IDENTIFICATION OF WIRE ROPE ISOLATORS USING THE RESTORING FORCE SURFACE METHOD

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SUMMARY: The restoring force surface method offers an efficient and reliable identification of non-linear single-degree-of-freedom systems. The method may be extended to multi-degree-of-freedom systems but by loosing the key advantage of the method which lies in the two-dimensional representation for single-degree-of-freedom systems. An experimental application of the restoring force surface method is considered in the present paper. The structure investigated consists of wire rope isolators mounted between a load mass and a base mass. These helical isolators were found to be characterised by a non-linear behaviour. The results obtained are discussed in details and the advantages and drawbacks of the method are underlined.

KEYWORDS: non-linear systems – identification – restoring force surface – isolators.

INTRODUCTION

The importance of diagnosing, identifying and modelling non-linearity is recognised since a long time but it is only recently that non-linear theory is beginning to be applied for structural dynamic design. Identification of non-linear systems ranges from methods which simply detect the presence or type of a non-linearity to those which seek to quantify the dynamic behaviour through a mathematical model. In this latter category lies the non-parametric scheme called the restoring force surface (RFS) method.

Masri and Caughey [1-2] laid down the foundations of the method and significant improvements were brought about since the original papers. Al-Hadid and Wright proposed a sensitivity approach for estimating the mass or modal mass [3]. Elaborated interpolation procedure used to overcome the problem of an inadequately covered state plane were presented by Worden and Tomlinson [4]. Duym and Schoukens designed optimised excitation signals in order to guarantee the quality of the fit by uniformly covering the phase plane [5]. They also used a local non-parametric identification of the non-linear force [6]. Kerschen *et al*. applied the method to beams characterised by bilinear and piecewise linear stiffness [7].

In this paper, the theoretical background of the RFS method is first recalled. Then, the method is applied to the benchmark proposed by the VTT Technical Research Centre of Finland in the framework of COST Action F3 working group on “Identification of non-linear systems”.

Finally, the results obtained using the RFS method are compared with those obtained by Marchesiello *et al.* with the conditioned reverse path method [8].

RESTORING FORCE SURFACE (RFS) METHOD

The RFS method is based on Newton's second law :

$$m\,\ddot{x}(t) + f(x(t),\dot{x}(t)) = p(t) \tag{1}$$

where $p(t)$ is the applied force and $f(x,\dot{x})$ is the restoring force, i.e. a non-linear function of the displacement and velocity. The time histories of the displacement and its derivatives, and of the applied force are assumed to be measured. In practice, the data must be sampled simultaneously at regular intervals. From equation (1), it is possible to find the restoring force defined as $f_i = p_i - m\,\ddot{x}_i$ where subscript i refers to the i^{th} sampled value. Thus, for each sampling instant a triplet $(x_i, \dot{x}_i, f_i)$ is found, i.e. the value of the restoring force is known for each point in the phase plane $(x_i, \dot{x}_i)$.

It is important to describe the system by a mathematical model. The usual way is to fit to the data a model of the form :

$$f(x,\dot{x}) = \sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij}\, x^i\, \dot{x}^j \tag{2}$$

Least-squares parameter estimation can be used to obtain the values of the coefficients a_{ij}. To have a measure of the error between the measured value x_i and the predicted value $\hat{x}_i$, the mean-square error (MSE) indicator is defined as :

$$MSE(x) = \frac{100}{N\, s_x^2} \sum_i (x_i - \hat{x}_i)^2 \tag{3}$$

where N is the total number of samples and s_x^2 is the variance of the measured input. Experience shows that an MSE value of less than 5% indicates good agreement while a value of less than 1% reflects an excellent fit. To determine which terms are significant and which terms can be safely discarded in equation (2), the significance factor is used :

$$s_q = 100 \frac{s_q^2}{s_x^2} \tag{4}$$

where s_x^2 corresponds to the variance of the sum of all the terms of the model and s_q^2 is the variance of the considered term. Roughly speaking, the significance factor represents the percentage of the contribution of the term to the model variance.

From the foregoing developments, it appears that the method requires to measure displacement, velocity, acceleration and force time histories at each degree of freedom. A pragmatic approach to the procedure demands that only one signal should be measured and the other two should be estimated from it. Numerical integration and/or differentiation may be adopted.

The differentiation can be carried out in the time domain or in the frequency domain. A polynomial can be fitted to N data points such that the point at which the derivative is required is at the centre. The analytic derivative of the fitted polynomial is then computed. This illustrates a possible way of differentiating in the time domain. However, it can be shown that numerical differentiation leads to an inaccurate estimation of the acceleration. Considerably more detailed discussion is available in reference [9].

The practical solution is to measure the acceleration and numerically integrate it to find velocity and displacement. Various methods for achieving integration exist : trapezium rule, Simpson's rule, integration in the frequency domain, and so forth. There are two main problems associated with the integration, i.e. the introduction of low- and high-frequency components. The trapezium rule only suffers from the introduction of low-frequency components and does not require the use of a low-pass filter. Furthermore, it is the simplest integration process and offers saving of time. For these reasons, the trapezium rule is considered throughout the paper.

Since the trapezium rule basically acts as an amplifier of the low-frequency components, the integrated signals are to be high-pass filtered. It should be noted that high-pass filtering with cut-off $n/(2N\,\Delta t)$ is equivalent to a polynomial trend removal of order n where N is the number of points and Δt the sampling interval [9]. Accordingly, choosing a cut-off frequency higher than 0 Hz immediately imposes the filtered signals to be of zero mean since a polynomial trend of order 0, i.e. a constant, is removed. This leads to an inaccurate estimation of velocity and displacement of asymmetrical systems [7].

DESCRIPTION OF THE BENCHMARK

The analysed data were chosen from those proposed by VTT Technical Research Centre of Finland within the framework of the European COST action F3 working group on "Identification of non-linear systems". The benchmark consists of wire rope isolators mounted between the load mass and the base mass (see Fig. 1). The helical wire rope isolators were found to be characterised by a non-linear behaviour.

The movement and forces experienced by isolators were measured. In particular, the acceleration of the load mass $\ddot{x}_2$, the acceleration of the bottom plate $\ddot{x}_{1b}$, the force F and the relative displacement between the top and bottom plates u_{12} were measured. The excitation produced by an electro-dynamic shaker corresponds to a white noise sequence, low-pass filtered at 400 Hz. It is worth pointing out that several excitation levels were considered going from 0.5 up to 8 Volts (V) and that a second test was carried out with another load mass (5.8 kg instead of 2.2 kg).

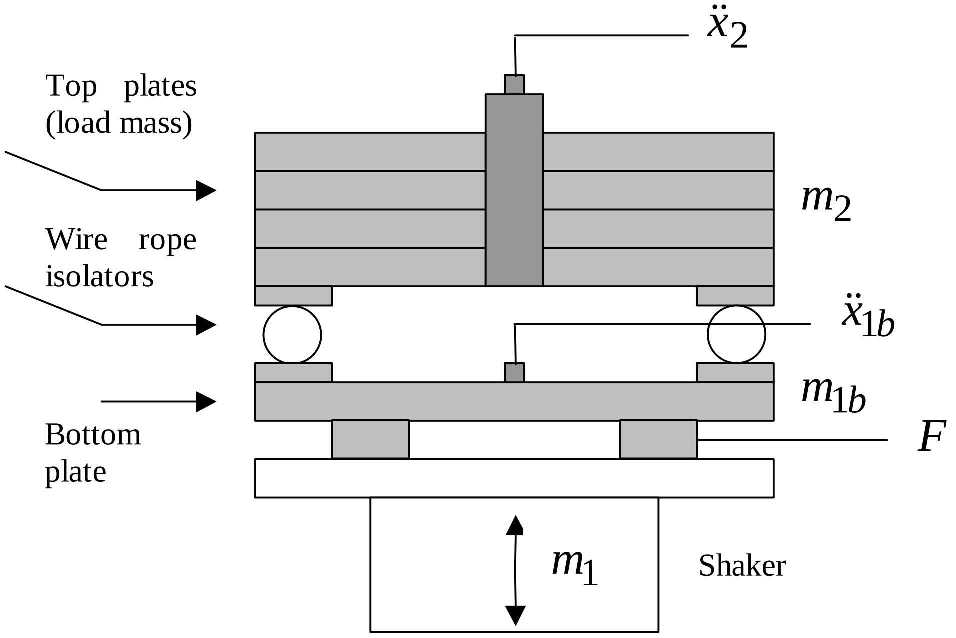

Fig. 1: Wire rope isolators (VTT benchmark)

RESULTS OF THE IDENTIFICATION

The RFS method has been introduced in this paper for the identification of single-degree-of-freedom systems while the structure under consideration is clearly a multi-degree-of-freedom system. However, writing Newton's second law for the load mass m_2 yields

$$m_2\ddot{x}_2 + f(\overbrace{x_2 - x_{1b}}^{u_{12}}, \overbrace{\dot{x}_2 - \dot{x}_{1b}}^{\dot{u}_{12}}) = 0 \quad (5)$$

This latter equation can be written in the equivalent form

$$m_2\ddot{u}_{12} + f(u_{12}, \dot{u}_{12}) = -m_2\ddot{x}_{1b} \quad (6)$$

Equation (6) may be viewed as a single-degree-of-freedom system with a base acceleration.

Prior to the identification, the RFS approach offers a very interesting way of visualising the non-linearity through the stiffness and damping curves. The stiffness (damping) curve represents the evolution of the restoring force as a function of the displacement (velocity) only. Fig. 2 illustrates the stiffness curves for the different excitation levels considered. The restoring forces corresponding to the 0.5 V, 1 V and 2 V levels [Fig. 2(a), Fig. 2(b), Fig. 2(c)] may be assumed to be linear since the stiffness curves are almost straight lines. However, the data corresponding to the 1 V level [Fig 2(b)] will not be considered since the stiffness curve is very noisy. The non-linear behaviour may be observed for the remaining levels, i.e. 4 V, 8 V and 4 V (m_2=5.8 kg) [Fig. 2(d), Fig. 2(e), Fig. 2(f)]. It is also observed that the non-linearity is characterised by a softening effect. Another feature to underline is the symmetry of the stiffness curves which means that the non-linear stiffness is odd. The damping curves could also be represented. But due to the slight participation of the damping in the system response, these curves are quite noisy and the damping is assumed to be linear in the following.

The foregoing procedure helped in the decision of the model in the sense that a softening symmetrical non-linearity in stiffness has been detected. Accordingly, a possible model for the restoring force may be written as follows

$$f(u_{12}, \dot{u}_{12}) = au_{12} + b\dot{u}_{12} + c\left|u_{12}\right|^{a} \operatorname{sign}(u_{12}) \quad (7)$$

where parameters a, b, c and a have to be estimated. The RFS method allows to identify a, b and c using a least-squares scheme provided that the non-linearity exponent a is known. Since it is not the case, the MSE of the identification procedure is computed for different values of a (Fig. 3). Several comments can be made regarding this figure:

1. A peak appears for a value of a equal to 1 which means that a linear model is fitted to the data. Thus, a linear model is not sufficient to capture the dynamics since the MSE is much smaller for other values of the exponent.
2. The MSE reaches its minimum value for a equal to 1.5. This value, corresponding to an MSE equal to 2.11 %, is adopted for the identification procedure.

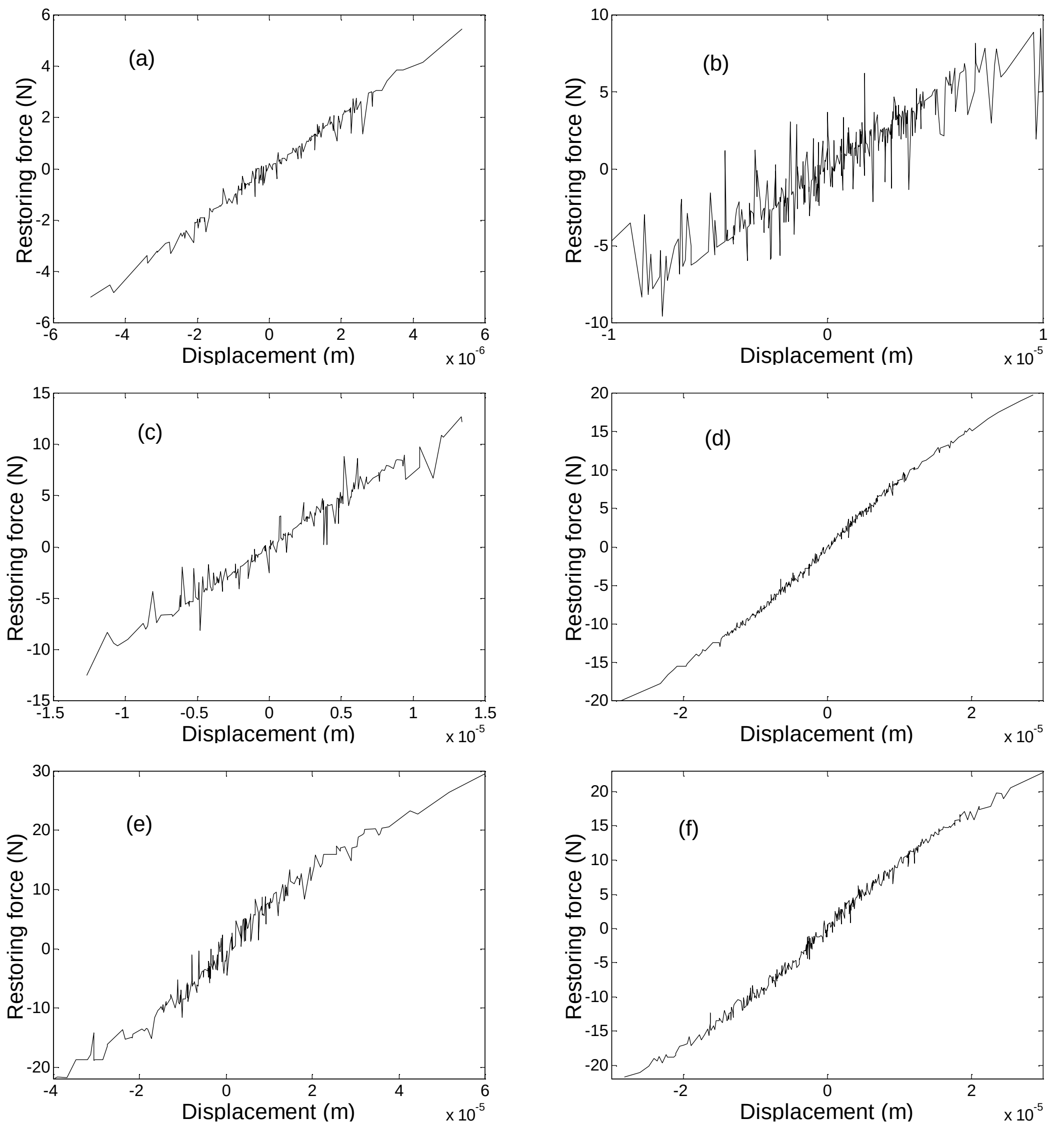

Fig.2: Stiffness curves (a) 0.5 V (b) 1 V (c) 2 V (d) 4 V (e) 8 V (f) 4 V (m_2=5.8 kg)

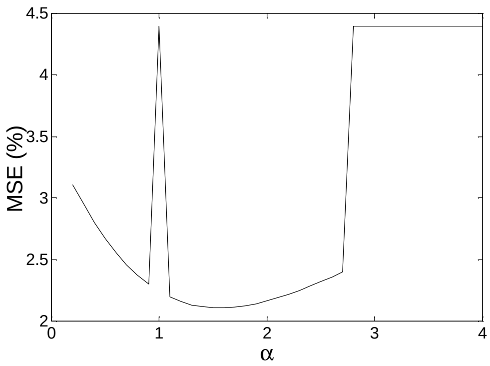

Fig. 3: Evolution of the MSE as a function of the non-linearity exponent

It should be noted that the identification procedure considers the data for all excitation levels in a single step. The parameters of the identified model are given in Table 1. The negative sign corresponding to the non-linear stiffness corroborates the presence of a softening effect. To confirm that the identification has provided reliable results, the measured restoring force is compared to the restoring force computed by the identified model. Fig. 4 represents this comparison for two levels, i.e. 4 V and 8 V levels. It can clearly be seen that the fit is almost perfect for the 4 V level while for the 8 V, some slight distortions are present.

There is a simple explanation to these distortions. Fig. 5 compares the stiffness curves for the 4 V and 8 V levels. This figure underlines that the superimposition between both curves is not perfect while it should. This is probably due to the measurement noise as well as errors introduced by the signal processing.

Table 1: Identification results

	a (N/m)	b (Ns/m)	c (N/m$^{1.5}$)	a
Identified parameters	$1.09\ 10^{6}$	183.44	$-8.52\ 10^{7}$	1.5

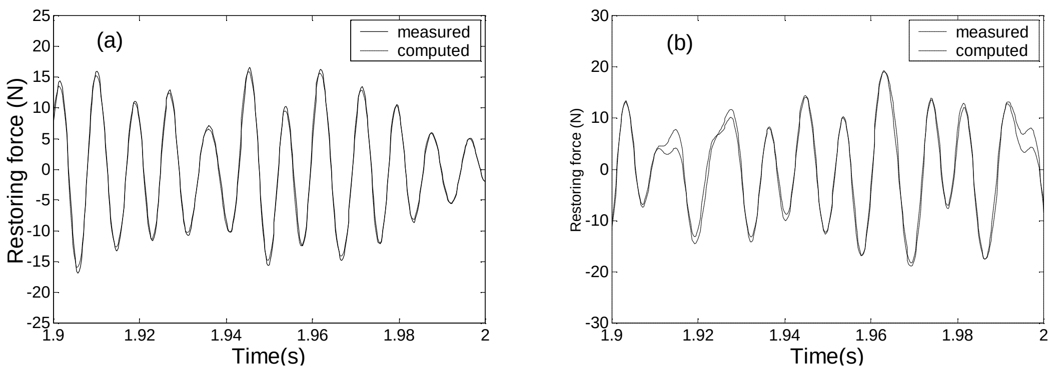

Fig. 4: Comparison between the measured and computed restoring forces (a) 4 V (b) 8 V

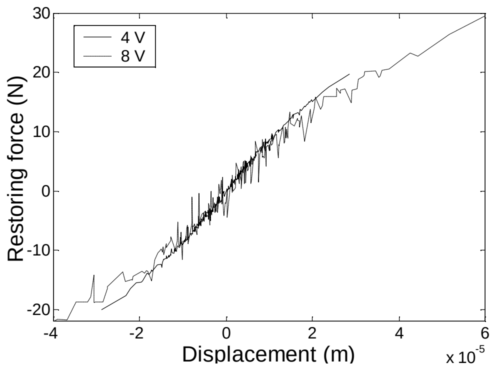

Fig. 5: Comparison between the stiffness curves for the 4 V and 8 V levels

COMPARISON WITH THE CONDITIONED REVERSE PATH (CRP) METHOD

The wire rope isolators were previously studied at the University of Torino using the conditioned reverse path (CRP) method [8]. This method was introduced by Richards and Singh [10] and is now emerging as a useful tool for the identification of multi-degree-of-freedom non-linear systems. The description of this technique is beyond the scope of this paper and the reader is referred to [10] for further information.

Although the CRP and RFS methods are different (frequency domain versus time domain), similar results were obtained:

1. the same non-linearity was identified, i.e. a softening $|u_{12}|^{1.5}\text{sign}(u_{12})$ non-linearity ;
2. on the one hand, the CRP method identified a value equal to $-5.5\ 10^7$ N/m$^{1.5}$ which is slightly higher than the one found with the RFS method ($-8.5\ 10^7$ N/m$^{1.5}$). On the other hand, the frequency of the underlying linear system is slower in the CRP method (108 Hz versus 112 Hz). Thus, it seems that the softening effect due to a lower linear frequency in the CRP method compensates for the stiffening effect due to a less negative non-linear stiffness.

CONCLUSION

The identification of wire rope isolators using the restoring force surface method has been considered in the present paper. Basically, the method is defined for single-degree-of-freedom systems. However, the experimental application has clearly demonstrated that the method is also well suited for non-linearity localised between two degree-of-freedom. The restoring force surface method is appealing by its simplicity and reliability but the need for numerical integration and for filtering may introduce errors in the estimation of signals.

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