# **Compressor and Turbine Blade Design by Optimization** Olivier Leonard, André Rothilde and Pierre Duysinx

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# 1. Abstract

Compressor and turbine blade design involves thermodynamical, aerodynamical and mechanical aspects, resulting in an important number of iterations. Inverse methods and optimization procedures help the designer in this long and eventually frustrating process. In this paper an optimization procedure is presented which solves two types of twodimensional or quasi-three-dimensional problems: the inverse problem, for which a target velocity distribution is imposed, and a more global problem, in which the aerodynamic load is maximized.

## 2. Keywords

Turbomachines Design, Shape Optimization, Mathematical Programming Algorithms

### 3. Introduction and optimization procedure

The internal flow fields encountered in turbomachines are three-dimensional, viscous and unsteady. Because of the computational cost of the analysis of such numerical problems it is of common use to consider an axisymmetric flow during the design step. In a classical quasi-three-dimensional approach the full 3D problem is replaced by a series of 2D problems. The first 2D problem is the so-called through-flow problem. It is solved in the meridional plane containing the axis of the machine, and provides the radial evolution of pressure, temperature and velocity triangles, as well as the shape of the axisymmetric stream surfaces. The second type of 2D problem is solved on each of these axisymmetric surfaces, namely the flow around the rotor and stator blade sections is computed. This paper deals with the design of blade sections on these so-called blade-to-blade surfaces.

Several methods are used for this design problem. Most designers still adopt a '*direct' approach*, evaluating the performance of the actual geometry, and modifying it in function of computational results, according to empirical rules or to their own experience. This approach can be very time consuming, and even very unefficient in some cases. It is clear that more powerful design strategies can be obtained by using optimization and/or inverse methods, which allow for the direct generation of geometry design achieving given performance.

*Inverse codes* are based on physical methodologies that deduce the blade shape modifications from the required performance. This results in a computation effort comparable to what is needed for a flow analysis. But this physical link between performance and modification may block the convergence if the target performance is somewhat unphysical. This problem may be solved at the cost of relaxing the shape control [1].

*Optimization procedures* represent an alternative approach to inverse methods for blade design. Optimization procedures are based on the assemblage of 3 basic components: a parameterization system, a flow solver and an optimization algorithm. Here the sensitivity analysis is made with a finite difference approach. The 3 components remain independent from each other and they are quite easy to integrate into a common design chain.

Compared to inverse codes, the optimization approach is more flexible, because constraints of many types (related to mechanical responses or to geometrical properties) can be imposed. On another hand the fact that regular curves can be used for the shape parameterization is also a favorable point for the exploitation of the optimized shape. As sensitivity analysis is performed with a finite difference method, the optimization approach necessitates only minor changes and it can be used as a black box. Moreover as the flow solver is always used in the analysis mode, convergence properties of the design are less affected by existence and convergence difficulties of the inverse problems. However because of the finite difference method, the computation effort is generally heavier in our optimization approach than for an inverse method, which takes full benefits of our knowledge of the physics of the problem.

### 4. Shape parameterization

The parameterization of a blade section is a crucial point. It has to guarantee a reliable representation with as few parameters as possible. Inspired by the work performed in structural shape optimization [2], we considered in this study Bézier curves that have well-known smoothness properties. A blade section is represented in the blade-to-blade plane using two separate Bézier curves, one for the suction side and one for the pressure side. Experience has shown that a number of 9 control-points per side is a good compromise between the quality of the representation and the computation effort. For each control points the design variables to be optimized are the azimutal  $\theta$  positions, while their *x* positions are fixed. Because of the separation into 2 distinct curves, additional continuity conditions have to be imposed at the leading edge to insure C<sup>1</sup> and C<sup>2</sup> continuity (tangent and curvature continuity). The C<sup>1</sup> condition can be explicitly taken into account with a design variable linking. The satisfaction of the curvature continuity requires the introduction of an equality constraint into the design optimization procedure.

#### 5. The preliminary identification problem

The first problem is to find the best approximation of a given curve in our parameterization system, namely to find the position of the control point that render the best fit of a given target profile. Indeed a good initial guess of the aerodynamic optimization can be found among geometry profiles documented in the literature. This preliminary identification problem is attacked as an optimization problem, i.e. the minimization of the distance between the Bézier curves and a certain number of data points describing the known geometry. For this problem the distance is measured with an Euclidean norm which gives more weight to the highest gaps while the smallest are not neglected. We use a Sequential Quadratic Programming algorithm [3] to solve this minimization problem. This algorithm has shown to be so robust that the initial control points may even correspond to a simple flat plate. For most of the examples 50 iterations were necessary to obtain a very close representation of the given geometry.

### 6. The design problems





Figure 1: Velocity along the profiles

Figure 2: Shape of the initial and of the final profiles

In the real design problem one looks for the geometry that optimizes the aerodynamic performance of the blade. The design variables are the Bézier control points, the stagger angle, as well as the direction of the tangent at the leading edge. In this preliminary work a fast Q3D flow solver has been used. It is based on a potential model for incompressible flows and a Martensen approach [4] based on vortices distributed on the blade contour. Sources have been added in the blade channel to mimic the effects of compressibility, radial shift and stream tube thickness variation.

The present work has allowed to test an optimization package, CONLIN, which was initially developed for structures. CONLIN is actually a set of optimization algorithms based on convex linearization techniques [5] and dual methods of mathematical programming [6]. The objective function and the inequality constraints are approximated with the mixed variable linearization scheme while the equality constraints are treated as linear, resulting in a solution strategy based on a primal-dual formulation.

The first problem which is addressed is known as the inverse problem. The designer prescribes a target velocity distribution according to aerodynamic criteria. The objective function is the 4-norm distance between the actual velocity distribution and the target. From our numerical experience, the 4-norm is sufficient to give enough weight to the biggest gaps. This objective function is highly non-linear in terms of the design parameters. Furthermore the function is generally non-monotonous and we observe an oscillatory convergence history. To overcome this problem a move-limits strategy is used. Severely unrealistic shapes are avoided during the iteration history, which could provide 'crazy' velocity distributions and lead to the divergence of the procedure. Figure 2 shows the results for a turbine blade optimization. A target velocity distribution (drawn with dotted lines) corresponding to a known geometry is imposed (see Figure 1). The dashed curve is the initial velocity distribution, and the solid curve is the result after about 40 iterations, which means a total of about 800 flow evaluations included the finite difference runs. The convergence is excellent, even though the optimized geometry (solid line) is not exactly the geometry which provided the target velocity (dotted line), as shown in figure 2, which shows that the solution is not unique.

The second problem to be investigated consists in maximizing the aerodynamic load imposed to the blade. When formulated in terms of a global objective, the optimization algorithm has much more freedom for modifying the blade geometry. However in order to get realistic designs one must add several design constraints, such as a bound on the maximum velocity peak, a restriction on the sign (negative) of the curvature of the suction side velocity distribution, a bound on the thickness of the blade profile, a maximum value for the diffusion factor (for compressor blades), a

maximum value for the peak velocity divided by the downstream velocity (for turbine blades), a minimum distance from the leading edge before separation of the boundary layers... Figures 3 and 4 illustrate the problem for a compressor blade, resulting in an increase of 13 % of the loading (the dashed curves correspond to the initial blade while the solid lines are related to the optimized blade).





Figure 3: Velocity along the profiles



#### 7. Future developments

Presently the parameterization strategy is reconsidered as B-splines are tested. With B-splines the whole blade shape is represented with one single curve. We are also investigating a multi-point optimization strategy, i.e. minimizing the blade profile losses for a set of incidences instead of only one flow configuration corresponding to the design point. In the near future we will link CONLIN to a fast Q3D Euler flow solver so that compressibility effects will be more correctly simulated.

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#### 9. References

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