A study of histograms of wavelet coefficients of the Mars topography to determine its scaling properties

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Introduction and Short Review on the Scaling Properties of Mars Topography

Previous works about the scaling properties of Mars topography show two distinct scaling regimes. The scale break and the scaling exponents H vary from one paper to another:

Method	**Small scales**	**Large scales**
Power spectral density [1]	$H \approx 1.2$ (< 10 km)	$H \approx 0.2 - 0.5$
Variance of a wavelet transform [4]	$H \approx 1.25$ (< 24 km)	$H \approx 0.5$
Statistical moments [5]	$H \approx 0.76$ (< 10 km)	$H \approx 0.5$
Wavelet leaders method [2]	$H \approx 1.1$ (< 15 km)	$H \approx 0.7$

Let us notice that the three first methods are based on along-track measurements, which implies that the 2D part of the topography field has not been taken into account. The wavelet leaders method has allowed to perform the first complete 2D study of the surface roughness of Mars. The presence of specifics regions such as craters can disrupt this last method and thus the interpretation of the scalings of some signals can be more difficult.
In this work, a recent method based on the use of histograms of wavelet coefficients is used to avoid these scaling problems [3]. This poster presents the first results for the small scales. We use the MOLA data, using the 128 pix/deg map (`pds-geosciences.wustl.egu`).

The Wavelet Profile Method (WPM)

Let us denote by c_λ the wavelet coefficient of a function $f : \mathbb{R}^n \mapsto \mathbb{R}$, associated to the dyadic cube $\lambda = [k2^{-j}, (k+1)2^{-j})^n$ ($j \in \mathbb{N}$ and $k \in \{0, \ldots, 2^j - 1\}^n$).
One has that $c_{\lambda_j(x)} \sim 2^{-H_f(x)j}$ as $j \to +\infty$, where $\lambda_j(x)$ is the unique dyadic cube of size 2^{-j} that contains x and $H_f(x)$ is the scaling exponent of f at the point x. Let us set

$$\nu_f(h) = \lim_{\epsilon \to 0^+} \limsup_{j \to +\infty} \frac{\log \# E_j(1, h+\epsilon)(f)}{\log 2^j}, \tag{1}$$

where

$$E_j(C,h)(f) = \{\lambda \in \Lambda_j : |c_\lambda| \geq C 2^{-hj}\}$$

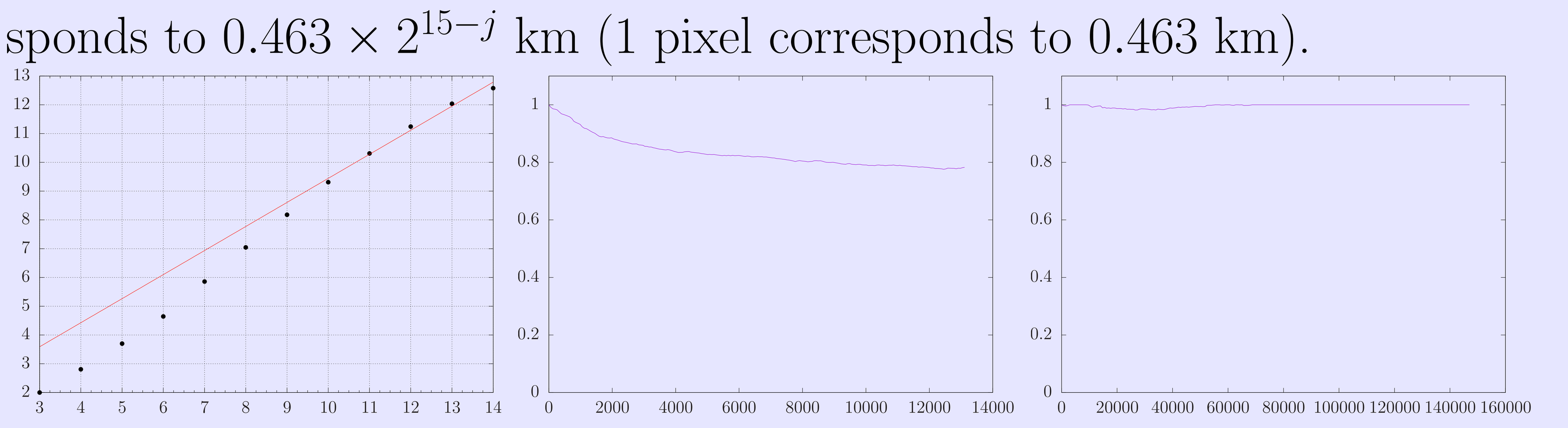
and Λ_j is the set of dyadic cube of size 2^{-j}. If a signal f has the same scaling exponent H at each point, one has that $\nu_f(h) = -\infty$ if $h < H$ and $\nu_f(h) = n$ if $h \geq H$. More generally, the smallest h such that $\nu_f(h) = n$ represents the *dominant scaling exponent* of f, noted H_f. Moreover, theoretically, the constant 1 in Equality (1) is arbitrary but in practice, it is not the case because we have only a finite number of wavelet coefficients. Here is a algorithm to approximate H_f [3]:

- For a fixed $h > 0$, we first do a linear regression, noted $\nu_f^C(h)$, on the function
$$j \mapsto \log_2 \# E_j(C,h)(f). \tag{2}$$
- Secondly, we construct the function $C \mapsto \nu_f^C(h)$. If h is larger than H_f, then it should exist an interval I for which the values $\nu_f^C(h)$ with $C \in I$ are close to n. The typical length of I is the median of $(|c_\lambda|/2^{-hj})_{j,k}$ (see [3]). The smaller h verifying this property is an approximation of H_f.

In this poster, we do a linear regression of the function (2) on the small scales j (< 15 km).

1D Analysis

These figures represent the classical steps of the WPM. The scale j corresponds to $0.463 \times 2^{15-j}$ km (1 pixel corresponds to 0.463 km).



$j \mapsto \log_2 \# E_j(6000, 0.8)(f)$ $C \mapsto \nu_f^C(0.8)$ $C \mapsto \nu_f^C(1.06)$

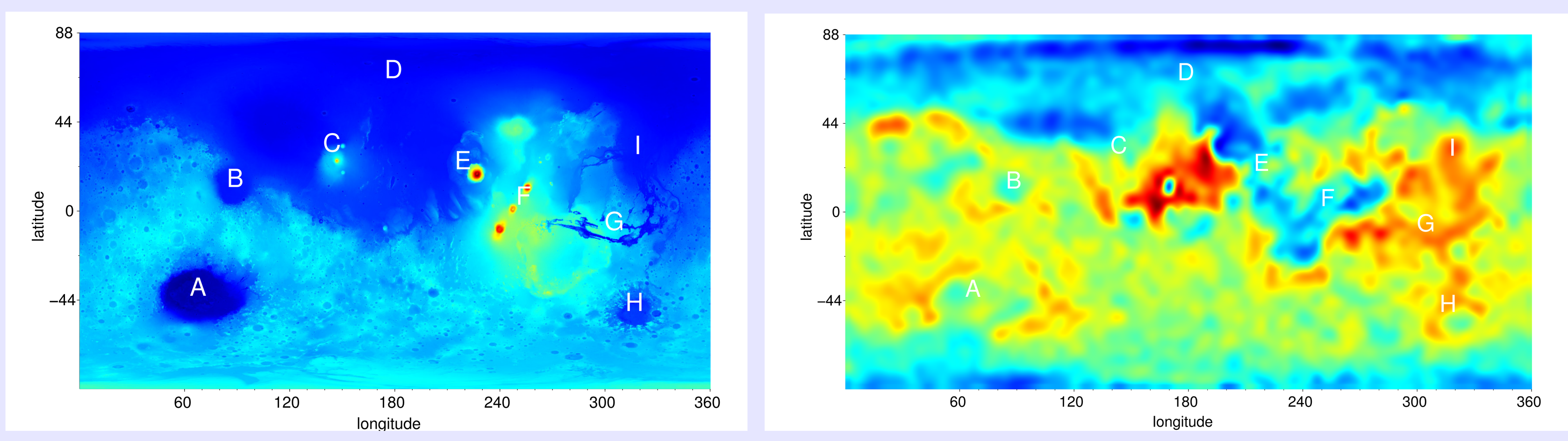
Results obtained for the longitudes (l) and latitudes (L) :

	mean (l)	mean (L)
H for the small scales	1.22 ± 0.14	1.07 ± 0.26

The difference between latitude and longitude may indicate a slight anisotropy of the surface roughness of Mars.

2D Analysis

The map is gridded into tiles of 1024×1024 pixels, corresponding to windows of $8° \times 8°$ on Mars. The spatial distribution of the scaling exponents at small scales shows that the most distinctive features of Mars can be recovered.



topographic map spatial distribution of H (WPM with wavelet leaders)

Regions of interest : Hellas Planitia (A), Isidis Planitia (B), Elysium Mons (C), Vestitas Borealis-Northern plains (D), Olympus Mons (E), Tharsis (F), Valles Marineris (G), Argyre Planitia (H) and Acidalia Planitia (I).

Conclusion

This works shows that the Wavelet Profile Method is well-suited for studying the scaling properties of planetary surfaces. We still have to refine the method, study the multifractality and the large scales, and compare the results with results of some classical methods.

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