Nonlinear Energy Pumping: A New Paradigm for Vibration Isolation

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Abstract—We discuss passive nonlinear energy pumping from a linear (main) mechanical structure to a weakly coupled, local, passive nonlinear energy sink (NES). We show that the NES can be designed to effectively absorb vibrational energy from the main structure in a one-way, irreversible fashion. We demonstrate the occurrence of pumping cascades, where an appropriately designed NES passively extracts energy sequentially from a number of modes of the main (linear) structure, interacting individually with each mode before moving to the next. Experimental results confirm our theoretical findings. The applications of the nonlinear energy pumping phenomenon to the problem of vibration and shock isolation will be discussed.

I. Introduction

The tuned vibration absorber is an effective device for vibration suppression in many mechanical systems including bridges and buildings (see [1] for a survey). However, this passive device is effective over a narrow band of frequencies and is incapable of robustly suppressing broadband transient disturbances. In order to overcome this limitation, the nonlinear energy pumping phenomenon [2-3] is investigated in this paper. In essence, nonlinear energy pumping corresponds to the one-way and irreversible channeling of vibrational energy from a primary system to a passive nonlinear energy sink (NES) where it localizes and diminishes in time due to damping dissipation.

Numerical simulations are first performed and illustrate that the underlying dynamical phenomenon governing energy pumping is a resonance capture on a 1:1 resonance manifold of the system [4]. However, this manifold is not compatible with the NES being initially at rest, and it cannot therefore be directly excited at time t=0 when an impulse is applied only to the primary system. The numerical simulations reveal that transient bridging orbits, compatible with the initial conditions, bring the motion into the domain of attraction of the resonance manifold. It is also shown that resonance capture cascades, where a NES passively extracts energy sequentially from a number of modes of the main structure (i.e., it interacts individually with each mode before moving to the next), may also occur in the case of a multi-degree-of-freedom primary system.

Experimental measurements on a system comprised of a damped linear oscillator coupled to an essentially nonlinear attachment demonstrate that as much as 86% of the energy imparted to the system by the impulsive input force can be absorbed and dissipated at the NES.

II. Basic Mechanisms for Energy Pumping: Numerical Results

We consider a two-degree-of-freedom system composed of a damped linear oscillator which is coupled to an essentially nonlinear (nonlinearizable) attachment

$$\begin{aligned} &\ddot{y} + \varepsilon\lambda_1\dot{y} + \varepsilon\lambda_2(\dot{y}-\dot{v}) + \omega^2 y + \varepsilon(y-v) = 0 \\ &\ddot{v} + \varepsilon\lambda_1\dot{v} + \varepsilon\lambda_2(\dot{v}-\dot{y}) + Cv^3 + \varepsilon(v-y) = 0 \end{aligned} \tag{1}$$

Weak coupling and damping are assured by requiring that ε<<1, and all other variables are assumed to be O(1) quantities. Variables y and v refer to the displacement of the primary system and of the NES, respectively. In Figs. 1-3, we depict the transient responses of the two oscillators for $\lambda_1=0.1$, $\lambda_2=0.7$, $\varepsilon=0.1$, $C=5$, $\omega^2=0.9$ and initial conditions $y(0)=v(0)=\dot{v}(0)$ and $\dot{y}(0)=\{0.5,1.1,1.5\}$, respectively; these initial conditions correspond to impulsive excitation of the primary system. To facilitate the interpretation of the resulting dynamics, the instantaneous percentage of the total energy carried by the NES and the instantaneous frequencies of the two oscillators are represented in addition to the displacement signals.

Looking at Fig. 1, one sees that both oscillators perform damped free oscillations, and no energy pumping occurs since most energy is stored in the directly excited primary system. This numerical simulation indicates that, for fixed system parameter values, energy pumping in the weakly coupled system takes place above a specific value of the initial energy level. By increasing the magnitude of the impulsive excitation to 1.1 (cf. Fig. 2), we see that energy transfer from the primary system to the NES takes place; energy pumping is activated. By further increasing the initial impulse to 1.5 (cf. Fig. 3), the two basic mechanisms for energy pumping can clearly be distinguished. From 0 to 50s, energy is transferred between the primary system and the NES. However, this energy transfer is not irreversible as, around t=25s, energy is released from the NES back to the primary system. Clearly, a nonlinear beating phenomenon occurs. It is very interesting to note that this beating phenomenon plays the role of a transient bridging orbit and enables the motion to reach the domain of attraction of the 1:1 resonance manifold. Indeed, this manifold is not compatible with the NES being at rest, and it cannot therefore be directly excited at time t=0. Once the nonlinear beating phenomenon has triggered the 1:1 resonance capture, an irreversible and complete transfer of energy from the primary system to the NES occurs between 50 and 150s; the envelope of the NES displacement barely decreases with time whereas that of the primary system decreases almost linearly. During this particular regime, the NES is capable of tuning itself in order to engage in a 1:1 resonance interaction with the linear oscillator; this is nicely evidenced by the wavelet transform in Fig. 3. Eventually, as much as 79% of the input energy is dissipated by the NES.

To provide an indication of the complex, multi-frequency transitions that can occur in coupled oscillators with essential nonlinear local attachments, we now increase the number of degrees-of-freedom of the linear subsystem to 2 and examine the system

$$\begin{aligned}&M\ddot{y}_1+\lambda_1\dot{y}_1+Ky_1+K_c(y_1-y_2)=F(t)\\&M\ddot{y}_2+\lambda_1\dot{y}_2+Ky_2+K_c(y_2-y_1)+\varepsilon(y_2-v)=0\\&m\ddot{v}+\lambda_2\dot{v}+\varepsilon(v-y_2)+Cv^3=0\end{aligned}\quad(2)$$

In Fig. 4, we depict the transient response of the NES for $K=2000$, $K_c=1000$, $\varepsilon=300$, $C=1e6$, $M=0.75$, $m=0.25, \lambda_1=0.35, \lambda_2=0.5$. F(t) is a half sine pulse whose amplitude and duration are equal to 160N and 0.02s, respectively. The graph showing the instantaneous percentage of energy carried by the NES shows that vigorous energy transfers take place between the primary system and the NES. In addition, looking at the instantaneous frequency of the NES computed by the wavelet transform, the multi-frequency content of the transient response is evident. The dashed lines represent the natural frequencies of the primary system and indicate that the NES acts as a passive, broadband boundary controller which absorbs, confines and eliminates energy from the two modes of the primary system. A detailed study of resonance capture cascades is performed in references [5,6].

III. Energy Dissipation using a Nonlinear Energy Sink: Experimental Results

The experimental fixture built to examine the nonlinear energy pumping phenomenon is depicted in Fig. 5. It consists of two single-degree-of-freedom oscillators connected by means of a linear coupling stiffness. The left oscillator (the linear subsystem) is grounded by means of a linear spring, whereas the right one (the nonlinear energy sink) is grounded by means of a nonlinear spring with essential cubic nonlinearity. To dissipate the pumped energy, a grounded viscous damper exists in the NES.

The experimental fixture comprises two “cars” (masses) made of aluminum angle stock which are supported on a straight air track, along with various connecting hardware and transducers. Most of the components of the apparatus appear in Fig. 5. Not visible in that figure are the long-stroke electrodynamic shaker used to excite the linear subsystem by means of the rod which contacts the force transducer on the left end of the linear subsystem. The shaker was driven by a signal generator and amplifier through a switching arrangement which allowed a single, brief pulse to be created, resulting in an isolated transient force on the initially stationary linear subsystem. The essential stiffness nonlinearity was realized by a thin wire, with no pretension, clamped at both of its ends and performing transverse vibrations at its center. The geometric nonlinearity of the system considered produces, to the leading order of approximation, a cubic stiffness nonlinearity with coefficient $C=EA/L^3$.

The eigenfrequency of the linear subsystem, and the viscous damping factors of both subsystems, were estimated by performing experimental modal analysis. The mass, linear stiffness and damping coefficient of the linear subsystem were estimated to be M=0.834 kg, K=993N/m, $\varepsilon\lambda$=0.129Nsec/m, corresponding to the eigenfrequency ω_0=35.63 rad/s and the viscous damping ratio ζ=2.3x10^{-3}; hence, the linear oscillator is lightly damped. The mass and damping parameters of the NES, and the linear coupling stiffness connecting the two subsystems, were estimated by performing experimental modal analysis of the linear single-degree-of-freedom oscillator that results when the coupling stiffness is grounded and the nonlinear spring is disconnected. These parameters were estimated to be m=0.393kg, εc=0.454Nsec/m, ε=114N/m. The stiffness characteristic of the nonlinear spring of the NES was identified by performing a series of static tests wherein known displacements were imposed to the sink mass and the corresponding restoring forces created by the wire were measured. A cubic polynomial was fitted to the experimental data, yielding $f(x)=166x+1.36\times10^{7}x^{3}$.

Experimental results for the response of the system to a transient load are depicted in Fig. 6. A force was applied to the linear subsystem with the entire system initially at rest, and was approximately a half-sine pulse of 6.25 ms duration and 55 N peak amplitude; this forcing level is typical of the strong excitation that is required to induce nonlinear energy pumping in the system under consideration. To perform comparisons between measured results and theoretical predictions, an independent numerical simulation of the transient response of system with the experimentally measured force and zero initial conditions was carried out. A comparison between the experimental and theoretical acceleration time series of the two subsystems is shown, from which very good agreement is noted.

The energy input $E_i(t)$ is plotted in Fig. 7(a); both experimental and simulated data are provided, with good agreement between them. In Fig. 7(b) experimental and simulated estimates of the energy measure $E_{NES}(t)$ (energy portion dissipated at the NES) are depicted, again with good agreement. From these results it is determined that, eventually, 86% of the total input energy is absorbed and dissipated at the NES; this estimate was obtained by considering the asymptotic limit of $E_{NES}(t)$ for $t>>1$. This experimental result demonstrates that the NES is an effective mechanism for passively absorbing and dissipating a significant portion of impulsively generated vibrational energy of the structure to which it is attached.

Similar experiments were conducted with a nonlinear energy sink of the same configuration connected to a 2-DOF linear primary structure with natural frequencies of 5.92 and 9.92 Hz and 1% damping in each (linear) mode. The excitation was again a force pulse, applied to the first mass (i.e., the mass not connected to the NES). Figure 8(a) shows the displacement response following this excitation; beating is clearly evident, and may be seen most easily in the NES response prior to time $t=5\,\mathrm{s}$, where the envelope of the signal demonstrates characteristic slow modulation which can be shown to correspond to partially reversible energy exchanges between the primary system and the nonlinear energy sink.

That the NES interacts with both modes of the linear primary structure is shown by computing the instantaneous frequency content of the sink response using the wavelet technique previously described. The result is shown in Fig. 8(b), where several higher frequencies appear before interaction with the first mode becomes dominant. For the form of excitation considered, transient forcing of only the first mass, it has proven difficult to excite the second mode of the linear subsystem to levels sufficient to induce energy pumping from that mode as well as the first. Nonetheless, these results confirm that a single NES with fixed parameter values is capable of interacting with multiple modes, at multiple frequencies.

IV. Conclusions

Even though the systems considered in this work possess rather simple configurations and small numbers of degrees of freedom, they exhibit interesting passive energy transfer properties. Indeed, it is possible to transfer a significant portion of the energy of the primary system to the nonlinear attachment and dissipate it locally. Moreover, when the domain of attraction of the 1:1 resonance manifold is reached, this transfer of energy is irreversible, and there is no transfer back to the primary system.

Experimental measurements on a two degree-of-freedom system comprised of a damped linear oscillator coupled to an essentially nonlinear attachment have also demonstrated that as much as 86% of the energy imparted to the primary system by the input force can be absorbed and dissipated at the nonlinear attachment.

We believe that nonlinear energy sinks will find application in diverse problems in engineering and physics, including vibration and shock isolation of machines and structures, seismic mitigation, packaging, and instability (such as limit cycle oscillation or flutter) suppression.

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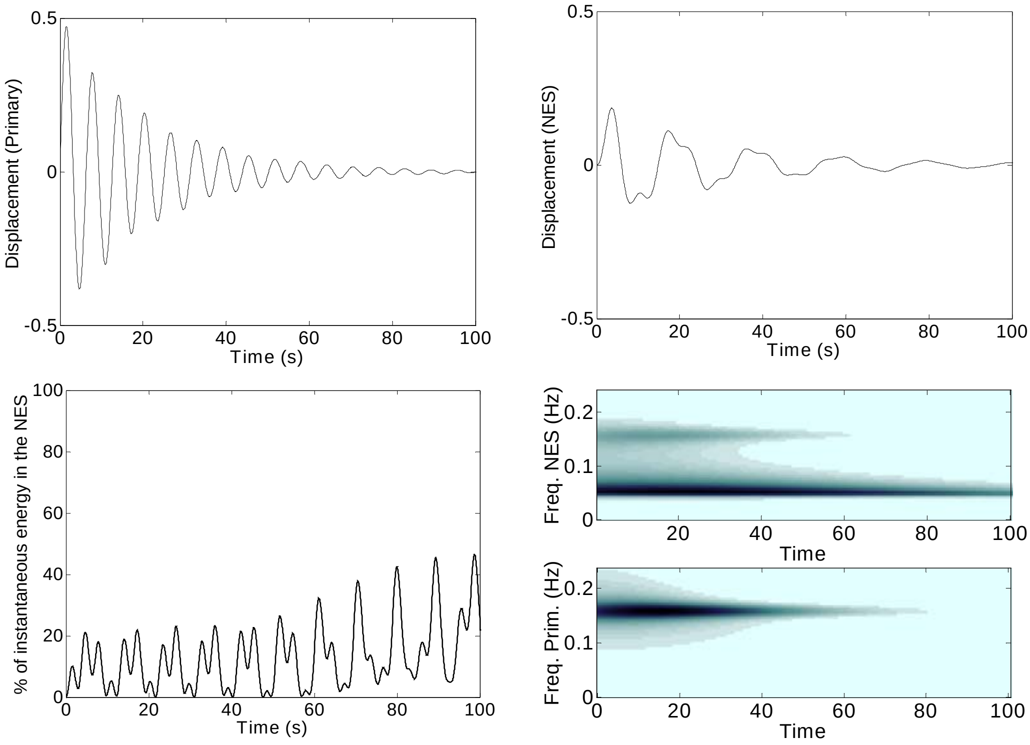

Fig. 1. Numerical simulation of a one degree-of-freedom primary system with an essentially nonlinear attachment ($\dot{y}(0) = 0.5$)

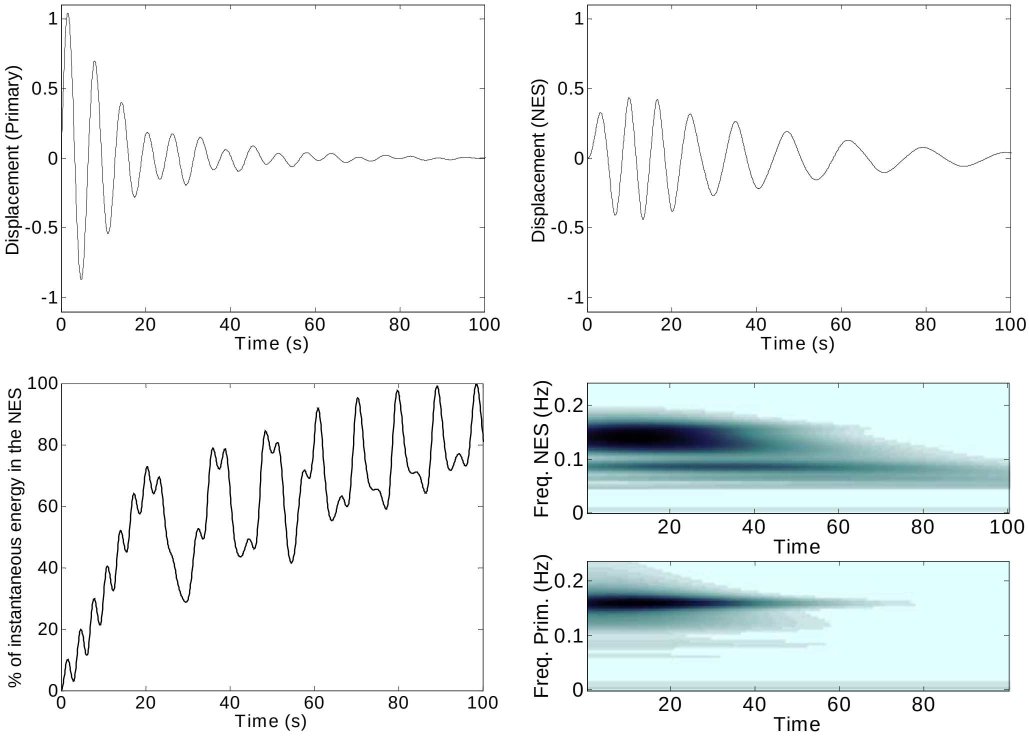

Fig. 2. Numerical simulation of a one degree-of-freedom primary system with an essentially nonlinear attachment ($\dot{y}(0) = 1.1$)

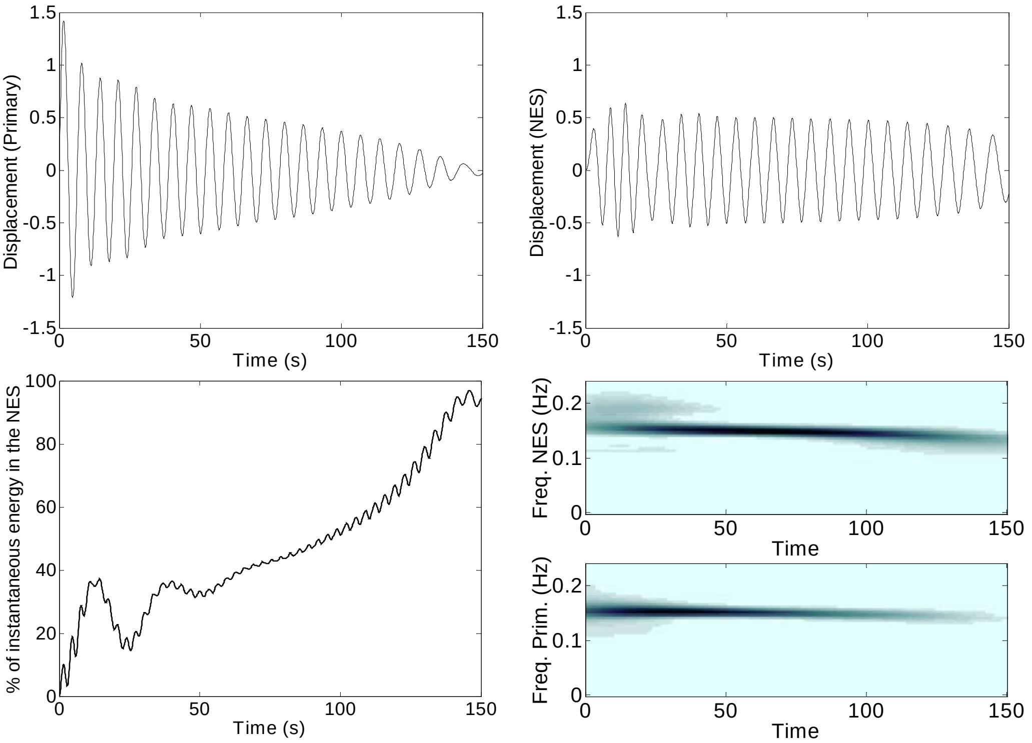

Fig. 3. Numerical simulation of a one degree-of-freedom primary system with an essentially nonlinear attachment ($\dot{y}(0)=1.5$)

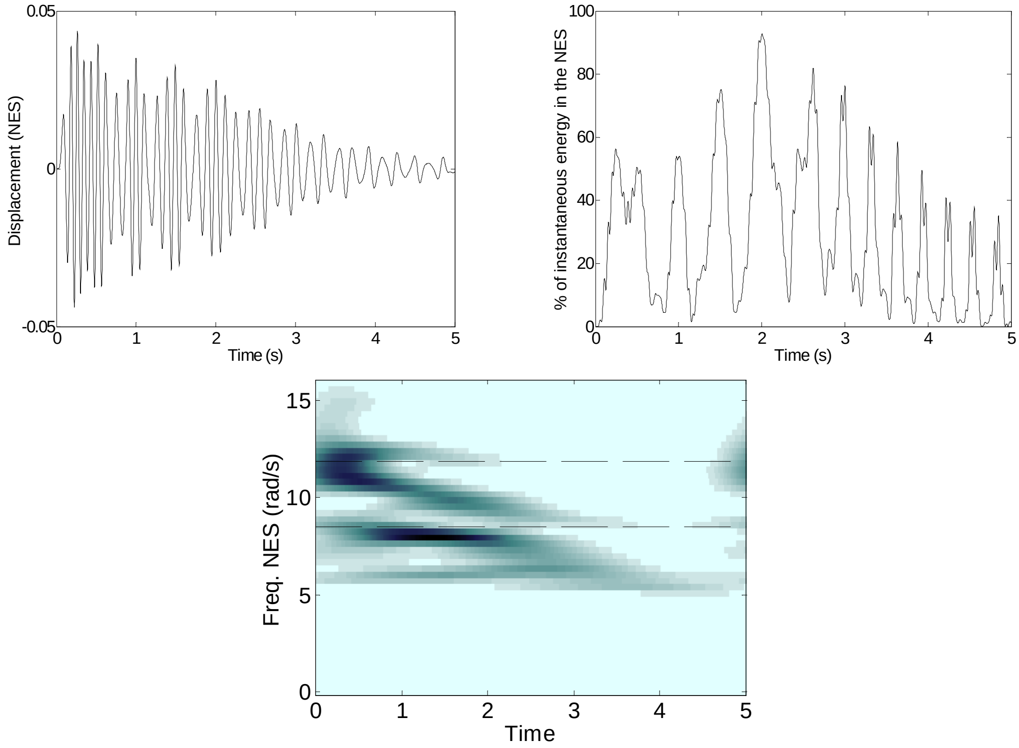

Fig. 4. Numerical simulation of a two degree-of-freedom primary system with an essentially nonlinear attachment

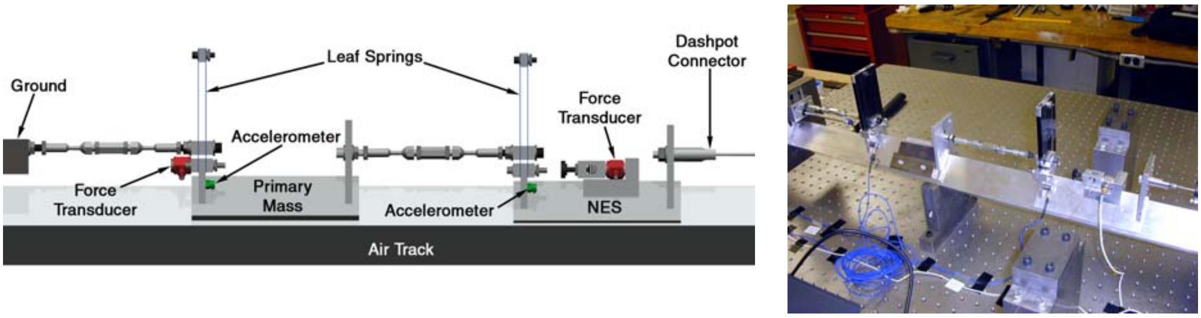

Fig. 5 Experimental fixture for nonlinear energy pumping from a SDOF primary system.

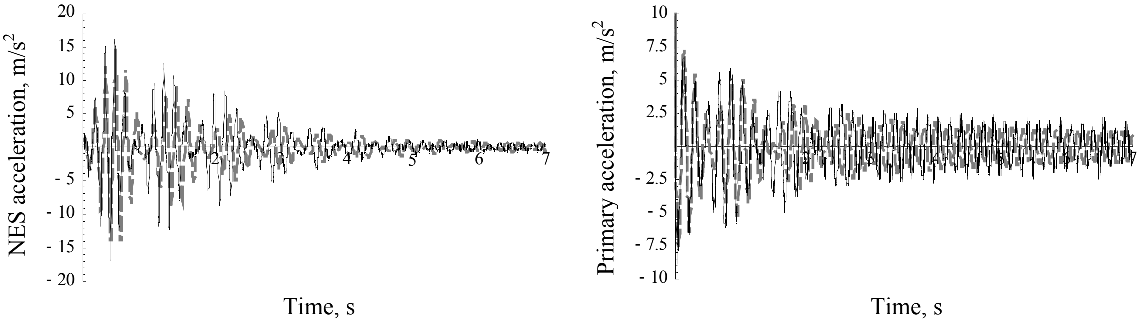

Fig. 6. Experimental ——— and numerical — — — acceleration time series of (a) the linear subsystem (directly excited) and (b) the NES.

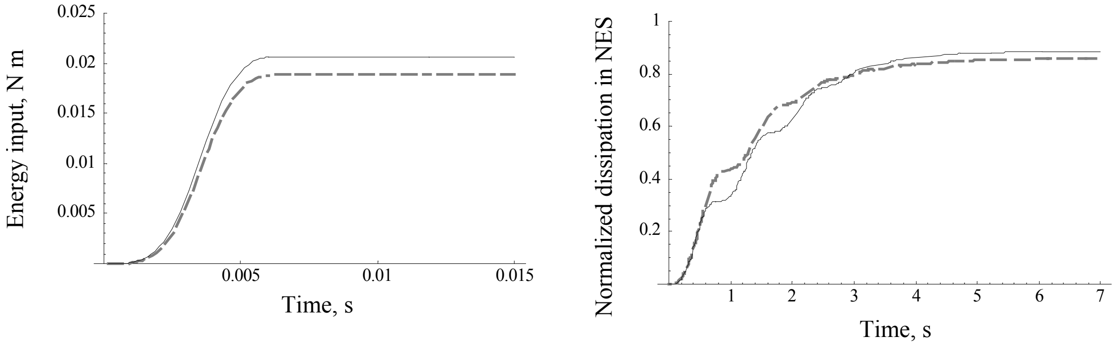

Fig. 7 (a) Instantaneous input energy and (b) energy dissipated at the NES; experimental data ———, numerical data — — —.

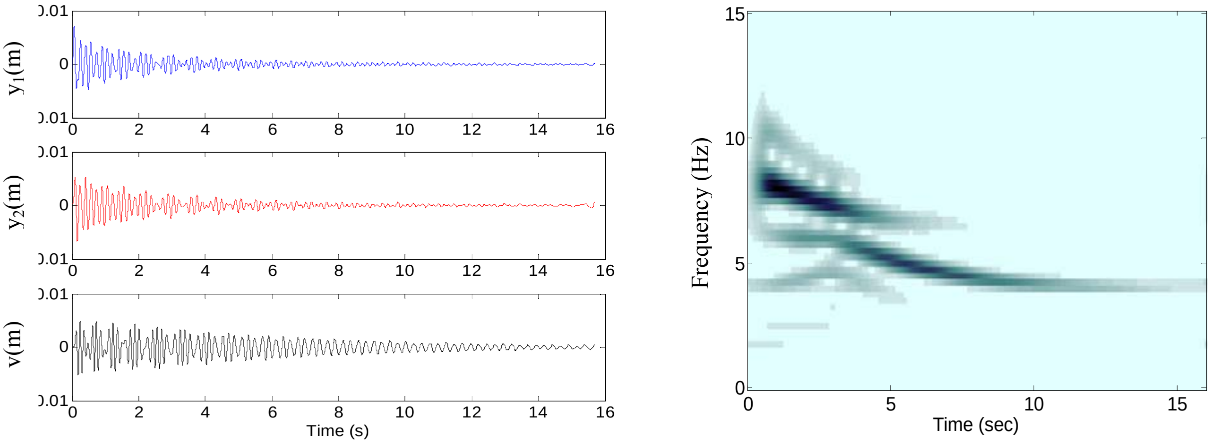

Fig. 8 (a) Displacements of the 2-DOF primary system (y_1, y_2) and the NES mass (v) following transient loading of primary mass 1 (experimental data); (b) Right plot: Instantaneous frequency content of NES response (v)