Using Targeted Energy Transfer to Stabilize Drill-string Systems

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ABSTRACT

Torsional vibration of the drill strings used in drilling oil and gas wells arises from a complex interaction of the dynamics of the drilling structure with speed-dependent effective rock-cutting forces. These forces are often difficult to model, and contribute substantially to the problems of controlling the drilling operation so as to produce steady cutting. We consider here the use of a nonlinear energy sink (NES) , an attachment which has been shown to be effective in reducing or even eliminating self-excited motions in van der Pol and aeroelastic systems. The NES is a completely passive, inherently broadband vibration absorber capable of attracting and dissipating vibrational energy from primary structures, in this case nonlinear discontinuous models of drill-string systems. In this paper we describe a prototypical drill string-NES system, briefly discuss some of the analytical and computational tools suitable for its analysis, and then concentrate on mathematical results on the efficacy of the NES in this application and their physical interpretation.

1 INTRODUCTION

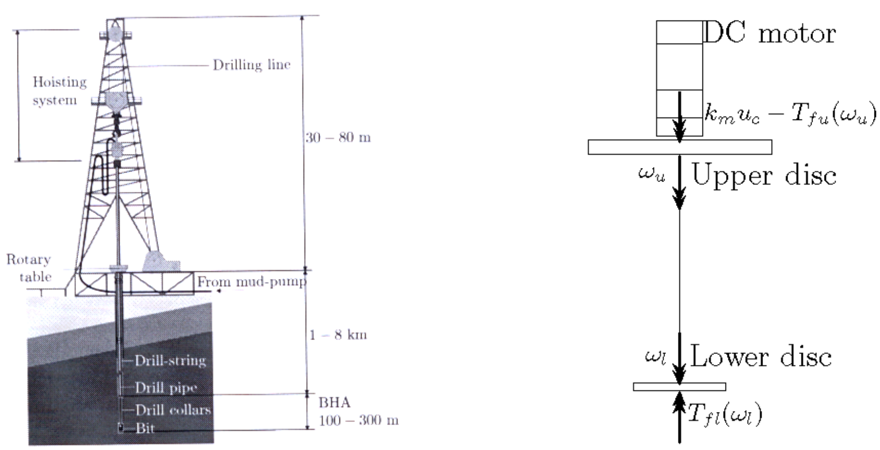
In drilling deep oil and gas wells, it can be difficult to maintain smooth cutting at the bit-rock interface. Among other disturbances, the rock is not homogeneous, the unavoidable friction is complicated by fluid flow, and spatially asymmetric, time-varying forces may be introduced to "steer" the well bore. All of these effects interact with the drill string, which itself can be several thousand meters long, to produce a dynamic environment which can exhibit inadequate stability.

In this paper we address the problem of improving the dynamics of the drill-string system by adding a passive, spatially localized attachment, a nonlinear energy sink (NES). The phenomenon we seek to exploit is that of targeted energy transfer, or "energy pumping," which has received extensive treatment in the dynamical systems literature in recent years. The NES here takes the form of a discrete torsional oscillator comprising a disk coupled to the drill string through an essentially nonlinear spring and a viscous damper. Mathematically similar devices of various configurations have been shown to be effective in suppressing transient response to broadband external loads and in reducing or eliminating self-excited vibrations in a van der Pol oscillator and in aeroelastic systems. The present use bears some similarities to the latter class of applications, but differs qualitatively in the large spatial separation between the driving motor and the drill bit.

Following a precise statement of the problem to be studied, we review some of the numerical challenges in modeling a drill-string system. Most of these are found to arise from the need to efficiently compute dynamic responses in the presence of friction and related discontinuities. A bifurcation diagram depicting the behavior of the system over a realistic range of inputs is produced, and serves to illustrate the influences of the NES parameters on this system. A numerical study and the resulting NES design is next described. Finally, a detailed analysis of a drill string and attached NES is presented, including some remarks on the robustness of the passive control achieved with this device.

2 PROBLEM DESCRIPTION

Deep wells for the exploration and production of oil and gas are drilled with a rotary drilling system (Figure 1(a)). A rotary drilling system creates a borehole by means of a rock-cutting tool, called a bit. The torque driving the bit is generated at the surface by a motor with a mechanical transmission box. Via the transmission, the motor drives the rotary table that consists in a large disk acting as a kinetic energy storage unit. The medium to transport the energy from the surface to the bit is a drill-string, mainly consisting of drill pipes. The drill-string can be up to 8km long. The lowest part of the drill-string is the bottom-hole-assembly (BHA) consisting of drill collars and the bit.



(a) (b)

Figure 1: (a) Schematic view of the rotary drilling system [1] / (b) Model of the drill-string system

This structure may undergo different kind of vibrations (torsional, bending, axial vibrations; hydraulic vibrations) during the drilling operation. In this paper, however, our efforts are mainly devoted to vibration mitigation of torsional vibrations. Many studies were undertaken to gain improved knowledge of the origins of those vibrations [2−7]. It was established that the cause for torsional vibration is the stick-slip phenomenon due to the friction force between the bit and the well [3−5]. Moreover, according to some other results, the cause of torsional vibrations is velocity weakening in the friction force (i.e., Stribeck effect) due to the contact between the bit and the borehole [6, 7]. As a result, friction plays a very important role in the occurrence of limit cycling in drill-string systems.

Figure 1(b) shows a prototypical drill-string system. As only torsional vibrations are considered, lateral movements of the system are constrained. This results in a two-degree-of-freedom system with generalized coordinates **q** = [θ_u α]^T, with α = θ_l - θ_u, and the equations of motion established in [8, 9] are :

$$\begin{cases} J_u\dot{\omega_u} - k_\theta\alpha + T_{fu}(\omega_u) & = & k_m u \\ J_l(\ddot{\alpha} + \dot{\omega_u}) + T_{fl}(\omega_u + \dot{\alpha}) + k_\theta\alpha & = & 0, \end{cases} \quad \text{where } \omega_u = \dot{\theta_u} \text{ and } \omega_l = \dot{\theta_l}. \tag{1}$$

The friction modeling in the set up is considered in [8−10], and the friction torques acting on each disc are given by the following set-valued force laws:

$$T_{fu}(\omega_u) \in \begin{cases} T_{cu}(\omega_u)sgn(\omega_u) & for \quad \omega_u \neq 0, \\ [-T_{Su}, T_{Su}] & for \quad \omega_u = 0. \end{cases} \tag{2}$$

$$T_{fl}(\omega_l) \in \begin{cases} T_{cl}(\omega_l)sgn(\omega_l) & for \quad \omega_l \neq 0, \\ [-T_{cl}(0^-), T_{cl}(0^+)] & for \quad \omega_l = 0. \end{cases} \tag{3}$$

$$T_{cu}(\omega_u) = T_{Su} + b_u\,|\omega_u| \tag{4}$$

$$T_{cl}(\omega_l) = T_{cl} + (T_{Sl} - T_{cl})e^{-|\omega_l/\omega_{Sl}|^{\delta_{sl}}} + b_l\,|\omega_l|\,. \tag{5}$$

and the complete model (1-5) now constitutes a differential inclusion.

Finally, it must be mentioned that an experimental set-up of the prototypical drill-string system was built at Eindhoven University of Technology. Its parameters, listed in Table 2, were identified using a nonlinear least-squares technique [9]. These parameters are used for all the numerical simulations carried out in the present study.

J_u, J_l	Rotating inertia of the upper and lower disc
k_m	Motor constant
T_{fu}, T_{fl}	Friction torque at the upper and lower disc
k_θ	Torsional spring stiffness
θ_u, θ_l	angular displacements of the upper and lower disc
$\alpha = \theta_l - \theta_u$	relative angular displacement
u (u_c)	Input voltage at the DC-motor

TABLE 1: Definitions

Parameter	Units	Estimated Value	Parameter	Units	Estimated Value
J_u	$[kgm^2/rad]$	0.4765	T_{sl}	$[Nm]$	0.2781
k_m	$[Nm/V]$	4.3228	T_{cl}	$[Nm]$	0.0473
T_{su}	$[Nm]$	0.37975	ω_{sl}	$[rad/s]$	1.4302
b_u	$[Nms/rad]$	2.4245	δ_{sl}	$[-]$	2.0575
k_θ	$[Nm/rad]$	0.0775	b_l	$[Nms/rad]$	0.0105
j_l	$[kgm^2/rad]$	0.0414			

TABLE 2: Parameter estimation

3 NUMERICAL SIMULATION OF A TWO-DEGREE-OF-FREEDOM DRILL-STRING SYSTEM

The objective is to study the steady-state behavior of a flexible rotor system with friction, modeled by a set-valued force laws (equations (2-5)). Such a system belongs to the class of discontinuous nonlinear dynamical systems and requires the use of appropriate methods for simulation and analysis.

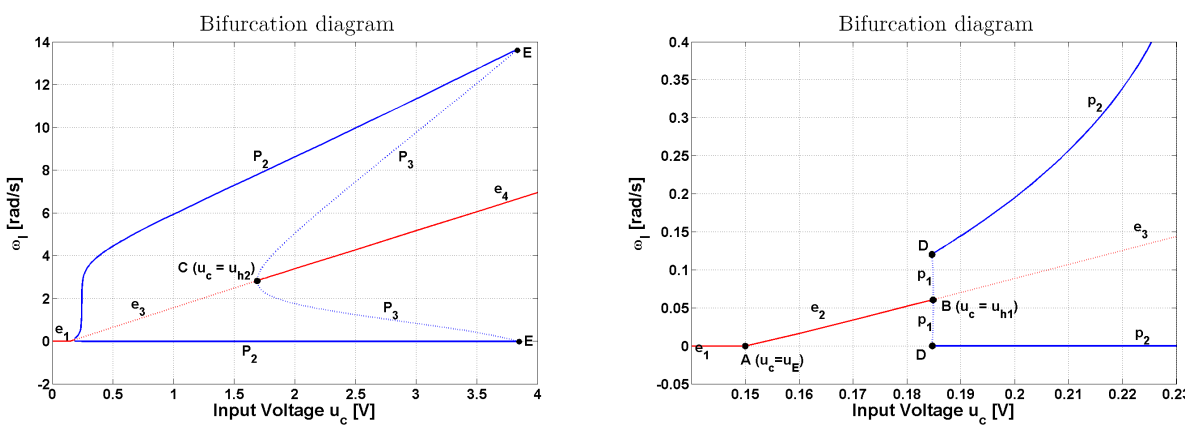
While pursuing numerically solutions of differential inclusions, a range of computational problems can be encountered around a discontinuity. Therefore, a number of techniques exists for the numerical solution of differential inclusions. Let us note, for example, the **smoothing method** consisting in the approximation of the sign function (in equations (2-3)) by differentiable functions. However, its main disadvantage is the fact that it generally results in stiff differential equations which can seriously compromise the computational efficiency.

The solution to these drawbacks is the use of the **switch model** whose main goal is to approximate a differential inclusion by a number of sets of ordinary differential equations. This model is explained in great detail in [11], but its basic idea can be briefly presented here. In order to numerically integrate the differential inclusion, this model introduces a boundary layer around the hypersurface (Σ) generated by the discontinuity in state space. From a general point of view, this layer is divided into several subspaces respectively characterizing transversal intersections and both attractive and repulsive sliding modes. In the boundary layer around the attractive sliding mode, the vector field is adapted such that an exponential convergence to the sliding mode is attained, thereby avoiding numerical instabilities. The main disadvantage of this model is the rapidly increasing complexity of the logical structure with increasing number of switching boundaries. In this case, more sophisticated simulation methods can be used like event-driven integration and time-stepping methods [9].

The integration process may reveal the presence of steady-state solutions such as equilibrium points and periodic solutions. Therefore, in order to obtain accurate approximations of these solutions and their related stability, specific numerical methods and stability analysis tools have to be considered. *Stable equilibrium points* can be easily computed using numerical simulation, but a more general method consists in the resolution of the algebraic inclusion of the discontinuous system. Moreover, the related stability (local and global) can be determined using Lyapunov's indirect and direct method (see reference [9]). While several numerical methods can be used to find *periodic solutions*, **the shooting method** is the most popular one and was used in this study. Its main advantage lies in the possibility to find stable and unstable periodic solutions. On the other hand, its main drawback is the need of an initial guess (for the iterative process) close enough to the solution, as it does not find a periodic solution but only refine an already good guess. The local stability of the periodic solutions has been determined using Floquet Theory. For further details about the properties and the fields of application of all the considered methods and theories, the reader may refer to [8−12].

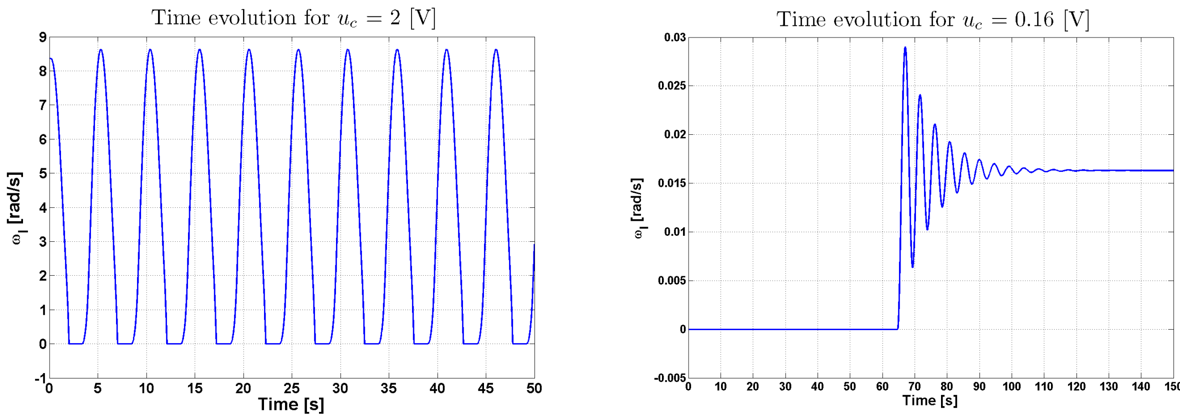
The association of the previous methods with continuation methods enables us to compute the bifurcation diagram of the system. The arclength continuation method is used in this study and is based on the association of a prediction step and a correction step as explained in reference [12]. Throughout this study, the terminology for the bifurcation diagrams is the same. The periodic solutions, which give rise to limit cycle oscillations (LCO), and the equilibrium branches correspond to 'p' and 'e', respectively. Solid and dotted lines are used to refer to stable and unstable branches, respectively.

The bifurcation diagram corresponding to system (1-5) with the parameters of Table 2 is depicted in Figure 2. One can observe that, for an input voltage at the DC motor $u_c < u_E$, an equilibrium set exists. This set, indicated by branch e_1, reduces to an equilibrium point at point A and progresses as an equilibrium branch e_2 of isolated equilibria (Figure 3(b)). Point B, for which $u_c = u_{h1}$, represents a subcritical Hopf bifurcation point. Both an unstable periodic branch p_1 consisting of limit cycles without stick-slip and an unstable equilibrium branch e_3 originate from point B. The unstable periodic branch p_1 is connected to a locally stable periodic branch p_2 at the point D, which represents a fold bifurcation point. As shown in Figure 3(a), the periodic branch p_2 consists of locally stable limit cycles, which represent torsional vibrations with stick-slip. As a result, point D corresponds to a discontinuous fold bifurcation. A similar analysis can be achieved for points C (subcritical Hopf bifurcation point) and E (discontinuous fold bifurcation point) as well as for the remaining branches.



(a) Bifurcation diagram (b) Close-up of the low voltage range

Figure 2: Bifurcation diagram of the primary system.



(a) Stick-slip phenomena for $u_c = 2$ V

(b) Equilibrium point for $u_c = 0.16$ V

Figure 3: Illustration of stick-slip phenomenon (2 V) and equilibrium point (0.16 V).

4 SUPPRESSION OF FRICTION-INDUCED LIMIT CYCLING BY MEANS OF A NONLINEAR ENERGY SINK : A PARAMETRIC STUDY.

In order to mitigate the vibrations in mechanical systems, active or passive methods can be used. In this study, passive methods are considered. One of their advantage is that they can be achieved using relatively minor structural modifications. Moreover, these methods are interesting, because their performance is remarkable and they do not need any external energy supply to work.

4.1 Addition of the Nonlinear Energy Sink to the Drill-String System

The tuned mass damper (TMD) is a simple and efficient device, but it is only effective when it is precisely tuned to the frequency of a vibration mode. Another limitation of this device occurs when several modes of a structure participate in the system response. Indeed, in this case, the TMD cannot absorb the vibrations of more than one mode of the primary system, a feature which clearly limits its efficiency. To overcome the limitations of the TMD, an essentially nonlinear attachment (i.e., characterized by the absence of a linear term in the force-displacement relation), termed a nonlinear energy sink (NES), has been introduced. As shown in [13, 14], an NES is characterized by two remarkable properties:

1. Targeted energy transfer (i.e., an irreversible energy flow) from a primary structure to an attached NES can be achieved.
2. The NES has no preferential resonance frequency, which makes it capable of resonating with any mode of the primary structure through isolated resonance captures. Moreover, and this is especially interesting from a practical point of view, the NES can engage in transient resonance either with isolated structural modes of the primary structure, or with a set of modes, drawing energy from each before engaging the next. In essence, the NES can be designed to be a passive, adaptive, broadband boundary controller, which, although local in space, can affect the global dynamics of the primary structure to which it is attached.

While the use of an NES seems promising for passive control of vibrations, the increasing complexity of the dynamical behavior of the new systems created must be put forward, because of the strongly nonlinear characteristic of these systems.

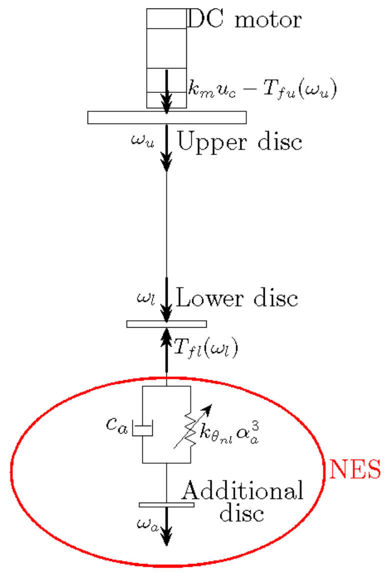


Figure 4: Schematic representation of the drill-string system with an NES.

In Figure 4, the addition of the NES to the primary system corresponds to the introduction of a third degree-of-freedom in the primary system. The vector of generalized coordinates becomes **q** = [θ_u θ_l θ_a]^T . This leads to some major modifications (detailed in reference [10]) in the differential equations (1) :

$$\begin{cases} J_u \dot{\omega}_u - k_\theta \alpha + T_{fu}(\omega_u) & = & k_m u \\ J_l(\dot{\omega}_u + \ddot{\alpha}) + k_\theta \alpha - k_{\theta_{nl}} (\alpha_a)^3 - c_a(\dot{\alpha}_a) + T_{fl}(\omega_u + \dot{\alpha}) & = & 0 \\ J_{add}(\ddot{\alpha}_a + (\ddot{\alpha} + \dot{\omega}_u)) + k_{\theta_{nl}} (\alpha_a)^3 + c_a(\dot{\alpha}_a) & = & 0, \qquad \text{where } \alpha_a = \theta_a - \theta_l. \end{cases} \tag{6}$$

The configuration of the system with the NES gives rise to the modification of the state space vector considered for the initial system (**x** = [α ω_l $\dot{\alpha}$]^T). Two additional state space variables are considered (α_a $\dot{\alpha_a}$). Therefore, the vector field is modified, and some major modifications have to be applied to the switch model. Before going further in the analysis, the validation of the switch model was achieved to ensure the quality of the simulation results.

4.2 Determination of the New Parameters Introduced with the NES

The NES in the primary system introduces new parameters, namely the nonlinear stiffness $k_{\theta_{nl}}$, the additional disc mass inertia (rotating) J_{add} and the damping coefficient c_a.

The attribution of specific values to the NES parameters could be carried out using optimization methods. However, a parametric study is performed herein to determine those values. The current objective is not to find the best set of values for the NES, but rather to understand whether the introduction of the NES can enlarge the range of working input voltages for which stick-slip limit cycling is avoided. Therefore, an initial set of values is used and adapted to obtain the largest range of input voltages leading to stable equilibrium solutions and the avoidance of stick-slip limit cycling.

The common practice is to consider that the NES mass/inertia is approximatively around 5% of the total mass/inertia of the primary system. The dashpot of the NES is such that its viscous damping coefficient has a value close to that of the lower disc. Finally, the determination of the nonlinear (cubic) stiffness is based on the linear stiffness of the string of the primary system. The aim is to find a value of the nonlinear stiffness that creates an elastic torque of the same order than the one characterizing the primary system. The initial set of parameters is given by :

Parameter	Units	Estimated Value
J_{add}	$[kg\ m^2\ /\ rad]$	0.025895
c_a	$[N\ m\ s\ /rad]$	0.0105
k_{nl}	$[N\ m\ /\ rad^3]$	0.0025

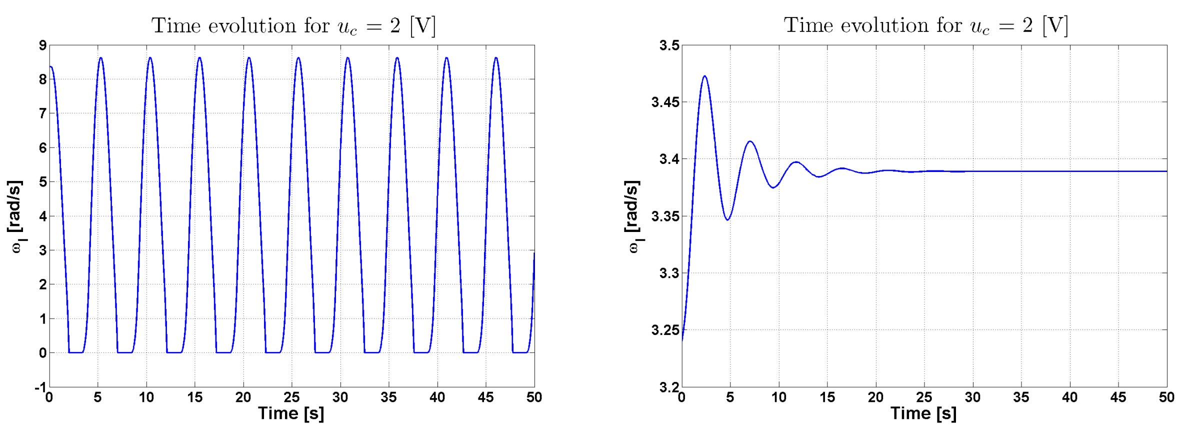
TABLE 3: Initial set of parameters.

Several simulations were carried out by modifying a single parameter in the initial set. This was achieved in order to assess the impact of the modification introduced on the dynamical behavior, the impact being revealed through bifurcation diagrams. All the results related to these numerical experiments are available in reference [10]. The simulations allow an essential property of nonlinear systems to be highlighted. Indeed, the combination of every single modification, leading to an improvement when acting alone, does not necessarily lead to an improved dynamical behavior when acting together. As this combination can be seen like a superposition, this clearly shows that the superposition principle is no longer valid in a nonlinear context. This limitation shows the difficulty to determine the best set of values using simple methods. The parametric study in [10] showed that the sets of parameters in Table 4 give interesting results in terms of the suppression of limit cycling:

Set	k_{nl}	J_{add}	c_a
n°	$N\ m\ /\ rad^3$	$kg\ m^2$	$N\ m\ s\ /\ rad$
1	0.002515	0.025895	0.0105
2	0.002515	0.025895	0.0210

TABLE 4: Results of the parametric study: two sets of parameters.

Figure 5 clearly shows that the presence of the NES stabilizes the drill-string system for an input voltage of 2 V. However, we note that the new equilibrium solution provided by the NES might not be the only steady-state solution for this particular voltage. This is discussed in the next section.



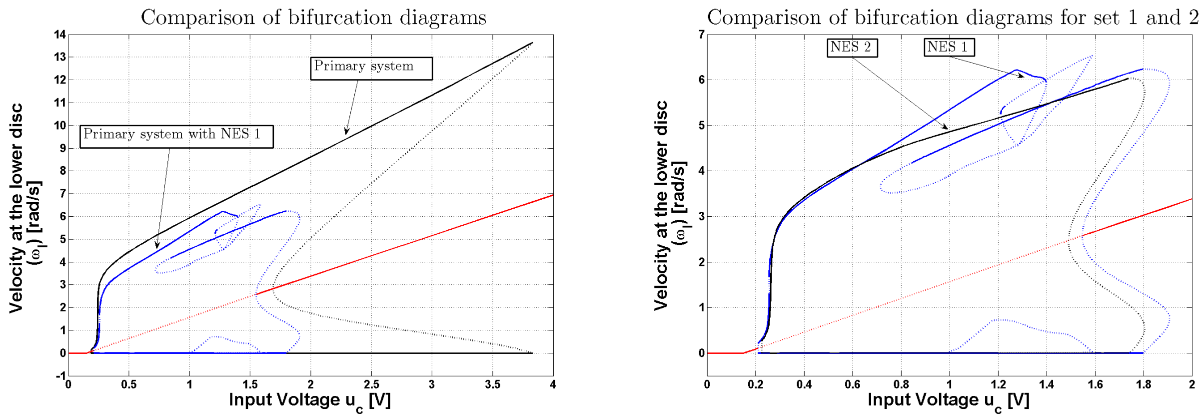
(a) Primary system without NES (b) Primary system with NES — set 1

Figure 5: Time evolution for a given input voltage at DC-motor u_c = 2 [V].

5 DETAILED ANALYSIS OF A DRILL-STRING SYSTEM WITH AN NES

The first step of this study investigated the possibility to mitigate torsional vibrations in the drill-string system using an NES. However, it is important to determine to what extent the nonlinear elastic torque introduced participates to the dynamics. In other words, we verify that the good suppression results obtained rely on the presence of the nonlinear string. To this end, the amplitudes of the elastic torque in the string of the primary system and in the NES were compared in [10]. According to this study, the torques were of the same order of magnitude, which demonstrates the importance of the nonlinear elastic torque.

5.1 Bifurcation Diagrams



(a) Primary system and absorbed system (set1)

(b) Absorbed system set 1 and set 2

Figure 6: Comparison of the bifurcation diagrams.

As previously mentioned, bifurcation diagrams are used to assess the impact of the NES on the global dynamics of the drill-string system. Graphical comparisons of such diagrams in Figure 6 clearly show the improvement of the dynamical behavior generated thanks to the NES. In order to gain further insight, three different criteria are considered :

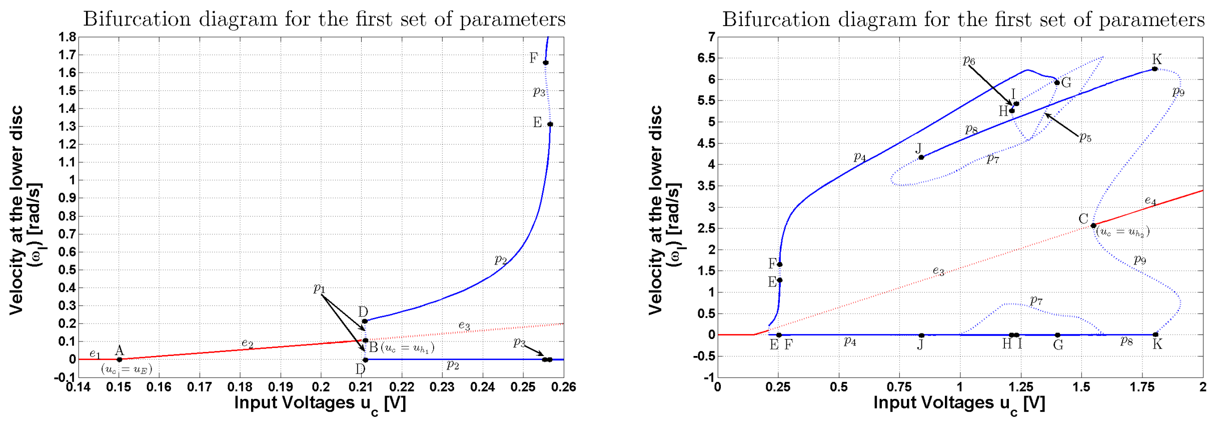
1. The percentile reduction of the range of voltages leading to unstable equilibria and their transformation into locally asymptotically stable equilibria.
2. The percentage of voltages leading to (locally or globally) stable equilibrium over the range [0 3.83] V.
3. The percentage of locally stable equilibrium solutions that are transformed into globally stable equilibrium solutions over the range [1.69 3.83] V (range related to the occurrence of locally stable equilibria in the primary system).

All the numerical values related to these criteria are listed in Table 5. The percentages show that the second set of parameters creates a wider range of voltages leading to only equilibrium solutions. Furthermore, the complete characterization of the bifurcation diagram for both sets can be achieved. Figure 7 shows the complexity of the dynamical behavior generated for the first set of parameter. On the other hand, Figure 8 illustrates that the resulting dynamics is much simpler for the second set, which should be related to a higher damping coefficient c_a. This is in agreement with what was observed in [15] where the NES was used for the suppression of aeroelastic instability.

These results demonstrate that the NES can stabilize the drill-string system in a relatively wide range of voltages. In addition, the smoothness of the resulting dynamics for the second set of parameters is certainly attractive from a practical viewpoint.

Observation	First Set	Second Set	Δ
1	11.04%	15.89%	4.85%
2	65.04%	66.95%	1.91%
3	94.95%	97.66%	2.71%

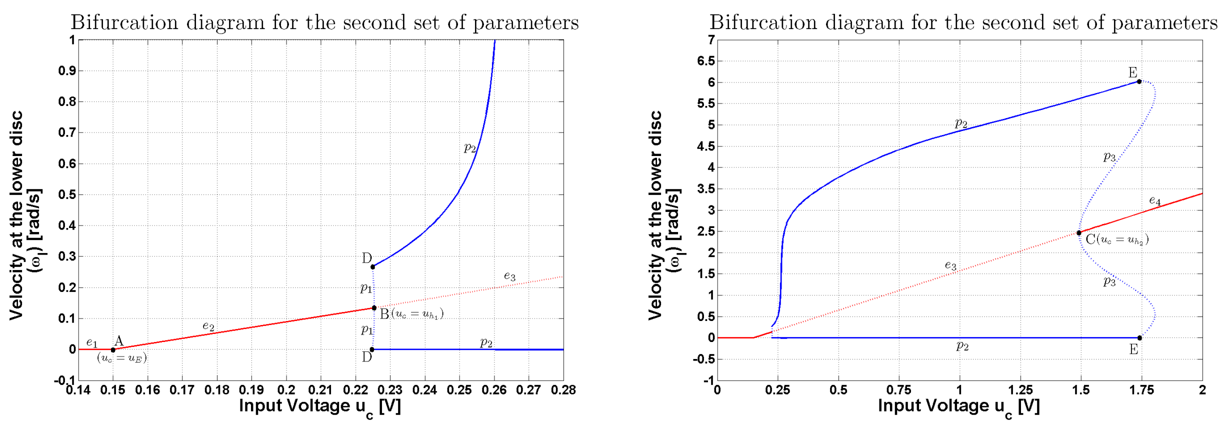
TABLE 5: Stabilization of the drill-string system (quantitative results).



(a) First part of the bifurcation diagram

(b) Second part of the bifurcation diagram

Figure 7: Bifurcation diagram for the first set.



(a) First part of the bifurcation diagram

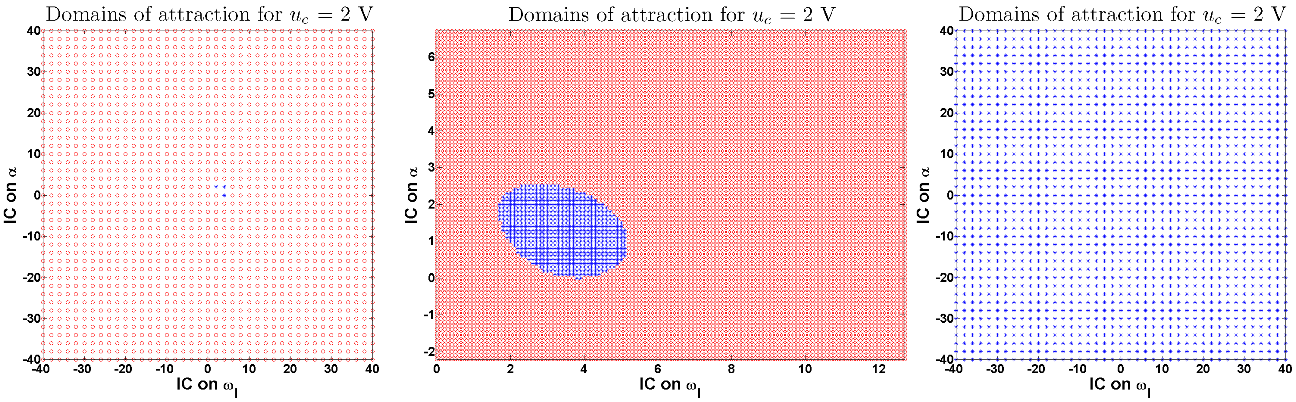
(b) Second part of the bifurcation diagram

Figure 8: Bifurcation diagram for the second set.

5.2 Domains of Attraction

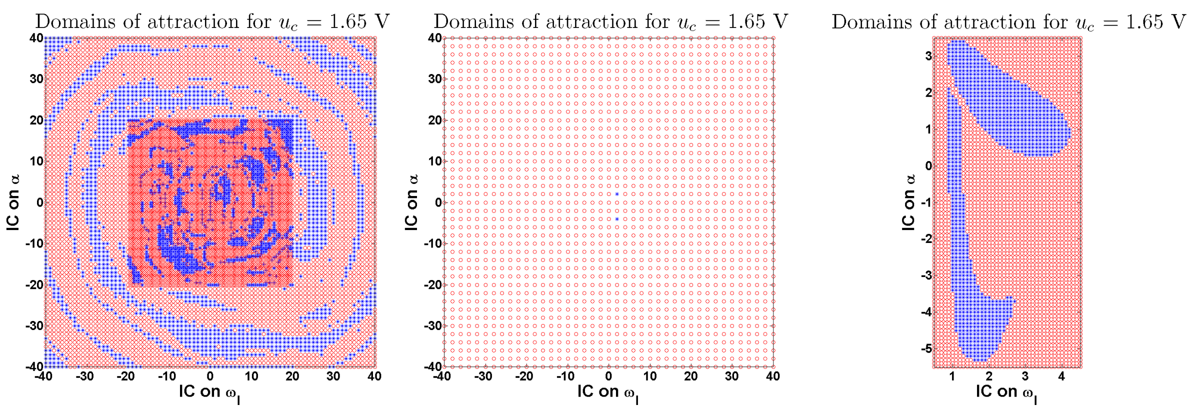
One key feature of nonlinear systems is the fact that several (quasi-)periodic solutions and/or equilibrium points may coexist. According to this property, the steady-state behavior of a nonlinear system may be characterized by the presence of two stable steady-state solutions (e.g., a stable equilibrium point and a stable periodic solution). The evolution toward one or the other depends on the initial conditions imposed to the system. For a constant input voltage, we may therefore converge to different kinds of steady-state solutions depending on the initial conditions. It is therefore meaningful to represent the domains of attraction of each kind of steady-state solutions (equilibria or periodic solutions).

The system composed of the drill-string and the NES is characterized by 5 state-space variables. This means that the domains of attraction lie in a 5-dimensional space. This is obviously impossible to represent. Therefore, only two state-space variables are considered; i.e., the velocity at the lower disc ω_l and the deformation of the string of the primary system α. All the other state variables are set to 0.



(a) Primary system without NES (b) Primary system without NES: close-up (c) Primary system with NES (set 2)

Figure 9: Domains of attraction for u_c = 2 V. Red circles and blue stars correspond to stable limit cycle oscillations and stable equilibrium solutions, respectively.



(a) Primary system with NES (set 1) (b) Primary system with NES (set 2) (c) Primary system with NES (set 2): close-up

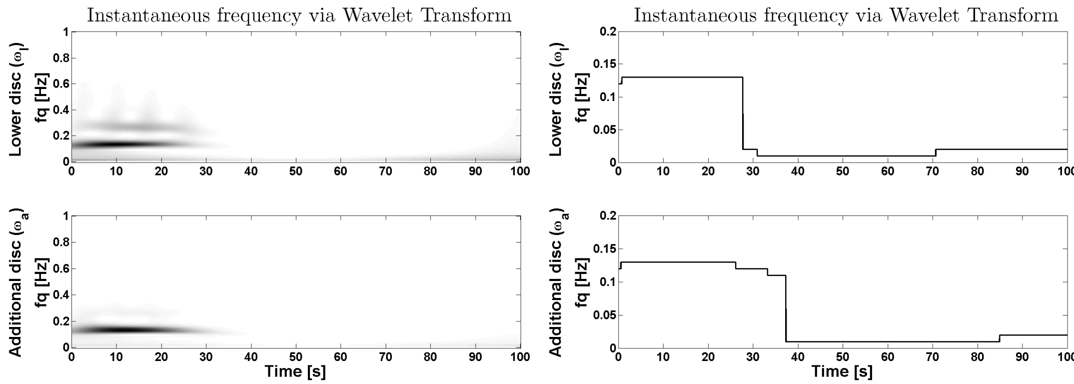
Figure 10: Domains of attraction for u_c = 1.65 V. Red circles and blue stars correspond to stable limit cycle oscillations and stable equilibrium solutions, respectively.

Two voltages are considered, and the domains of attraction of the system without and with NES are computed. At u_c = 2 V (Figure 9), it is clear that the locally asymptotically stable equilibria characterizing the system without NES are changed into globally asymptotically stable equilibria with the introduction of the NES.

At u_c = 1.65 V, the primary system undergoes a stable LCO without any stable equilibrium point (not shown here). When the NES is introduced, the LCOs are transformed into locally asymptotically stable solutions for both sets, as depicted in Figure 10. Furthermore, the complexity of Figure 10(a) in comparison with Figures 10(b) and 10(c) confirms the practicality of the NES related to the second set of parameters.

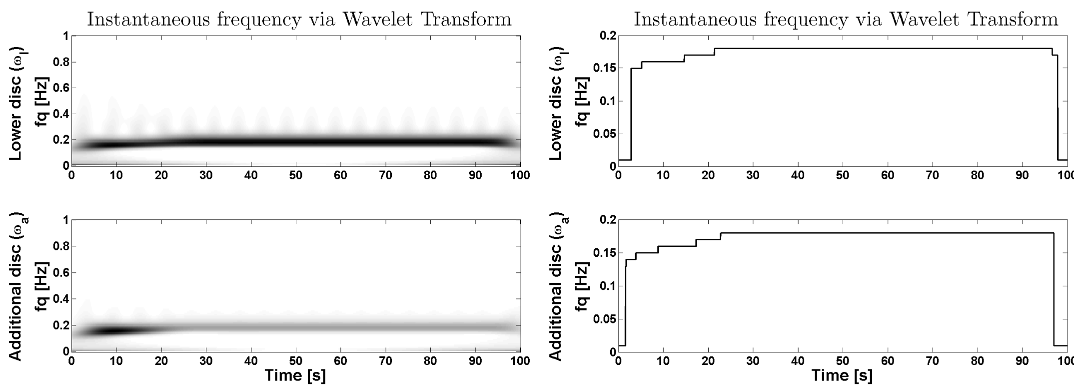
5.3 Wavelet Transform

The purpose of this section is to apply the wavelet transform to the time series to demonstrate possible resonance captures between the primary system and the NES. The wavelet transform (WT) is a suitable technique to analyze the temporal evolution of the dominant frequency components of nonlinear signals. The comparison of the instantaneous frequency of the velocity at the lower disc and at the NES provides a robust means of verifying the occurrence of resonance captures, or frequency locking during the transient dynamics.



(a) Shaded representation (b) Maximum frequencies

Figure 11: Instantaneous frequency via wavelet transform (u_c = 2 V) parameter set 1



(a) Shaded representation (b) Maximum frequencies

Figure 12: Instantaneous frequency via wavelet transform (u_c = 1.5 V) parameter set 2

Figures 11 and 12 depict the instantaneous frequencies of the velocity at the lower (ω_l) disc and at the NES (ω_a) for voltages leading to an equilibrium point (2 V) and a periodic solution (1.5 V), respectively. These figure seem to confirm the resonance capture and the locking in the transient phase. In fact, the NES, which has no preferential resonant frequency, tunes itself to the frequency of the developing instability. This resonance phenomenon leads to an energy flow from the drill-string system to the NES. The energy is confined in the absorber, where it is eventually dissipated by the dashpot.

6 CONCLUDING REMARKS

Self-sustained vibrations may appear in mechanical systems for various reasons. They may limit the performance of such systems or even cause their failure. In this study, the focus was on friction-induced vibrations in drill-string systems. As a benchmark we consider a rotor-dynamics system with (set-valued) friction and flexibilities.

This paper investigated the possibility of passively mitigating these instabilities using a nonlinear absorber characterized by an essential stiffness nonlinearity. The motivations for using a nonlinear energy sink are:

1. the possibility of resonating with any mode of the primary structure through isolated resonance captures.
2. the possibility of realizing targeted energy transfer from the primary structure to the NES.

The parametric study demonstrated that the NES can eliminate completely the instabilities in a relatively wide range of input voltages at the DC-motor generating the driving torque. In addition, the NES can also reduce the amplitude of the remaining limit cycles in the regions where complete elimination is not possible. According to all these results, one can conclude that the addition of an NES to a drill-string system clearly improves the global dynamical behavior of the system and substantially extends its domain of operation.

Because a complete suppression of the instabilities in the whole range of voltages investigated was not possible, there is still much work to be done. Future research should therefore perform a more exhaustive search of the design space of the NES using optimization methods to get even broader suppression results. Furthermore, an experimental demonstration is necessary to validate the developments. Finally, the comparison of the present results with the performance achieved with active control would be an interesting contribution.

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