Toward a Fundamental Understanding of the Hilbert-Huang Transform in Nonlinear Structural Dynamics

G. Kerschen(1), **A.F. Vakakis**(2), **Y.S. Lee**(3), **D.M. McFarland**(3), **L.A. Bergman**(3)

(1) Department of Materials, Mechanical and Aerospace Engineering, University of Liège, Belgium
E-mail: g.kerschen@ulg.ac.be

(2) Division of Mechanics, National Technical University of Athens, Greece
Department of Mechanical and Industrial Engineering (adjunct), University of Illinois at Urbana-Champaign, U.S.A.
E-mail: vakakis@central.ntua.gr, avakakis@uiuc.edu

(3) Department of Aerospace Engineering, University of Illinois at Urbana-Champaign, U.S.A.
E-mail: yslee4,dmmcf,lbergman@uiuc.edu

ABSTRACT

The Hilbert-Huang transform (HHT) has been shown to be effective for characterizing a wide range of nonstationary signals in terms of elemental components through what has been called the empirical mode decomposition. The HHT has been utilized extensively despite the absence of a serious analytical foundation, as it provides a concise basis for the analysis of strongly nonlinear systems. In this paper, we attempt to provide the missing link, showing the relationship between the EMD and the slow-flow equations of the system. The slow-flow model is established by performing a partition between slow and fast dynamics using the complexification-averaging technique, and a dynamical system described by slowly-varying amplitudes and phases is obtained. These variables can also be extracted directly from the experimental measurements using the Hilbert transform coupled with the EMD. The comparison between the experimental and analytical results forms the basis of a nonlinear system identification method, termed the slow-flow model identification method, which is demonstrated using numerical examples.

1 INTRODUCTION

Nonlinear structural dynamics has been studied for a relatively long time, but the first contributions to the identification of nonlinear structural models date back to the 1970s [1]. Since then, numerous methods have been developed because of the highly individualistic nature of nonlinear systems. A large number of these methods were targeted to single-degree-of-freedom (SDOF) systems, but significant progress in the identification of multi-degree-of-freedom (MDOF) lumped parameter systems was realized during the last ten years. However, it is fair to say that there is no general analysis method that can be applied to all systems in all instances. For a review of the literature on the subject, the reader is invited to consult the textbook [2] or the recent overview [3].

The idea of using slow-flow dynamics for nonlinear system identification dates back to Feldman who exploited the Hilbert transform. SDOF systems were first studied in the FREEVIB method [4], and the generalization to 2DOF systems soon followed [5]. The proposed procedure is one of the most successful approaches to tracking the varying nature of vibration of a large class of nonlinear systems thanks to the extraction of backbone curves from experimental data. Alternative approaches for slow-flow-based identification were developed, in particular the Wigner-Ville approach described by Feldman and Braun [6] and the wavelet transform [7-9]. Using the Gabor transform, Bellizzi *et al.* [10] related the slow-flow dynamics to the concept of coupled nonlinear modes.

Because multicomponent signals do not admit a well-behaved Hilbert transform, the Hilbert-Huang transform (HHT) was introduced in [11]. It has been shown to be effective for characterizing a wide range of signals in terms of elemental components, termed intrinsic mode functions (IMFs), through what has been called the empirical mode decomposition (EMD). Several applications of this technique to structural dynamics recently appeared. For instance, Yang *et al.* [12,13] used it for modal analysis and were able to relate the IMFs to the modal properties, giving a clear interpretation of the outcome of HHT in linear dynamics.

The strength of the method lies in its ability to deal with nonlinear and nonstationary data (see, e.g., [14-15]), despite the absence of a serious analytical foundation under those assumptions. In this paper, we attempt to provide a fundamental understanding of the HHT in nonlinear structural dynamics by linking its outcome to the slow-flow dynamics. The slow-flow model is established by performing a partition between slow and fast dynamics using the complexification-averaging technique, and a dynamical system described by slowly-varying amplitudes and phases is obtained. These variables can also be extracted directly from the experimental measurements using the Hilbert transform coupled with the EMD. The comparison between the experimental and analytical results forms the basis of a nonlinear system identification method, termed the *slow-flow model identification* (SFMI) method. The SFMI method can be viewed as a generalization of Feldman's FREEVIB approach to MDOF systems. In addition, it identifies the parameters of the equations of motion, something which was not considered in [4,5].

2 COMPLEXIFICATION-AVERAGING METHOD

The complexification-averaging (CX) method, which was introduced by Manevitch [16], establishes the governing equations of the slow flow of structural systems. To illustrate the method, a damped Duffing oscillator is considered

$$\ddot{x} + c\dot{x} + kx + Cx^3 = 0 \quad \text{with} \quad x(0) = X,\ \dot{x}(0) = 0 \tag{1}$$

A complex change of variable, $\psi(t) = \dot{x}(t) + j\omega x(t)$, is performed such that

$$x = \frac{\psi - \psi^*}{2j\omega},\ \ \dot{x} = \frac{\psi + \psi^*}{2},\ \ \ddot{x} = \dot{\psi} - j\omega\frac{\psi + \psi^*}{2} \tag{2}$$

where ω is the frequency which best describes the system response. Equation (1) becomes

$$\dot{\psi} - j\omega\frac{\psi + \psi^*}{2} + c\frac{\psi + \psi^*}{2} + k\frac{\psi - \psi^*}{2j\omega} + C\left(\frac{\psi - \psi^*}{2j\omega}\right)^3 = 0 \tag{3}$$

A partition of the dynamics into slow and fast components is realized, $\psi(t) = \varphi(t)e^{j\omega t}$; the motion is approximated by a single fast-frequency component with modulated amplitude and phase. By averaging out the fast-frequency component $e^{j\omega t}$, equation (3) is transformed into

$$\dot{\varphi} + \frac{j\omega\varphi}{2} + \frac{c\varphi}{2} - \frac{jk\varphi}{2\omega} - \frac{3jC}{8\omega^3}|\varphi|^2\varphi = 0 \tag{4}$$

To extract the envelope and phase variables, the complex amplitude $\varphi(t)$ is expressed in polar form, $\varphi(t) = a(t)e^{j\beta(t)}$,

$$\dot{a} + ja\dot{\beta} + \frac{j\omega a}{2} + \frac{ca}{2} - \frac{jka}{2\omega} - \frac{3jCa^3}{8\omega^3} = 0 \tag{5}$$

The real and imaginary parts of this equation are

$$\dot{a} + \frac{ca}{2} = 0,\ \dot{\beta} + \frac{\omega}{2} - \frac{k}{2\omega} - \frac{3Ca^2}{8\omega^3} = 0;\ a(0) = X\omega,\ \beta(0) = \frac{\pi}{2} \tag{6}$$

respectively. These equations describe the slow-flow dynamics and may be solved analytically in this case

$$a(t) = X\omega e^{-ct/2} \quad \text{and} \quad \beta(t) = \frac{3CX^2}{8\omega c}(1 - e^{-ct}) - \left(\frac{\omega}{2} - \frac{k}{2\omega}\right)t + \frac{\pi}{2} \tag{7}$$

The system response predicted by the CX method is given by

$$x(t) = \frac{a}{\omega}\sin(\omega t + \beta) \tag{8}$$

and is compared to the response computed using numerical simulation of the original equation of motion (1) in Figure 1. Four cases are considered, namely (i) a linear system $(C, k, X) = (0, 1, 1)$; (ii) a weakly nonlinear system $(C, k, X) = (1, 1, 0.2)$; (iii) a

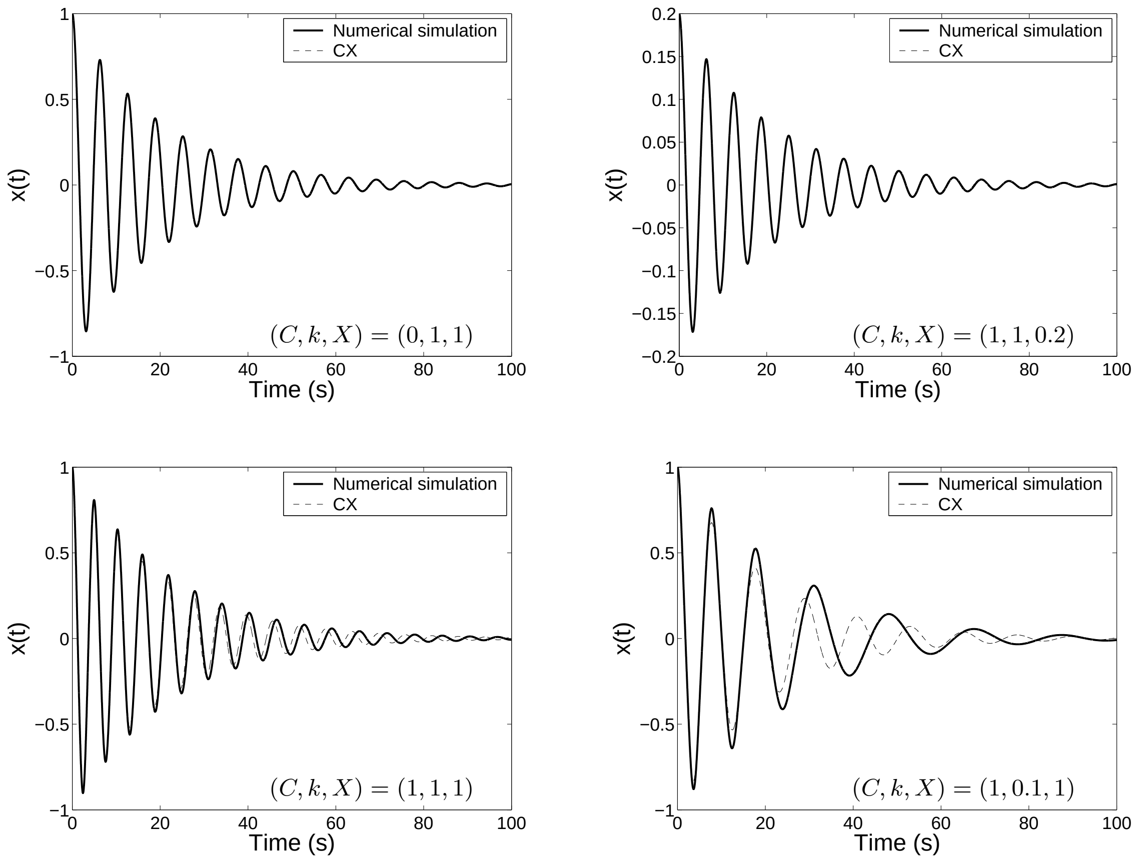

Figure 1: Approximation of the response of a damped Duffing oscillator using the CX method (c =0.1).

strongly nonlinear system $(C,k,X)=(1,1,1)$, and (iv) a system with an almost essential nonlinearity $(C,k,X)=(1,0.1,1)$; the dashpot constant is $c=0.1$. The agreement between predictions and simulations is perfect for the linear and weakly nonlinear systems. For the strongly nonlinear system, the agreement is excellent during the first few cycles, but deteriorates at the end of the signal. Finally, for system (iv), the predicted response significantly deviates from the actual response. This example gives an idea about the validity of model (6). We note that it is possible to substantially improve the predictive capability of this model by considering the time scaling

$$\dot{\tau}=\omega(A) \tag{9}$$

where A is the amplitude of the motion, prior to the application of the CX method. This is not further discussed here.

Moving now to a 2DOF system,

$$\begin{aligned} m_1\ddot{x}+c_1\dot{x}+k_1x+k_{12}(x-y) &= 0 \\ m_2\ddot{y}+c_2\dot{y}+k_2y+k_{12}(y-x)+Cy^3 &= 0 \end{aligned} \tag{10}$$

four complex variables are introduced, because the system response should possess two main components, $x(t)=x_1(t)+x_2(t)$ and $y(t)=y_1(t)+y_2(t)$, with frequencies ω_1 and ω_2, respectively,

$$\psi_1=\dot{x}_1+j\omega_1x_1, \quad \psi_2=\dot{x}_2+j\omega_2x_2, \quad \psi_3=\dot{y}_1+j\omega_1y_1, \quad \psi_4=\dot{y}_2+j\omega_2y_2, \tag{11}$$

By substituting this *ansatz* into (10), averaging separately over the two fast frequencies and expressing the complex amplitudes

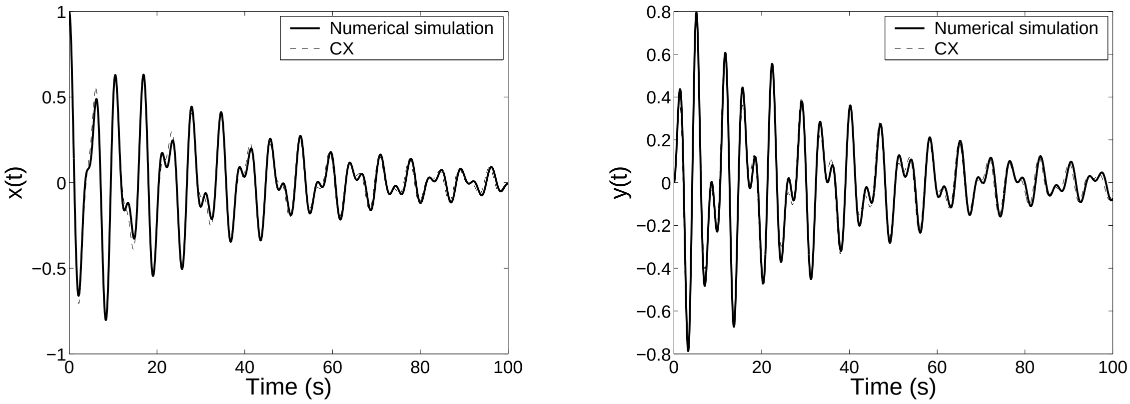

Figure 2: Approximation of the response of a 2DOF nonlinear system using the CX method.

in polar form, the slow-flow model is derived

$$
\begin{aligned}
&m_1\dot{a}_1 + \frac{c_1 a_1}{2} - \frac{k a_3}{2\omega_1}\sin(\beta_3-\beta_1) = 0\\
&m_1\dot{a}_2 + \frac{c_1 a_2}{2} - \frac{k a_4}{2\omega_2}\sin(\beta_4-\beta_2) = 0\\
&m_2\dot{a}_3 + \frac{c_2 a_3}{2} - \frac{k a_1}{2\omega_1}\sin(\beta_1-\beta_3) = 0\\
&m_2\dot{a}_4 + \frac{c_2 a_4}{2} - \frac{k a_2}{2\omega_2}\sin(\beta_2-\beta_4) = 0\\
&m_1(2\omega_1\dot{\beta}_1+\omega_1^2) - k\left(1-\frac{a_3\cos(\beta_3-\beta_1)}{a_1}\right) - k_1 = 0\\
&m_1(2\omega_2\dot{\beta}_2+\omega_2^2) - k\left(1-\frac{a_4\cos(\beta_4-\beta_2)}{a_2}\right) - k_1 = 0\\
&m_2(2\omega_1\dot{\beta}_3+\omega_1^2) - k\left(1-\frac{a_1\cos(\beta_1-\beta_3)}{a_3}\right) - k_2 - C\left(\frac{3a_3^2}{4\omega_1^2}+\frac{3a_4^2}{2\omega_2^2}\right) = 0\\
&m_2(2\omega_2\dot{\beta}_4+\omega_2^2) - k\left(1-\frac{a_2\cos(\beta_2-\beta_4)}{a_4}\right) - k_2 - C\left(\frac{3a_4^2}{4\omega_2^2}+\frac{3a_3^2}{2\omega_1^2}\right) = 0
\end{aligned}
\quad (12)
$$

Figure 2 shows the comparison between the system response predicted by the CX method

$$x(t) = \frac{a_1}{\omega_1}\sin(\omega_1 t+\beta_1) + \frac{a_2}{\omega_2}\sin(\omega_2 t+\beta_2) \quad \text{and} \quad y(t) = \frac{a_3}{\omega_1}\sin(\omega_1 t+\beta_3) + \frac{a_4}{\omega_2}\sin(\omega_2 t+\beta_4) \quad (13)$$

and the response computed using numerical simulation of the original equations of motion (10) for $k_1 = k_2 = k_{12} = m_1 = m_2 = 1$, $C = 2$, $c_1 = c_2 = 0.05$ and initial displacements $x(0) = 1$ and $y(0) = 0$. Because the nonlinear coefficient and the initial displacements are $O(1)$ quantities, a strongly nonlinear system is investigated. Satisfactory agreement between prediction and numerical simulation is observed throughout the responses of the two oscillators.

3 CHARACTERIZATION OF A MULTICOMPONENT SIGNAL

The Hilbert transform characterizes a signal $x(t)$ through the extraction of its envelope $A(t)$ and instantaneous phase $\Phi(t)$, $x(t) = A(t)\cos\Phi(t)$ [17]. It is based on the analytic signal $X(t)$ defined as

$$X(t) = x(t) + jH[x(t)] = A(t)\exp[j\Phi(t)] \quad (14)$$

where $H[\bullet]$ is the Hilbert transform, and j is the imaginary number. It follows that

$$A(t) = \sqrt{x(t)^2 + H[x(t)]^2} \quad \text{and} \quad \Phi(t) = \arctan\left[H[x(t)]/x(t)\right] \quad (15)$$

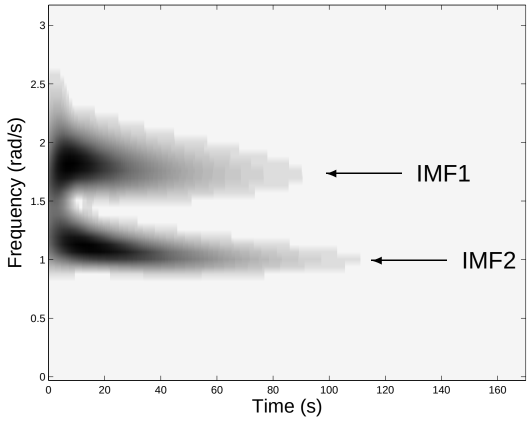

Figure 3: Wavelet transform applied to the response of a 2DOF nonlinear system.

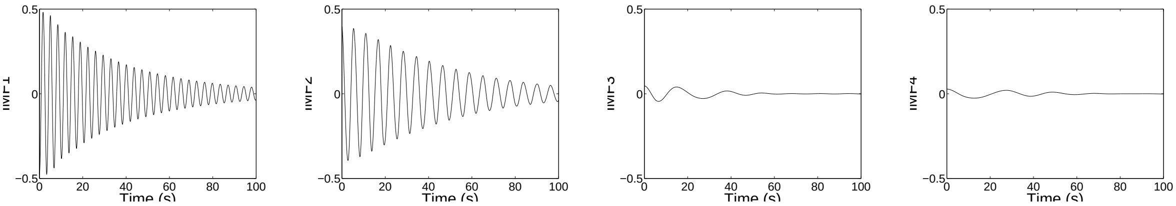

Figure 4: EMD applied to the response of a 2DOF nonlinear system.

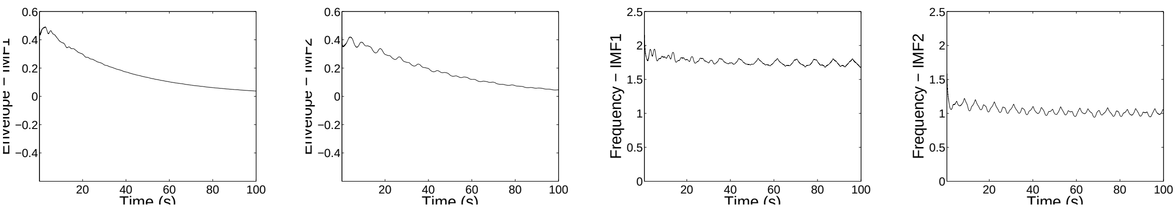

Figure 5: HHT applied to the response of a 2DOF nonlinear system.

The instantaneous frequency is the time derivative of the instantaneous phase. The investigation of the vibrations of nonlinear systems based on this procedure was proposed by Feldman [4].

For a multicomponent signal (i.e, a signal possessing multiple frequency components), the Hilbert transform cannot be applied because the assumption $x(t) = A(t)\cos\Phi(t)$ is no longer valid. An extension of the procedure in this case, the HHT, was developed by Huang and co-authors using a combination of EMD and the Hilbert transform [11]. The EMD decomposes a signal $x(t)$ in a sum of elemental signals, termed IMFs, such that the number of extrema and the number of zero-crossings are equal or differ by one and such that there is only one extremum between successive zero-crossings. As a result, the Hilbert transform can be applied to each IMF individually, giving a characterization of a multicomponent signal in terms of the amplitude and phase of its IMFs. A complete description of the method is beyond the scope of this paper; the interested reader may consult reference [11] for further details.

For illustration, the multicomponent signal $y(t)$ in Figure 2 is considered. Figure 3 depicts the wavelet transform of this signal, which reveals the presence of two dominant frequency components in the vicinity of the two natural frequencies of the linearized system, respectively. The application of the HHT begins with the decomposition of the signal in terms of its IMFs using EMD. The first 4 IMFs are displayed in Figure 4. The first 2 IMFs account for more than 99.5% of the total of variance in $y(t)$, which confirms that $y(t)$ can be approximated using a two-component signal. The Hilbert transform can now be safely applied to each of the first 2 IMFs. The final outcome of HHT in terms of amplitude and phase, or equivalently in terms of amplitude and instantaneous frequency, of the IMFs is shown in Figure 5.

4 SLOW-FLOW MODEL IDENTIFICATION METHOD

In the last two sections, the CX and HHT methods were described separately. Both approaches share a common basis by expanding a signal in a series of simple, monocomponent signals, which are related to the dominant frequency components of the signal:

- The CX method transforms the equations of motion of a nonlinear system into a set of approximate equations that govern the slow flow. Two equations, one for the amplitude and one for the phase, are derived for each modeled frequency component (note that there is *a priori* no restriction on the number of frequency components that can be taken into account). As a result, an expansion of the phase space occurs for multifrequency signals [compare, for instance, the dimension of the phase space of systems (10) and (12)].
- The HHT characterizes a signal through the envelope and phase of the elemental components, the IMFs.

The link between the methods is clear: *the equations derived using the CX method are the equations governing the amplitude and phase of the modeled IMFs. The CX method therefore provides a rigorous analytical framework for the HHT in nonlinear structural dynamics*. As further evidence, the envelope and phase of the response $y(t)$ of system (10) with $k_1 = k_2 = k_{12} = m_1 = m_2 = 1$, $C = 2$, $c_1 = c_2 = 0.05$ are computed separately using both methods and compared in Figure 6 for two different initial displacements, $x(0) = 0.5$ and $x(0) = 1$. There is an almost pointwise agreement for $x(0) = 0.5$; the agreement for $x(0) = 1$ is less good, but it is still satisfactory keeping in mind that a strongly nonlinear system is analyzed.

It is now demonstrated that the joint application of the methods enables us to develop a nonlinear system identification scheme, the SFMI method. Considering that the experimental response of a system with unknown physical parameters (e.g, the stiffnesses or the nonlinear coefficient) is measured, the HHT can be applied to the experimental data in order to extract the amplitudes a_i and phases β_i of the different frequency components. Based on the knowledge of the slow-flow model and the mass coefficients, the physical parameters can then be identified in a straightforward manner using a least-squares procedure.

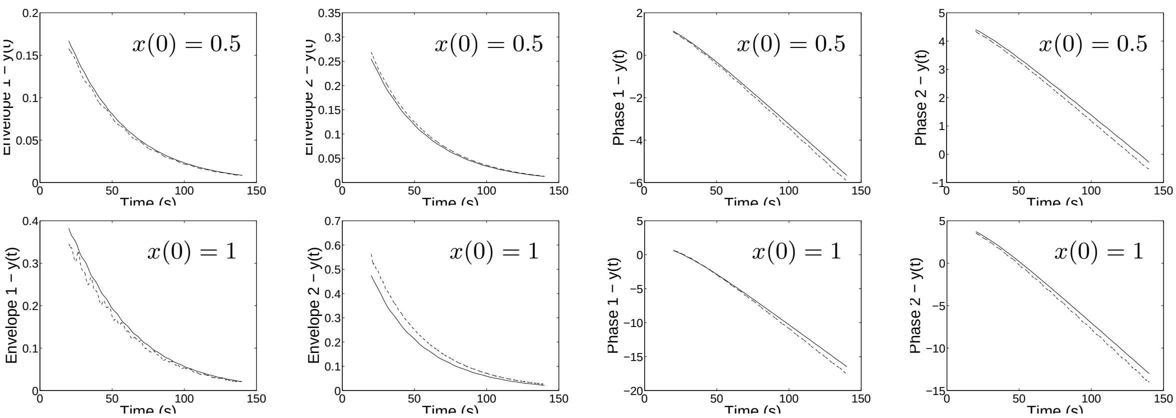

Figure 6: Envelope and phase of the two dominant components of signal $y(t)$ (solid line: CX method; dashed line: HHT).

	Initial conditions	k_1 (N/m)	k_2 (N/m)	k (N/m)	C (N/m^3)	c_1 (Ns/m)	c_2 (Ns/m)
Exact values	—	1	1	1	2	0.05	0.05
Identification	$x(0) = 0.5, y(0) = 0$	0.996	0.996	1.000	1.890	0.050	0.049
		0.4%	*0.4%*	*0.0%*	*5.5%*	*0.0%*	*2.0%*
Identification	$x(0) = 1, y(0) = 0$	0.976	0.973	1.000	1.986	0.048	0.052
		2.4%	*2.7%*	*0.0%*	*0.7%*	*4.0%*	*4.0%*
Identification	$x(0) = 0, y(0) = 1$	0.947	0.938	1.012	1.859	0.048	0.052
		5.3%	*6.2%*	*1.2%*	*4.0%*	*4.0%*	*4.0%*
Exact values	—	1	1	1	5	0.05	0.05
Identification	$x(0) = 1, y(0) = 0$	0.959	0.943	0.987	4.953	0.046	0.052
		4.1%	*5.7%*	*1.3%*	*0.9%*	*8.0%*	*4.0%*

TABLE 1: System identification results for the 2DOF system.

Taking system (10) as an example, its slow-flow model (12) can be recast into

$$\overbrace{\begin{bmatrix} -\frac{a_3}{2\omega_1}\sin(\beta_3-\beta_1) & 0 & 0 & 0 & \frac{a_1}{2} & 0 \\ -\frac{a_4}{2\omega_2}\sin(\beta_4-\beta_2) & 0 & 0 & 0 & \frac{a_2}{2} & 0 \\ -\frac{a_1}{2\omega_1}\sin(\beta_1-\beta_3) & 0 & 0 & 0 & 0 & \frac{a_2}{2} \\ -\frac{a_2}{2\omega_2}\sin(\beta_2-\beta_4) & 0 & 0 & 0 & 0 & \frac{a_2}{2} \\ \frac{a_3\cos(\beta_3-\beta_1)}{a_1}-1 & -1 & 0 & 0 & 0 & 0 \\ \frac{a_4\cos(\beta_4-\beta_2)}{a_2}-1 & -1 & 0 & 0 & 0 & 0 \\ \frac{a_1\cos(\beta_1-\beta_3)}{a_3}-1 & 0 & -1 & -\left(\frac{3a_3^2}{4\omega_1^2}+\frac{3a_4^2}{2\omega_2^2}\right) & 0 & 0 \\ \frac{a_2\cos(\beta_2-\beta_4)}{a_4}-1 & 0 & -1 & -\left(\frac{3a_4^2}{4\omega_2^2}+\frac{3a_3^2}{2\omega_1^2}\right) & 0 & 0 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} k \\ k_1 \\ k_2 \\ C \\ c_1 \\ c_2 \end{bmatrix}}^{\mathbf{x}} = -\overbrace{\begin{bmatrix} m_1\dot{a}_1 \\ m_1\dot{a}_2 \\ m_2\dot{a}_3 \\ m_2\dot{a}_4 \\ m_1(2\omega_1\dot{\beta}_1+\omega_1^2) \\ m_1(2\omega_2\dot{\beta}_2+\omega_2^2) \\ m_2(2\omega_1\dot{\beta}_3+\omega_1^2) \\ m_2(2\omega_2\dot{\beta}_4+\omega_2^2) \end{bmatrix}}^{\mathbf{b}} \quad (16)$$

The physical parameters are computed using the Moore-Penrose inverse

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \quad (17)$$

The validation of the SFMI method has been performed using 'virtual experiments' resulting from the numerical integration of the equations of motion of system (10). Four cases are analyzed, and the identified physical parameters together with their percentage error are listed in Table 1. Overall, the system is identified accurately since the percentage errors are almost all within 5 %.

5 CONCLUDING REMARKS

This paper focuses on the intimate relation between the outcome of the HHT and the slow-flow model with the aim of bringing a better understanding of this time-frequency decomposition in nonlinear structural dynamics. This has led to the development of the SFMI method, which identifies the physical parameters of a system from experimental data. The proposed method has interesting features:

- It is a 'linear-in-the-parameters' method and does not rely on nonlinear optimization techniques.
- It lends itself naturally to the identification of MDOF systems.
- It is based on the free response and does not require the measurement of the impulsive excitation.
- It has 'built-in characterization capabilities' since it exploits the Hilbert transform. In addition, the Hilbert transform gives sharper frequency and time resolutions compared to other time-frequency decomposition such as the wavelet and Gabor transforms.

The fact that the slow-flow model is an approximation of the true dynamics may be seen as a limitation. However, as demonstrated in this paper, this does not prevent one from identifying strongly nonlinear systems satisfactorily.

Finally, we note that the study is still at the exploratory stage. Several issues such as robustness to noise, nonparametric identification, identification with intermittent frequency components (harmonics) and experimental validation will be investigated in subsequent studies.

ACKNOWLEDGMENTS

The author G. Kerschen is supported by a grant from the Belgian National Fund for Scientific Research (FNRS) which is gratefully acknowledged.

REFERENCES

[1] Masri, S.F. and Caughey, T. K., A nonparametric identification technique for nonlinear dynamic problems, Journal of Applied Mechanics, Vol. 46, pp. 433-447, 1979.

[2] Worden, K. and Tomlinson, G.R., Nonlinearity in Structural Dynamics: Detection, Identification and Modelling, Bristol, Philadelphia, Institute of Physics Publishing, 2001.

[3] Kerschen, G., Worden, K., Vakakis, A.F. and Golinval, J.C., Past, present and future of nonlinear system identification in structural dynamics, Mechanical Systems and Signal Processing, in press (available via http://www.sciencedirect.com), 2005.

[4] Feldman, M., Nonlinear system vibration analysis using the Hilbert transform - I. Free vibration analysis method 'FREEVIB', Mechanical Systems and Signal Processing, Vol. 8, pp. 119-127, 1994.

[5] Feldman, M., Non-linear free vibration identification via the Hilbert transform, Journal of Sound and Vibration, Vol. 208, pp. 475-489, 1997.

[6] Feldman, M., Braun, S., Identification of non-linear system parameters via the instantaneous frequency: application of the Hilbert transform and Wigner-Ville technique, Proceedings of the 13th International Modal Analysis Conference (IMAC), Nashville, pp. 637-642, 1995.

[7] Staszewski, W.J., Identification of non-linear systems using multi-scale ridges and skeletons of the wavelet transform, Journal of Sound and Vibration, Vol. 214, pp. 639-658, 1998.

[8] Garibaldi, L., Ruzzene, M., Fasana, A. and Piombo, B., Identification of non-linear damping mechanisms using the wavelet transform, Mecanique Industrielle et Materiaux, Vol. 51, pp. 92-94, 1998.

[9] Argoul, P. and Le, T.P., Instantaneous indicators of structural behaviour based on the continuous cauchy wavelet analysis, Mechanical Systems and Signal Processing, Vol. 17, pp. 243-250, 2003.

[10] Bellizzi, S., Gullemain, P. and Kronland-Martinet, R., Identification of coupled non-linear modes from free vibration using time-frequency representation, Journal of Sound and Vibration, Vol. 243, pp. 191-213, 2001.

[11] Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.C., Tung, C.C. and Liu, H.H., The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, Proceedings of the Royal Society of London, Series A — Mathematical, Physical and Engineering Sciences, Vol. 454, pp. 903-995, 1998.

[12] Yang, J.N., Lei, Y., Pan, S.W. and Huang, N., System identification of linear structures based on Hilbert-Huang spectral analysis; Part 1: Normal modes, Earthquake Engineering and Structural Dynamics, Vol. 32, pp. 1443-1467, 2003.

[13] Yang, J.N., Lei, Y., Pan, S.W. and Huang, N., System identification of linear structures based on Hilbert-Huang spectral analysis; Part 2: Complex modes, Earthquake Engineering and Structural Dynamics, Vol. 32, pp. 1533-1554, 2003.

[14] Zhang, R.R., King, R., Olson L, and Xu, Y.L., Dynamic response of the Trinity River Relief Bridge to controlled pile damage: modeling and experimental data analysis comparing Fourier and HilbertHuang techniques, Journal of Sound and Vibration, Vol. 285, pp. 1049-1070, 2005.

[15] Peng, Z.K., Tse, P.W. and Chu, F.L., An improved Hilbert Huang transform and its application in vibration signal analysis, Journal of Sound and Vibration, Vol. 286, pp. 187-205, 2005.

[16] Manevitch, L.I., Complex representation of dynamics of coupled oscillators in Mathematical Models of Nonlinear Excitations, Transfer Dynamics and Control in Condensed Systems, New York, Kluwer Academic/Plenum Publishers, pp. 269-300, 1999.

[17] Ziemer, R.E. and Tranter, W.H., Principles of Communication: Systems, Modulation and Noise, Boston, Houghton Mifflin, 1976.