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Discussion

Wei-Cheng Cui, Visitor, Shanghai Jiao Tong University

In the current methodology of assessing the reliability of ship structures, the limit states are described in three different levels: hull girder, stiffened panels and unstiffened plates. The ultimate limit states of stiffened panels are very important in deriving the reliability of ship structures. Many research works have been reported including the authors' own contributions. However, in comparison with existing works, this paper seems to be the most comprehensive and application directed. The authors are complimented in making such an achievement. It is really a joy for me to read this paper. By reading through the paper, I have the following comments and questions on the authors' work.

(1) As far as the loading conditions and the potential failure modes are concerned, this paper has made a great step forward and the results are readily implemented into the design rules. This sets a good example of how to conduct fundamental research to a level suitable for application.

(2) Based on some arguments, the paper has defended the use of simply-supported boundary conditions. However, the real situation will never be simply supported nor clamped. In the authors' experience or opinion, how much would be the maximum effect of boundary condition in typical loading conditions and in extreme loading conditions.

(3) The approach adopted in this paper for combined loading is basically empirical in nature, but as a matter of fact, analytical solutions might be found by using an elastic large deflection solution in conjunction with rigid plastic analysis. In Cui et al (2000), such an analytical solution has been derived for simply-supported stiffened panels under a combination of longitudinal stresses, transverse stresses and lateral pressure.

(4) The load sequence effect is briefly mentioned by assuming that the lateral pressure is applied first. However, in some cases, the lateral pressure may be applied additionally and which leads to final failure such as due to impact. I would like, therefore, to ask the authors if they have investigated different load sequences and if so, how much could it be to the ultimate strength of stiffened panels?

(5) Another point is about the influence of the initial distortion on the effect of lateral pressure. In the present formulations, both lateral pressure and initial deflection will decrease the ultimate strength. When the lateral pressure is larger than a certain amount, for example, 0.035 MPa given in the present paper, the effect of lateral pressure on the ultimate strength is very significant. However, in reality, when the initial deflection and the lateral pressure are of the opposite sign, the lateral pressure may tend to suppress the tendency to buckle. Have the authors considered this effect?

(6) My final point on the paper is to ask the authors what they are going to do next? In this paper, they have thoroughly covered the major problems of structural design of stiffened panels and the results are readily implemented into the design rules. Are there any significant problems related to the ultimate strength of stiffened panels to be investigated?

In conclusion, I congratulate the authors on producing such an excellent and comprehensive paper on ultimate limit states of ship stiffened panels and I believe it is a significant and very useful contribution in this aspect.

Additional reference

Cui, W. C., Wang, Y. J., and Pedersen, P. T. (2000). Strength of ship stiffened panels under combined loading, *J. of Ship Mechanics*, Vol. 4, No. 3, pp. 59-86.

Philippe Rigo, Member, and **Catalin Toderan**, Visitor, University of Liege

We would like to congratulate the authors for this very interesting and comprehensive paper. As we can see, this paper is a remarkable extension of the study on ultimate strength assessment of plating between stiffeners presented at SNAME 2000 by Paik et al. The sophisticated method presented today accommodates all potential applied load components and provides very useful information for designers and researchers. We have read the paper with great interest and would like to make some comments.

(1) Concerning panel boundary conditions, the authors make some idealizations for practical purposes of analysis. The edges of panels are considered to remain straight even up to panel collapse and no deflection and rotational restraints along the edges are considered. It is well known that rotational restraints could significantly affect the result of analysis. We estimate that ultimate strength of bottom panels subjected to predominant lateral pressure load is underestimated by the present method. Have the authors some plan to consider the torsional rigidities of support members in a future development? For practical purposes, we suggest to consider two types of rotational restraints: simply supported and clamped. The choice of boundary conditions for a given panel could be made as a function of dominant load.

(2) The post-welding residual stress is included using a simplified distribution, generally accepted for steel welded structures. Could the authors indicate some recommendations for user purpose concerning the level of residual stress to consider (tension and compression)?

(3) The authors explain that the load effect can typically be calculated using classical theory of structural mechanics or the linear elastic finite-element method. The load effect includes stresses and deformations. The stress components acting on the stiffened panel are included in the present method. On the other side, the level of linear deformation is comparable with the level of initial post-welding deflection, so it seems to be obvious to expect an influence of these deformations on the ultimate strength of stiffened panel. Is the effect of such linear deformation considered for all collapse modes?

(4) Many methods dedicated to assess ultimate bending moment of hull girders were developed in the past years. Performing an ultimate bending moment analysis involves a computation of average stress-average strain curve of the panel. For this purpose, an estimation of average strain corresponding to ultimate strength of a stiffened panel could be very useful. Could the present method estimate this average strain?

(5) If any, could the authors define the limits of validity of their formulations in terms of slenderness, aspect ratio, plate thickness, initial imperfections and so on?

(6) We would like to know the position of the authors about the feasibility to implement their new formulations in the rules of a Classification society. Are the presented formulations suitable without any change? If no, what are the authors' plan to ease their use?

(7) The present formulations were developed for steel structures. Is it possible to extend their use to aluminum stiffened panels? If yes, what will be the limitations?

This paper is a remarkable contribution in a very complex area of the ship structure analysis. Nowadays, it is clear that use of more accurate and reliable methods in ship rules becomes more and more a necessity. We believe that the presented method is a step further in the right direction and we fully support this new approach.

Eivind Steen, Visitor, **T. K. Østvold**, Visitor, and **Sverre Valsgård**, Member

The authors present a very comprehensive paper on the buckling strength of stiffened panels and grillages as a followup to their SNAME 2000 paper on unstiffened plates. The paper is broad in the sense that it embraces combined loads, several different types of buckling modes and effects from geometrical imperfections and residual stresses. This makes it difficult to discuss the paper in full since the topics are so numerous; however, we will focus on a few items which we think are among the more important ones. Our discussions are linked to present practice and principles used in Classification rules and guidelines.

We understand that the comparisons with test results presented in the paper are based on measured values for yield stress and geometrical imperfections/residual stresses. The application of a buckling model in a code is different and it would be interesting to hear how ALPS/ULSAP buckling models are meant to match relevant test results in such a setting, i.e., whether a mean value or lower bound strength value is the output?

As far as we can tell from the paper the combined in-plane load situation is not directly treated theoretically for any of the five failure modes. Instead, interaction is determined by means of quadratic equations expressed in terms of the in-plane specified load components scaled by the corresponding ultimate load capacities for each of the load components acting separately. The interaction equations are not given much attention in the paper, and the authors refer to a 1999 paper by Paik et al for further documentation. We could not see that the interaction equations are treated in the referenced paper, and it would be interesting to get more detailed information about the background for selecting this set of interaction equations. It would also be interesting to hear the authors' opinion on the feasibility to utilize consistent buckling theory directly on problems involving the full range of load combinations.

Failure mode I in the presented procedure generally allows the stiffeners to buckle in an overall/global out-of-plate plane mode. The concept of not allowing the stiffeners to buckle out-of-plane when subjected to destabilizing in-plane loads (compression and shear) is the very basis in the present classification rules and most international codes. One reason for the present DNV rule approach is that the consequence of accepting a certain type of buckling has to be dealt with in the design procedure. This is not the case for stiffener buckling. Thus without the present rule limitations, too small stiffener designs may be accepted leading to flexible panels with load redistribution to neighboring panels/girders in the hull. It would be interesting to hear the authors opinion on whether they think the present practice used by Classification societies are sound or too conservative in this respect? Comments and background for the use of weighted average yield stress in Mode I in case of different yield stress in plate and stiffener would be helpful.

The presented calculation procedure incorporates the effects of in-plane biaxial bending. As far as we can determine this is not achieved in the underlying mathematical models, but rather by defining a set of guidelines on how the procedure should be used in such cases. Comments are appreciated.

Ultimate shear buckling is independent of stiffener proportions. How does the procedure accounts for weak/small stiffeners, i.e., for cases where the overall panel shear buckling strength is lower than the buckling strength of the plate between stiffeners?

Coupling of buckling modes is essentially a nonlinear phenomena and it is not clear to us how the authors deal with this in the model, apart from presenting formulas for ideal elastic buckling stress (eigenvalues) of a hinged stiffener. Comments on this would be helpful.

The presented model includes the lateral pressure effect on the in-plane capacities. The lateral pressure enters the procedure at several levels and it is difficult to follow the arguments. We fully agree with the authors that this field needs more study before a final conclusion can be made.

The paper lacks comparisons with existing codes and rules used by different classification societies etc. Such information would highlight the differences and would improve the understanding of the discussions.

The philosophy of the ALPS/ULSAP is clearly a step in the direction of applying more advanced methods in the design of ship structures with a very complicated set of equations as a basis. As such, it is definitely not suited for hand calculations. In general we support the idea of applying more sophisticated approaches for buckling and ultimate strength of stiffened panels. All parties can benefit from this in the form of possible material savings and the improved measure of the actual safety level of ships.

Pentti Kujala, Member

The authors are to be thanked again for a comprehensive addition to the current literature related to the analysis of the ultimate strength of ship structures. In the paper the authors develop new analysis tools for limit state design of stiffened panels and grillages as a logical continuation of the previous publications of the same authors. As stated by the authors the consistent determination of safety of ship structures requires evaluations of the "true" ultimate limit state characteristics. The value of the paper is on the systematic analysis of the various possible collapse modes for stiffened panels and grillages together with detailed comparisons with existing experimental and FE analysis database.

The obtained formulations are, however, fairly sophisticated for an ordinary designer, e.g., working in the shipyard. Therefore my question is if there is any possibility to simplify the used approach so that more user-friendly equations can be obtained?

My other question is related to the analysis of the ultimate strength of the shell structures of ice-strengthened ships. The damage statistics and laboratory experiments have indicated that the collapse of the transverse frames due to the tripping is the dominating failure mode (e.g., Varsta et al 1978, Kujala 1991), see Fig. 28. The frames are located at the bow area of the ship, therefore the angle between the web of the frame and shell plating is 20 deg.

In the case shown in Fig. 28, the collapse under ice pressure loading is caused by a vertical force due to the variation of the longitudinal stress along the frame. Therefore, I am interested to hear the authors' comments on the failure mecha-

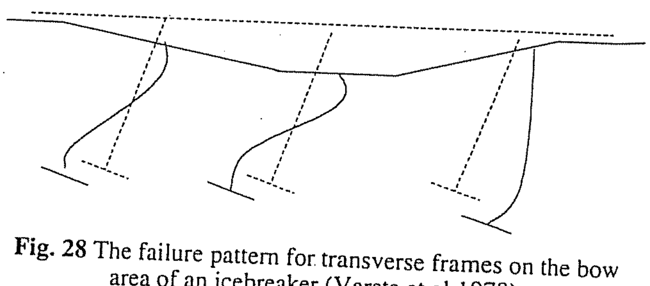

Fig. 28 The failure pattern for transverse frames on the bow area of an icebreaker (Varsta et al 1978).

nism of this type of "tilted" frame and the applicability of their approach to analyze this kind of failure mechanisms.

Additional references

Kujala, P. (1991). Damage statistics of ice-strengthened ships in the Baltic Sea 1984–1987, Winter Navigation Research Board, Research report No. 50, Helsinki, Finland.

Varsta, P., Droumev, I. V. and Hakala, M. (1978). On plastic design of an ice-strengthened frame, Winter Navigation Research Board, Research report No. 27, Helsinki, Finland.

Paul A. Frieze, Member

The authors are to be congratulated on producing yet another comprehensive paper that addresses the ultimate strength behavior and design of stiffened panels. In the light of this, it is perhaps not surprising that this raises a number of questions.

Figure 3 illustrates the various failure modes that a stiffened panel can suffer. While agreeing in principle that a range of failure modes exist and have to be addressed, I am a little unhappy in the way the authors have described and illustrated these. For example, Mode I shown in Fig. 3(a) is described as overall (grillage) buckling. Overall grillage buckling can involve both the longitudinal and transverse stiffeners and not, as illustrated in Fig. 3(a), just the longitudinal stiffeners between transversals. The failure mode illustrated in Fig. 3(a) is, in my opinion, just a variation of that shown in Fig. 3(c), namely, plate-induced failure of a longitudinally stiffened panel but with stocky plating that does not buckle. Further, the failure modes shown in Figs. 3(d) and (e) appear to be identical except that the former includes plate buckling. Both appear to relate to flat-bar stiffeners. While accepting that the plate-induced failure of a longitudinally stiffened panel can occur perhaps without any overall out-of-plane response as illustrated in Fig. 3(b), it is most unlikely that stiffener web buckling (Fig. 3(d)) or stiffener-induced failure (Fig. 3(e)) of a longitudinally stiffened panel will occur without any overall out-of-plane response. Mode IV failure might be better illustrated by the use of a deep Tee longitudinal stiffener in which just the web buckles.

In describing the Perry-Robertson approach to determine stiffened panel beam-column strength,is described as the distance to the extreme fiber. In my experience, this definition would normally be used in relation to the stiffener extreme fiber but rarely in relation to the plating extreme fiber. In relation to plating, one would normally assumewas the distance to the mid-plane of the plating.

In discussing panel boundary conditions, reference is made to the use of edges remaining straight in-plane through continuity with adjacent plate panels. While this is attractive, and indeed was widely adopted for many years when stiffened steel bridge design was under intense investigations following the collapse of several bridges in the early 1970s, mechanical tests have demonstrated that this assumption does not hold true especially in the presence of lateral pressure, see Dean (1975).

In dealing with shear under combined edge shear stress and lateral pressure for Mode I failure as quantified by equation (6), the authors claim that plate panel aspect ratio can be ignored because the shear strength of long or wide plates is greater than that of square plates. This contradicts completely the findings of other finite element and experimental studies which are incorporated in, for example, the U.K. steel bridge code, BS 5400: Part 3, where the longer the panel the lower its strength. The authors' findings are therefore of concern.

In relation to Fig. 13, perhaps I misunderstand the diagrams and/or the text but Fig. 13(b) is described as illustrating a rigid web with finite rotational restraint. The diagram seems to portray a flexible web.

When considering Mode V failure, the authors assume that the effect of rotational restraint by the plating can be ignored. From a design viewpoint, this was confirmed by recent independent studies by Chou et al (1999a, 1999b, 2000a, 2000b) on flat-bar and bulb flat stiffeners. These comprehensive studies are recommended reading.

For Mode VI failures, this appears to coincide exactly with the Mises-Hencky yield condition since buckling is completely ignored. Accordingly, it is not clear why the authors are not using the Mises-Hencky yield condition in its original form. That is, why set to zero when both the longitudinal and transverse applied stresses are compressive when there appears to be no good reason of doing so?

Figure 16 presents comparisons between predicted and experimental results for plates subjected to longitudinal compression and lateral pressure. I understand the tested plates by Yamamoto et al were not welded. The predicted strengths are presented for both zero and 10% longitudinal residual stress ratios. Not surprisingly, the axial capacities of the former are stronger than that of the latter. SPINE results are also presented and include residual stress effects. The SPINE results are generally significantly stronger than the nonwelded test results. This suggests that SPINE considerably overestimates strength. Can the authors please elaborate on this?

Figure 19 highlights apparent weaknesses in the Perry-Robertson approach as applied to stiffened panels when treated as beam-columns. Is it possible that in pursuing this approach, the authors have concentrated on using a nondimensional slenderness parameter based upon the Euler buckling stress for a stiffener and associated plating? For the stiffened plate geometries where the major discrepancy occurs, namely, small stiffeners, it can be expected their critical buckling stresses when treated as an orthotropic plate between transverse frames will be significantly higher than those found from considering the simplified geometry (stiffener plus associated plating).This is seen in Fig. 19 where the Mode I strength considerably exceeds the Mode III strength. Accordingly, could I suggest the Perry-Robertson solution might be improved by using the orthotropic buckling stress for the longitudinal stiffened panel in place of the Euler stress when calculating λ.

The text describing Fig. 20 seems to suggest that the ultimate strength interaction relationship generally follows the Mises-Hencky yield condition. This does not seem to be reflected in the figure.

While acknowledging the comprehensive nature of the paper, the level of detail presented by the authors, particularly when dealing with the details of the failure modes, including not inconsiderable repetition in places, does not make for easy reading. Perhaps more of the detail could have been confined to appendices. Notwithstanding, the authors are to be congratulated on a splendid effort in bringing together as much data and results as they have.

Additional references

Chou, S. K. G. et al (1999a). Local critical buckling of flat-bar-stiffened plating, *The Structural Engineer*, Vol. 77, No. 19, 5 October.

Chou, S. K. G. et al (1999b). Design of flat-bar stiffened plating against local buckling, *The Structural Engineer*, Vol. 77, No. 22, 16 November.

Chou, S. K. G. and Chapman, J. C. (2000a). Local critical buckling of bulb-flat stiffened plating, *The Structural Engineer*, Vol. 78, No. 8, 18 April.

Chou, S. K. G. et al (2000b). Design of bulb-flat stiffened plating, *The Structural Engineer*, Vol. 78, No. 18, 19 September.

Dean, J. A. (1975). The collapse behavior of steel plating subject to complex loading, Ph.D. Thesis, University of London (Imperial College of Science and Technology).

Masahiko Fujikubo, Visitor, Hiroshima University

I congratulate the authors on presenting the ultimate strength formulations for stiffened panels and grillages, which highly contribute to the development of ultimate limit state design of ships and many types of offshore structures. Here, I would like to discuss some aspects in strength formulations, particularly the effects of continuity of plating on the ultimate strength.

As described in "Panel Boundary Conditions" of the text, when predominantly compressive loads are applied on a continuous stiffened panel in the stiffener direction, one panel tends to buckle up and the adjacent panel tends to deflect down. This means the deflection in PIF (Plate Induced Failure) mode and that in SIF (Stiffener Induced Failure) mode develop simultaneously in two adjacent panels. According to our FEA of a continuous stiffened panel using the so-called double-span model (Fujikubo et al 1999), the deflection in PIF mode (Mode III in the authors' paper) increases in one panel after the local panel buckling between stiffeners. However, the ultimate strength is eventually reached when the stiffener fails in the adjacent panel even when the local panel slenderness is large, that is, SIF is a trigger to the collapse of continuous stiffened panels.

In the authors' study, SIF is excluded from the Mode III prediction based on the Perry-Robertson approach, and only PIF is taken into account. The ultimate strength corresponding to PIF is evaluated for the isolated stiffened panel that is simply supported along the edges, Fig. 3(c). In the buckling tests or analyses of such isolated panels, compressive loads are generally applied at the height of the neutral axis of a whole cross section. Hence, except for the case of stiffeners having very large initial deflection towards panels, PIF takes place after the local panel buckling because of a shift of the neutral axis towards stiffeners. On the other hand, in the continuous stiffened panels, the panel in SIF mode resists the deflection of the adjacent panel in PIF mode. Also, the loading point is variable in cross section under the forced compressive displacement as assumed in the double-span model. As a result, SIF may become the more dominant failure mode. How do the authors consider such an effect of continuity of plating on Mode III failure under predominantly compressive loads?

Figure 19 shows that the Perry-Robertson formula gives only too pessimistic predictions of ultimate strength over the wide range of stiffener geometry, particularly when SIF is assumed. The reason for this is not clear from the given results. According to our study, the Perry-Robertson approach assuming SIF gives reasonable predictions of ultimate strength for the stiffener sizes, smaller than those corresponding to Modes IV and V, when the panel effective width is properly taken into account.

For Mode II, simply-supported edge conditions are assumed for the local plate panels. When lateral pressure acts on the continuous stiffened panel, each adjacent local panel deflects to the direction of lateral pressure loading. The panel can be regarded as if it were clamped along the edges when the compressive loads are applied. According to the numerical studies on continuous ship bottom platings (double bottom) under combined compression and lateral pressure (Fujikubo et al 1999), both compressive buckling and ultimate strengths become significantly larger than those calculated for the simply-supported isolated panels under the same lateral pressure.

In the authors' study, the importance of considering Mode I failure is highlighted taking the VLCC deck structure as an example. This is a quite informative result in the ultimate strength calculation of a stiffened grillage under lateral pressure. However, for other types of structure, like double bottoms, Mode II is a more important failure mode, and thus the above-mentioned effect of continuity should be taken into account.

Additional reference

Fujikubo, M., Yao, T. and Khedmati M. R. (1999). Estimation of ultimate strength of ship bottom plating under combined transverse thrust and lateral pressure, *J. of the Society of Naval Architects of Japan*, Vol.186, pp. 621–630.

Authors' Closure

To start with, we thank all the discussers for their kind remarks regarding our paper. We thank them even more for the time they spent developing their various discussions.

Regarding the first question of *Prof. Cui*, our comments to *Dr. Frieze* may be referred to. When a structure approaches its ultimate strength, it generally involves large deformation and local plastic deformation. Starting from initial yielding at some point of the structure, the structure goes through a stage of elastic-plastic deformation, which is very difficult to precisely analyze using only theoretical tools. A combination of large deformation elastic theory with rigid plastic analysis is promising in the sense that the difficulties of treating elastic-plastic deformation are avoided, although the common intersection region for the two solutions, namely elastic and rigid plastic analyses, may not necessarily be the ultimate strength.

As long as the magnitude of lateral pressure is relatively small, the loading sequence does not affect the ultimate strength. However, for very large lateral pressure, the panel can of course collapse under lateral pressure alone. In this regard, the present approach considers a criterion of ultimate strength under lateral pressure loads. The approach presented is not, however, normally suitable for impact load response where structural dynamics will play an important role.

Whether lateral pressure pushes the plate to deform in the direction of initial deflection depends on many factors. Under some circumstances, these two factors can cancel each other, as Prof. Cui points out. Our assumption that both lateral pressure and initial deformations will have the same effect of reducing strength is conservative but this treatment is practical for design purposes.

Our next project is to investigate structural degradation such as corrosion and fatigue cracking damages and their effects on strength. We are developing formulations that may be used as design tools. Also, we aim to use them to develop rules.

As for the question of *Dr. Rigo* et al on the effect of elastically restrainededges on the buckling and ultimate strength of plates, our last year's SNAME annual meeting paper (Paik et al 2000) deals with this problem to some extent. As to the effect of plate edges being assumed to remain straight, and its effect on ultimate strength particularly with lateral pressure effects, we refer to the discussion of Dr. Frieze. This, and the observations of *Prof. Fujikubo* on interacting effects in continuous stiffened panels are interesting and may need additional study. Regarding the second question of Dr. Rigo et al on the post-welding residual stress, again please refer to our last year's SNAME annual meeting paper.

Regarding the third question of Dr. Rigo et al, for structural design, both load effects and resistance must be quantified. The former can be obtained from either the classical theory of structural mechanics or the linear elastic finite-element method, while the latter can be predicted by the strength formulations presented in this paper. The present approach accommodates the initial imperfections in the form of initial deflection and residual stresses as parameters of influence.

The authors consider that the fourth question of Dr. Rigo et al is very important. For a more accurate computation of the ultimate strength of a structure system, it is of crucial importance to better understand the entire failure history of every structural element before and after it reaches ultimate strength. Formulations for the entire failure history including the average stress-strain relationships are described in our textbook of Paik & Thayamballi (2002), and will also hopefully be the subject of an additional paper at an appropriate venue.

Dr. Rigo et al ask about the limitations of validity of our formulations (slenderness, aspect ratio, plate thickness, initial imperfections, etc.). Many of these formulations are derived using analytical methodologies; they are applicable to different material, different geometrical proportions and other parameters included in them. Some parts of the study are based on extensive nonlinear FEA and experiments. For these formulas, it is always good to keep in mind the ranges over which such analyses were made, and the limitations introduced by the analysis methods and modeling techniques. In general, much of the development is aimed at present day merchant ship structures. Application to any "general case" must therefore be carefully made.

As for the sixth question of Dr. Rigo et al, the present formulations provide relatively more accurate design results. They are sophisticated and may not be appropriate for hand calculations. We also note that computers are widely used in modern designs. We have coded our formulations in a computer program called ALPS/ULSAP. We have confirmed that the ultimate strength based structural optimization for a large complex ship can be realized using ALPS/ULSAP. As to the inclusion of such formulations in classification society rules, this is, of course, our goal. However, such research as this will invariably need additional experience and validation, quantitative assessment of associated uncertainties, and, of course, some simplification and a development of consensus in order to be successfully integrated into design standards.

Regarding the last question of Dr. Rigo et al, in principle, many of our formulas are applicable to aluminum panels. However, it should be noted that the aluminum alloys are different from steels. Aluminum alloys do not show a clear yield point, but have a significant strain hardening. The yield strength of aluminum alloys in a heat-affected zone may be smaller than that of base metal. We have not thus far studied applications to aluminum in any depth, and so are unable to be any more definitive in this regard.

We thank the various discussions of *Dr. Steen* and his colleagues. For comparisons with mechanical test results, and verification of our formulas, we used measured values of material yield stress and initial imperfections. Since yield stress and initial imperfections are treated as parameters of influence, a designer can use those values they think appropriate. We recommend the use of the values specified by classification societies rules, together with appropriately and consistently defined safety factors to achieve a required target reliability level. The quantification of strength uncertainties thus remains important.

Our interaction equations are based on the results of a series of nonlinear finite-element analyses. While various types of plate ultimate strength interaction relationships under combined loads have been suggested in the literature, most of them for plating may be generalized as the following

$$\left(\frac{\sigma_{xav}}{\sigma_{xu}}\right)^{c_1} + \alpha\left(\frac{\sigma_{xav}}{\sigma_{xu}}\right)\left(\frac{\sigma_{yav}}{\sigma_{yu}}\right) + \left(\frac{\sigma_{yav}}{\sigma_{yu}}\right)^{c_2} + \left(\frac{\tau_{av}}{\tau_u}\right)^{c_3} = 1 \quad (30)$$

where a, c_1, c_2, c_3, = coefficients.

Some examples of the constants used for combined biaxial compression in equation (30) by different investigators are indicated in Table 9. Figure 29 plots equation (30) with various constants indicated in Table 9 when $\tau_{av} = 0$. Figure 30 compares the ultimate strength interaction curve using the constants of Paik et al (2001) with nonlinear finite-element results for a simply supported long thick plate under biaxial compression or tension. It is apparent that the approximations suggested in the present study are not unreasonable. For stiffened panels typical of ship structures, the ultimate strength interaction relationships are similar to those for plating. This assumption is based on our insights developed through a series of nonlinear finite-element analyses under combined loads.

Most design standards presume that stiffeners do not fail prior to plating between them. It is also our own view that the supporting members should be designed to fail after the plating they support fails. By doing this, a ship's hull will collapse in a progressive manner, and keep a certain level of structural redundancy. Drastic loss of strength for a wide area of structure should be avoided as this will endanger the global integrity of a hull. Achieving this in design is a function of not only the strength formulations (and load aspects) but also the safety factors. We are not in general in favor of a "one hoss shay"' approach to design wherein many or all failure modes apparently occur simultaneously, not only because it is very difficult to "exactly" quantify the uncertainties in a design code, but also because different failure modes are associated by different consequences.

When the orthotropic plate theory is applied, the stiffeners may be smeared into the plate. In this case, we use the weighted average value of yield stress for plating and stiffeners. This does not have a clear physical meaning, but is an approximation

Table 9 Examples of the constants used in Equation (30) for biaxial compressive loading

References	Constants used in Eq. 4.55
BS5400 (2000)	$c_1 = c_2 = 2$, $\alpha = 0$; both σ_{xav} and σ_{yav} are compressive
Valsgard (1980)	$c_1 = 1$, $c_2 = 2$, $\alpha = -0.25$ for $a / b = 3$; both σ_{xav} and σ_{yav} are compressive
Dier & Dowling (1980)	$c_1 = c_2 = 2$, $\alpha = 0.45$; both σ_{xav} and σ_{yav} are compressive
Stonor et al (1983)	$c_1 = c_2 = 1.5$, $\alpha = 0$; (lower bound) $c_1 = c_2 = 2$, $\alpha = -1$; (upper bound) [both σ_{xav} and σ_{yav} are compressive]
Paik et al (2001a)	$c_1 = c_2 = 2$, $\alpha = 0$; both σ_{xav} and σ_{yav} are compressive (negative) $c_1 = c_2 = 2$, $\alpha = 0$; either σ_{xav} or σ_{yav} or both are tensile (positive)

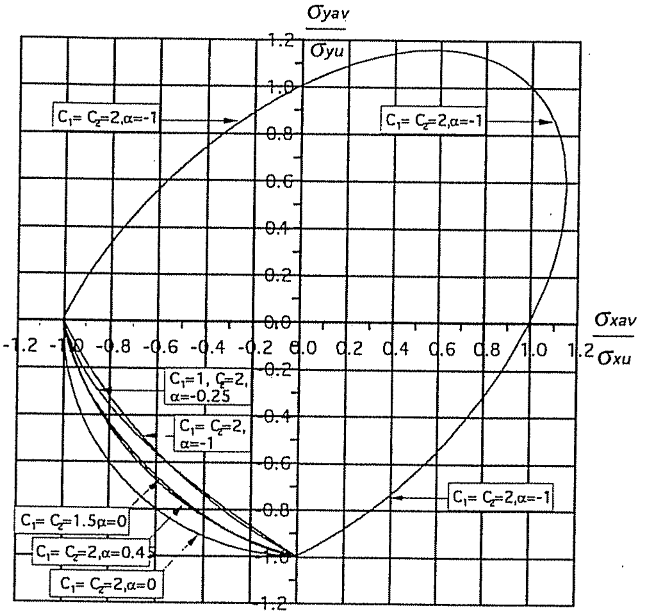

Fig. 29 Various types of the plate ultimate strength interaction curves under biaxial loads

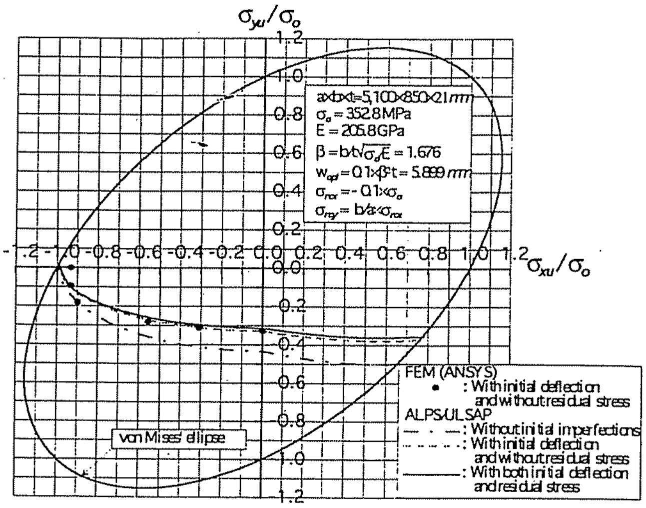

Fig. 30 Ultimate strength interaction curve of a simply supported long thick plate under biaxial compression or tension ($a / b = 6$, $t = 21mm$)

that gives more appropriate and practical results as long as the two yield stresses are not widely separated.

Dr. Steen et al point out that under in-plane bending, the failure history of individual stiffeners will be different from each other, with the most highly stressed stiffener failing first. We agree with this remark. Once a stiffener collapses, it sheds its load to its adjacent structures. The next most highly stressed stiffener collapses, either immediately or at a slightly higher load. In either case the stiffeners fail progressively, and the entire panel eventually reaches the ultimate limit state. It is reasonably conservative to say that the entire panel collapses if the most highly stressed stiffener collapses. Such conservatism seems advisable in a design content because of the variety of circumstances possible, and the associated uncertainties.

The present ultimate shear strength formulations approximately take into account the effect of stiffener proportions. The ultimate shear strength is either shear strength of the panel or the local strength of plating between stiffeners, whichever comes lower. Other failure modes such as diagonal tension failure are neglected. This is an area in which the present strength formulations can be improved in the future.

Our ultimate strength formulations are derived by considering stress redistribution of panels: they are not always directly related to the ideal elastic buckling stress. We have discussed all potential collapse modes and all possible interaction among them. There must be one primary and predominant mode which causes the collapse of the entire panel. In this regard, the present formulations have been developed based on the assumed presence of such a primary collapse mode and the interacting effects of the other potential modes is not accounted for. This is a valid assumption.

We agree with the comment of Dr. Steen et al that the effects of lateral pressure are not straightforward to deal with. The reason why the lateral pressure is treated at several levels is that the effect of lateral pressure on the panel ultimate strength is different depending on the type of in-plane load components. It is also noted that the lateral pressure is regarded as a secondary load component within the content of our treatment.

We share the opinion of Dr. Steen et al that it should be interesting to compare the present formulations with existing design standards. We present our comparisons in a separate paper (Paik & Kim 2002). We would like to remind that one should be very careful when interpreting his comparisons especially about the strength criteria of different classification societies. He should bear in mind that the rules are used to achieve designs similar to proven practice while our formulas provide a close prediction of strength of structure.

Although the shipbuilding industry has relied extensively on experience in the past, advanced analysis tools are now being recognized and widely applied in practice. There is now a commendable trend to improve our design standards through comprehensive R & D projects, which on a lighter note apparently seem to provide more complicated formulas than what we used to have! Compared to traditional formulas, these new formulas hopefully are more sophisticated and have a greater number of parameters. The industry will benefit from this sophistication, as Dr. Steen et al point out. Though the developed formulations are complicated, once they are coded into a computer program with good user interface, calculations can be made in a fraction of time. The flip side to this is that the greater complexity makes physical understanding more difficult. An appropriate balance thus needs to be found.

As for the first question of *Dr. Kujala*, we may refer to our reply to the last question of Dr. Steen et al The problem for panels with tilted stiffeners which Dr. Kujala raises is quite interesting. The present formulations are not directly applicable to panels with tilted frames. But with some approximations in terms of the effectiveness of the tilted support members, such application may be possible.

The authors thank *Dr. Frieze* for his various comments. We will try to improve the illustrations and figures representing the various collapse modes following his suggestions. We do note that these illustrations are meant to show certain idealizations of selected failure mode behavior; they do not necessarily represent what may occur in a given structural situation when considering all members, all possible modes of failure, and their interactions.

The authors are of the opinion that, normally, the difference of the ultimate strength is less than 1% even if the distance is taken from the neutral axis to the mid-plane of plating, compared to that with the distance to the extreme fiber.

To be consistent with the original Perry-Robertson formula, the authors took the distance to the extreme fiber of plating.

The authors agree with Dr. Frieze's opinion on the limitation related to the panel boundary conditions. It has been recognized that the ultimate strength of plating depends on whether or not the edge remains straight. The ultimate strength of plates with straight edges is, of course,. greater than that of plates whose edges are free to move in plane. When looking at the ultimate strength of plating between relatively small/weak stiffeners, the designer willneed to pay close attention to the plate edge condition. For stiffened panels bounded by relatively heavy support members (e. g., longitudinal girders, transverse frames), however, the condition with edges remaining straight may be relevant for the purpose of practical design and analysis. For plate panels between supporting members, we have discussed the influence of boundary restraint in ourlast year's SNAME annual meeting paper. Simple supports have been used in many design codes. That idealization is generally adequate and effective for most practical problems in merchant ships, though the real situation is not being fully represented. We plan to investigate the case where stiffener edges do not necessarily remain straight.

The present paper does not imply that the ultimate shear strength of square plates is greater than that of long or wide plates. Rather, the plate ultimate shear strength tends to decrease as the plate aspect ratio increases. Our conclusions are based on our analyses on designs typical in the shipbuilding industry. Figure 31 shows that the ultimate shear strength depends weakly on the plate's aspect ratio, especially for relatively thick plates. In this regard, the present formulations do neglect the effect of the plate aspect ratio for convenience, and related future improvements are, of course, possible.

The present paper deals with the lateral-torsional buckling (or tripping) based on Fig.13(c). This means that the stiffener web is considered to be undeformed while no rotational restraints exist between plating and stiffener web. Figure 13(b) indicates a case where the web is flexible while no rotational restraints exist between plating and web. These are two idealizations of the possible real behavior. The authors thank Dr. Frieze for citing additional latest literature to support and complement the findings ofthe present paper that the effect of rotational restraint by the plating can be ignored when considering Mode V failure.

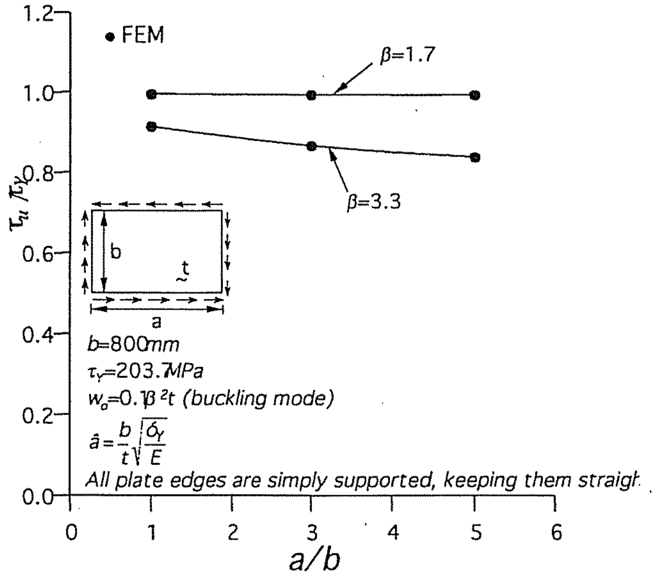

Fig. 31 Effect of the aspect ratio on the plate ultimate shear strength

The authors thank Dr. Frieze for the point related to Mode VI failure condition. While the difference between the two formulas noted above is not significant, his suggestion is not incorrect.

The SPINE results take into account the effect of membrane action on the large deflection behavior of plates with the edges assumed to remain straight. This may in part explain why the SPINE results are greater then the nonwelded test results. The comparisons provided are not extensive by any means, and additional studies are needed in order to be more definitive.

Regarding Dr. Frieze's suggestion that the Perry-Robertson solution might be improved by using the orthotropic buckling stress for the longitudinal stiffened panel in place of the Euler stress when calculating , we tried to obtain the corresponding results in a few cases. However, the predicted panel ultimate strength values so obtained are still too pessimistic as long as it is assumed that the ultimate strength is reached if the tip of the stiffener yields. In fact, the stiffened panel typically collapses either after the yielded region has expanded within the stiffener web or after the stiffeners deformed sideways.

In the text, it is implied that the ultimate strength interaction relationship between biaxial loads follows the von Mises-Hencky yield condition when axial tension is applied at both directions, but if the axial load is compressive in one direction, it does not follow the original von Mises-Hencky yield function.

It is not easy to include all of the derivations in a paper. We do thank Dr. Frieze for his helpful critical observations on the format of the paper, which we will certainly keep in mind for the future. We are currently in the process of preparing the manuscript of a related book, Paik and Thayamballi (2002), which we hope should be less constrained by space considerations.

The authors agree with the valuable insights of Prof. Fujikubo that in a continuous grillage the stiffener-induced failure can take place in one panel between transverse frames when the adjacent panel between transverse frames tends to show the plate-induced failure mode. This is because one panel may buckle up while the adjacent panel deflects down even if the stiffeners are arranged at the same side of the plating.

While the actual collapse mode will depend on the geometric proportions and material properties of the panel, the authors fully agree with the opinion of Prof. Fujikubo that the stiffener-induced failure is a trigger to the collapse of continuous stiffened panels. As we described in detail in the text, however, the Perry-Robertson type "stiffener induced failure" (i.e., at first yielding of the stiffener tip) may not always represent the actual ultimate limit state of the entire panel. The real stiffener-induced failure is in fact governed by lateral-torsional buckling of stiffener or buckling of stiffener web. The original idea of the Perry-Robertson formula assumes that the stiffener-induced failure occurs if the tip of the stiffener yields. This assumption may in some cases be too pessimistic in terms of the collapse strength predictions. Rather, plasticity may grow into the stiffener web as long as lateral-torsional buckling or stiffener web buckling does not take place, so that the stiffener may resist the further loading even after the first yielding occurs at the extreme fiber of the stiffener.

For the stiffened panel under combined axial loads and lateral pressure, the boundary condition atthe panel edges may be affected by the magnitude of lateral pressure. With larger magnitude of lateral pressure, the edges may behave more toward the clamped condition as long as the panel behaves as an elastic body. However, the panel edges may yield in the earlier stage of loading because of large out-of-plane bending moment which is caused by rotational restraints. Therefore, the rotational restraints at the panel edges will be weakened and may approach the simply-supported condition even before the ultimate strength is reached. In this regard, the present methodology presumes that the panel edges are simply supported even if lateral pressure is applied. This approximation gives somewhat pessimistic but usually adequate results.

Overall, we agree that the interactions between plate boundary conditions and related failure modes in continuous plate panels do need further study and more elaborate treatment, particularly in the context of failure analysis.

In closing, the authors would again like to thank all the discussers for their time and effort, which add value to the paper and serve to emphasize various aspects of the ultimate limit state design technology for ship stiffened panels and grillages. Finally, the authors would like to thank the SNAME Papers Committee members for their guidance related to this paper and the Society itself for having given the authors the opportunity to present this paper.

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Ultimate Limit State Design of Ship Stiffened Panels and Grillages

Jeom Kee Paik, Member, Pusan National University, Korea, **Anil Kumar Thayamballi**, Life Member, Chevron Shipping, **Bong Ju Kim**, Student Member, Pusan National University, Korea, **Ge Wang**, Member, **Yung Sup Shin**, Member, and **Donald Liu**, Member, ABS Houston, Texas

ABSTRACT

This paper is a logical sequel to the authors' 2000 SNAME Annual Meeting paper (Paik et al. 2000), and aims to deal with the advanced ultimate limit state design of stiffened panels and grillages that form parts of ship structures. In contrast, the previous paper dealt with the buckling and ultimate strength design of plate elements within such stiffened panels. In achieving a more advanced ultimate limit state design of stiffened panels and grillages, we are still confronted with a number of problem areas which have not been more completely solved. Related to such problem areas, this paper proposes a more advanced, yet design oriented ultimate limit state design methodology for ship stiffened panels and grillages. Possible failure modes involved in collapse of stiffened panels and grillages are first categorized. The ultimate strength of the stiffened panel under combined loads is calculated taking into account all of the possible failure modes and the interplay of various factors such as geometric and material properties, loading and post-weld initial imperfections. As is usual, the collapse of stiffened panels is considered to occur at the lowest value among the various ultimate loads calculated for each of the collapse patterns. The design oriented strength formulations developed accommodate all potential applied load components including biaxial compression / tension, biaxial in-plane bending, edge shear and lateral pressure loads. The fabrication related initial imperfections (initial deflections and residual stresses) are included in the developed strength formulations as parameters of influence. The validity of the proposed ultimate strength formulations is confirmed by a comparison with the nonlinear finite element solutions and mechanical collapse test results. Important insights developed from the present study are summarized. Recommendations are made with respect to needed future research.

NOMENCLATURE

Geometric Properties

a = length of plating, spacing between two adjacent transverses (frames)
A_x, A_y = cross-sectional areas of a single x- or y-stiffener with attached effective plating
a_e = effective 'length' of the plating corresponding to length a
a_{eu} = a_e at plate ultimate limit state
A_{xx}, A_{xy} = cross-sectional areas of a single x- or y-stiffener
b = breadth of plating, spacing between two adjacent longitudinals
B = breadth of the stiffened panel, spacing between two adjacent longitudinal girders
b_e = effective width of the plating corresponding to breadth b
b_{eu} = b_e at plate ultimate limit state
b_{fx}, b_{fy} = breadths of x- or y-stiffener flange
h_{wx}, h_{wy} = heights of x- or y-stiffener web
I_x, I_y = moments of inertia of x- or y-stiffener with attached effective plating