_{i+1} \tag{15}$$

$\boldsymbol{O}_{i-1}$ can be easily calculated by dropping the last m (number of outputs) rows of $\boldsymbol{O}_i$ computed from (13). Now the shifted state sequence $\hat{\mathbf{X}}_{i+1}$ can be calculated from:

$$\hat{\mathbf{X}}_{i+1} = \boldsymbol{O}_{i-1}^{\oplus} \mathbf{P}_{i-1} \tag{16}$$

The Kalman filter state sequences $\hat{\mathbf{X}}_i$ and $\hat{\mathbf{X}}_{i+1}$ being estimated, the system matrices **A** and **C** can be easily recovered by solving the following set of equations:

$$\begin{pmatrix} \hat{\mathbf{X}}_{i+1} \\ \mathbf{H}_{i,i} \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{C} \end{pmatrix} \hat{\mathbf{X}}_i + \begin{pmatrix} \mathbf{r}_w \\ \mathbf{r}_v \end{pmatrix} \tag{17}$$

where $\mathbf{H}_{i,i}$ is a block Hankel matrix with only one row of outputs. The set of equations is solved in a least squares sense, the Kalman filter residuals $\mathbf{r}_w$ and $\mathbf{r}_v$ being uncorrelated with $\hat{\mathbf{X}}_i$. It can be shown that the least squares solution gives an asymptotically unbiased estimate of **A** and **C**. At this point, the matrices **G** and $\mathbf{L}_0$ defined in (4) can be determined. The matrix $\mathbf{L}_0$ is determined as a sample covariance matrix of the output and **G** is computed via a reversed extended controllability matrix [2]. An important observation is that the identified covariance sequence $\mathbf{L}_i$ (4) determined by **A**, **G**, **C**, $\mathbf{L}_0$ should be a positive real sequence. If this is not true, it is not possible to obtain the innovation form (5). A way to ensure positive realness of the estimated covariance sequence is explained in [2]. However, this computation introduces a small perturbation on **G** and $\mathbf{L}_0$.

THE "STEEL-QUAKE" EXAMPLE

The two methods have been validated using experimental data proposed as benchmark in the framework of the European COST Action F3 "Structural Dynamics". The "Steel-Quake" structure is used at the Joint Research Centre in Ispra (Italy) to test the performance of steel buildings during earthquakes [8]. The different tests performed correspond to the undamaged and damaged states of the building.

Description and testing of the structure

Fig. 1: View of the Steel-Quake structure

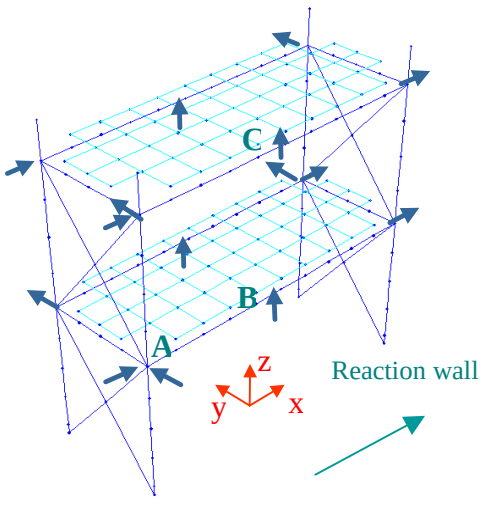

Fig. 2: Sensor and excitation configuration

The structure corresponds to a two-floor frame as depicted in Fig. 1. The main dimensions are 8 m × (4×2) m × 3 m. In the background, it can be observed the reaction wall which supports the 4 pistons (not present in the picture) used to deform the structure (on each side, on each stage) and to induce damage in the *x*-direction. Note that braces have been added in the plane parallel to the wall to reduce risk of collapse in that direction. Four excitation points were tested using impact hammer. Their locations are shown in Fig. 2 (points A(*x*), A(*y*), B(-*z*), C(-*z*); *x*, *y* and *z* indicating the three directions). Eight to ten hammer impacts were recorded for each test. The sensor configuration is the same for all the four tests (Fig. 2).

Structural identification

The ARMAV method (PEM)

The modal parameter estimation using the ARMAV method has already been carried out by the present authors in [3]. Therefore, only the results are presented. The results for the undamaged state are listed in Table 1.

The data-driven stochastic subspace method

The subspace method requires the selection of two important parameters: the number of block rows, *i*, in the Hankel matrix (6) and the number of used singular values in equation (13), i.e. the model order. Starting from a fixed number of block rows, the model order needs to be selected. Inspection of the singular values does not show a significant drop and consequently, this criterion is not of great use for the model order selection. Stabilisation diagrams are then needed in order to find the correct model order. For increasing model order, the stability of modal parameters is investigated and the order where the eigenfrequencies, damping ratios and mode-shapes remain stable is selected. This way, the physical modes can be distinguished from the spurious ones. The prediction errors (in percentage) are also inspected for increasing model order. These prediction errors are computed as:

$$\varepsilon = 100 \frac{1}{m} \sum_{c=1}^{m} \left[\sqrt{\frac{\sum_{k=1}^{N} \left(\mathrm{y}[k]_{\mathrm{c}} - \hat{\mathrm{y}}[k]_{\mathrm{c}} \right)^2}{\sum_{k=1}^{N} \left(\mathrm{y}[k]_{\mathrm{c}} \right)^2}} \right] \% \qquad (18)$$

where N is the number of samples and m the number of channels; $\mathrm{y}[k]_{\mathrm{c}}$ and $\hat{\mathrm{y}}[k]_{\mathrm{c}}$ are respectively the output and the one step ahead predicted output for the channel c. The selected model should lead to the smallest prediction errors. This study is performed for different selections of the number of block rows. Once the best model order is selected, the optimal number of block rows is also determined by use of a stability diagram.

As an example, the undamaged vibration data obtained from the "A(*y*) excitation" are analysed hereafter. The ten sensors in directions *x* and *y* are considered. Some investigations for different numbers of block rows show that the value 40 leads to a good choice for the model order. For example, in Fig. 3a, the stabilisation of the modal parameters is plotted against the model order for a number of block rows equal to 10. The chosen stabilisation criteria are 1% for frequencies, 5% for damping ratios and 1% for MAC values. The sum of the spectra of the 10 measured responses is also plotted in order to observe the localisation of stable modes. The plot of prediction errors (18) against model order allows to help in the selection of the model order. And finally, the stabilisation of the modal parameters is plotted against the number of block rows for the selected model order (Fig. 3b) and leads to the optimal number of block rows, which is equal to 14.

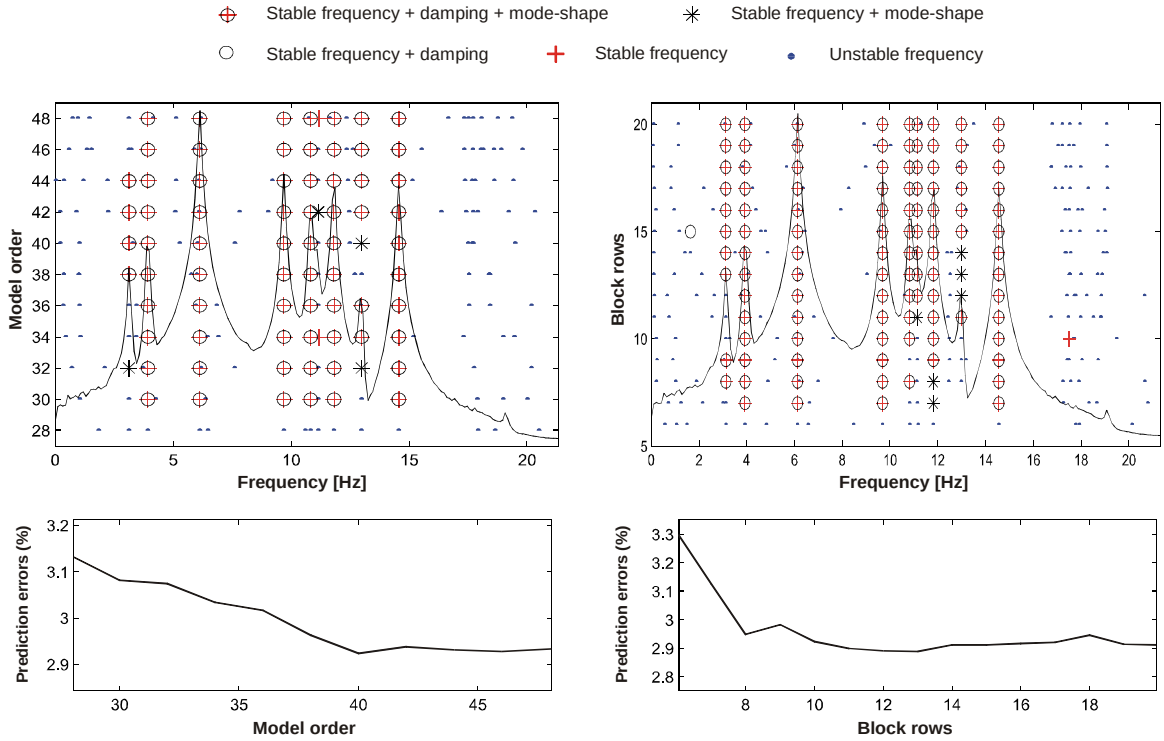

Fig. 3: Stability diagrams: (a) model order selection; (b) block rows selection

The estimated frequencies f_i and damping ratios ζ_i as well as their uncertainties are listed in Table 1 and refer to the undamaged structure. The *mean uncertainty* term indicates the average frequency and damping ratio uncertainties expressed in percent.

Table 1: Estimated modal parameters with their uncertainties for the *undamaged structure*

Mode-shape	ARMAV		Subspace	
	f_i (Hz)	ζ_i (%)	f_i (Hz)	ζ_i (%)
Bending 1X	3.1275 ± 0.0061	0.1309 ± 0.0978	3.1279 ± 0.0037	0.1437 ± 0.0705
Bending 1Y	3.9283 ± 0.0028	0.1010 ± 0.0415	3.9278 ± 0.0041	0.1186 ± 0.0593
Torsion 1	6.1288 ± 0.0055	0.1188 ± 0.0724	6.1290 ± 0.0034	0.1209 ± 0.0446
Bending 2Y	9.6872 ± 0.0064	0.0851 ± 0.0521	9.6874 ± 0.0038	0.0910 ± 0.0331
Bending 2X	10.8190 ± 0.0044	0.0978 ± 0.0404	10.8192 ± 0.0032	0.0943 ± 0.0286
2nd slab bending. 1	12.2853 ± 0.0167	0.4657 ± 0.1353	12.2817 ± 0.0149	0.4701 ± 0.1206
1st slab bending. 1	13.0804 ± 0.0157	0.4507 ± 0.1581	13.0758 ± 0.0123	0.4747 ± 0.1255
2nd slab torsion. 1	17.7089 ± 0.0119	0.3870 ± 0.0669	17.7075 ± 0.0159	0.3890 ± 0.0899
1st slab torsion. 1	19.0588 ± 0.0112	0.4095 ± 0.0655	19.0626 ± 0.0167	0.3912 ± 0.0876
Mean uncertainty	0.094 %	41.85 %	0.081 %	32.64 %

The identified mode-shapes of the undamaged structure are depicted in Fig. 4. Table 1 shows that both methods have very well identified the modal parameters. The stochastic subspace method gives estimates of modal parameters which are of same quality as the estimates obtained by the prediction error method (PEM) but with a smaller mean uncertainty. Moreover, the subspace technique gives quickly good results compared to the PEM which takes a long computation time. The same conclusions have been found for the analysis of the

damaged structure. Therefore, only the results obtained by the subspace method are presented. The modal parameters with their uncertainties are listed in Table 2.

Table 2: Estimated modal parameters with their uncertainties for the damaged structure

	Mode	f_i (Hz)	ζ_i (%)	Mode	f_i (Hz)	ζ_i (%)
Subspace method	1	2.691 ± 0.008	0.734 ± 0.296	6	10.648 ± 0.014	0.635 ± 0.220
	2	3.857 ± 0.007	0.329 ± 0.187	7	11.321 ± 0.017	0.701 ± 0.201
	3	6.079 ± 0.006	0.352 ± 0.092	8	15.023 ± 0.019	0.560 ± 0.201
	4	9.524 ± 0.005	0.203 ± 0.049	9	16.145 ± 0.024	0.973 ± 0.356
	5	9.913 ± 0.007	0.341 ± 0.092			

We note that the innovation form (5) allows to convert the identified stochastic subspace models into equivalent ARMAV models. Therefore, these converted models are used to estimate the uncertainties of the "subspace method" modal parameters. In order to achieve possible improvements in the estimation of the modal parameters, the identified stochastic subspace models were used as initial starting point in the prediction error method. However, no improvements have been observed in modal parameters estimates.

To assess the quality of the identified modal parameters, the model has to be validated. This is done by inspection of the predictions errors, which have to be white noise sequences. Through computation of the correlation functions of the prediction errors, the whiteness of the errors can be checked. We can also compare the spectra obtained by the model and by applying FFT to the data. Indeed, it is possible to estimate the spectrum based on an analytical expression including the identified state space matrices **A**, **G**, **C** and $\mathbf{L}_0$ [9]. The channel-2 spectra corresponding to the undamaged "A(y) excitation" analysis are plotted in Fig. 5.

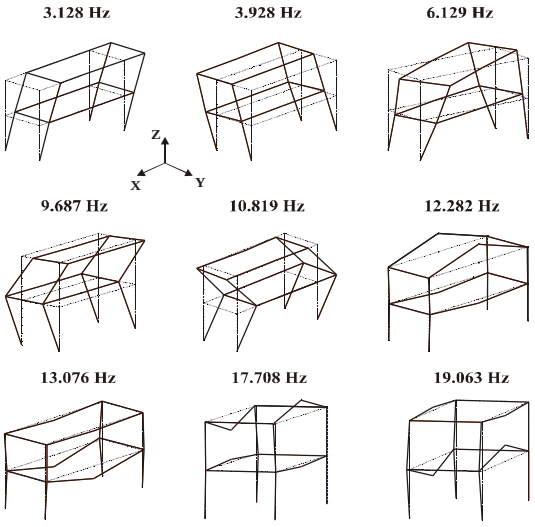

Fig. 4: Mode-shapes for the undamaged state

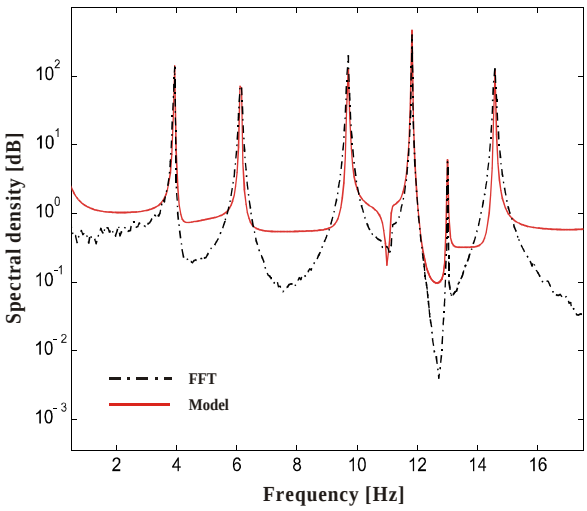

Fig. 5: Comparison of channel-2 spectra

From Fig. 5, it can be observed that the peaks are better identified than the valleys. As explained in [9], it can be understood by the implementation of the stochastic subspace algorithm. The method computes an asymptotically unbiased estimate of **A** and **C**. However, in order to obtain a stochastic state space model **A**, **G**, **C**, $\mathbf{L}_0$ that generates a positive real covariance sequence $\mathbf{L}_i$, a bias was introduced on **G** and $\mathbf{L}_0$.

CONCLUSIONS

In this paper, the application of the ARMAV and the data-driven stochastic subspace techniques in multi-channel structural identification has been presented and compared. They have demonstrated their capability to reach good estimates of frequencies and mode-shapes. The comparison reveals that the stochastic subspace method gives estimates of modal parameters which are of same quality as the estimates obtained by the prediction error method (PEM) but with a smaller mean uncertainty. Moreover, the subspace technique gives quickly good results compared to the PEM which is a highly non-linear minimisation procedure taking a long computation time, especially when the ARMAV model deals with many outputs. The subspace method has also the advantage with respect to PEM that it is computationally simple. Compared to subspace algorithms based on a covariance approach [10], the data-driven subspace method avoids the computation of the output covariances and allows in any case to obtain the innovation form of the state space which is necessary in the estimation of modal parameter uncertainties. However, the covariance-driven methods are more simple to implement and less time-consuming than the data-driven methods which imply a slower QR-factorisation step.

ACKNOWLEDGEMENT

This text presents research results of the Belgian programme on Inter-University Poles of Attraction initiated by the Belgian state, Prime Minister's office, Science Policy Programming. The scientific responsibility is assumed by its authors.

REFERENCES

1. Ljung, L., "System Identification – Theory for the User", Prentice-Hall, Englewood Cliffs, New Jersey, 1987.
2. Van Overschee, P., De Moor, B., "Subspace Identification for Linear Systems: Theory - Implementation - Applications", Dordrecht, Netherlands: Kluwer Academic Publishers, 1996.
3. Bodeux, J.B., Golinval, J.C., "ARMAV Model Technique for System Identification and Damage Detection", Proceedings of European COST F3 Conference on System Identification & Structural Health Monitoring, Madrid, June 2000.
4. Pandit, S.M., Mehta, N.P., "Data Dependent Systems Approach to Modal Analysis via State Space", Journal of Dynamic Systems, Measurement and Control, 1985.
5. Giorcelli, E., Fasana, A., Garibaldi, L., Riva, A., "Modal Analysis and System Identification using ARMAV Models", Proc. IMAC 12, Honolulu, Hawaii, 1994.
6. Andersen, P., Brincker, R., "Estimation of Modal Parameters and their Uncertainties", Proc. IMAC 17, Kissimmee, Florida, USA, 1999.
7. Peeters, B., De Roeck, G., "Reference-Based Stochastic Subspace Identification for Output-Only Modal Analysis", Mechanical Systems and Signal Processing, Vol. **13**(6), 1999, pp. 855-878.
8. Pascual, R., Molina, J., Golinval, J.C., "Short-term Scientific Mission Report", Cost F3 Action, Working Group 2, 1998.
9. Peeters, B., De Roeck, G., Andersen, P., "Stochastic system identification: uncertainty of the estimated modal parameters", Proc. IMAC 17, Kissimmee, Florida, USA, 1999.
10. Basseville, M., Abdelghani, M., Benveniste, A., "Subspace-Based Fault Detection and Isolation Methods – Application to Vibration Monitoring", Research report IRISA no 1143, 1997.