# The effect of heterogeneity on the agreement between analytical and numerical solutions describing the recession flow in unconfined hillslope aquifers

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## Abstract

Analytical approximations or linearized versions of the Boussinesq equation, describing recession flow in unconfined hillslope aquifers have been applied extensively in the past. One of the major assumptions in the conceptual formulation is isotropic homogeneity of the aquifer. Notwithstanding the effect of aquifer heterogeneity on recession flow has been pointed out [1], only recent studies are increasingly focusing on the quantification of this effect [2,3]. In the present research, the relevance of heterogeneity is being studied by analyzing the agreement between analytical approximations and the numerical solution of the receding flow in a layered unconfined aquifer with different domain configurations (horizontal, inclined and concave impervious bedrock). For the application of the analytical approximations use is made of the equivalent hydraulic conductivity concept.

## 1. Introduction

Analytical approximations or linearized versions of the Boussinesq diffusion equation are frequently used to predict the recession flow in hillslope aquifers. The isotropic homogeneous medium hypothesis is a basic assumption in the analytical approximations. Although offering practical advantages, the assumption is a strong simplification of reality and therefore only valid for well defined conditions [1,6]. Analysis of the impact of heterogeneity of the hydraulic conductivity on the aquifer response is a relevant research topic [1,2], and new theories for modelling groundwater flow in heterogeneous media are becoming increasingly available [4].

There are probably as many types of heterogeneous configurations as there are geological environments [5]. From the many broad classes of heterogeneity that exist, in the research presented herein, as depicted in Fig. 2, heterogeneity of the unconfined aquifer is made up by the succession of horizontal isotropic homogeneous layers, each having a different hydraulic conductivity The horizontal flow in such a composite aquifer can be calculated as the sum of the flows in each layer using the flux equation, or as the flow in an anisotropic homogeneous aquifer with thickness the sum of the thickness of the individual layers and hydraulic conductivity the equivalent hydraulic conductivity. Assuming $\Delta h$ as the head loss over a horizontal distance $\Delta x$, the discharge $q$ through a unit thickness of the system is the sum of the discharges through the layers. The specific discharge $v = q/D$ is therefore given by:

$$v = \sum_{i=1}^{n} \frac{K_i D_i}{D} \frac{\Delta h}{\Delta x} = K_x \frac{\Delta h}{\Delta x} \tag{1}$$

where $K_x$ is an equivalent horizontal hydraulic conductivity. Simplification yields:

$$K_x = \sum_{i=1}^{n} \frac{K_i D_i}{D} \tag{2}$$

Equation 2 provides the $K_x$ value for a single homogeneous but anisotropic formation, hydraulically equivalent to the layered system of homogeneous, isotropic geologic formations (see Fig. 2). Complete equivalency between the real heterogeneous medium and the fictitious homogeneous one is however impossible.

In the present research, the effect of heterogeneity on the recession flow in an unconfined hillslope aquifer is measured by comparing the predicted flows using at one hand an analytical approximation and the other hand a numerical solution of the general flow equation. The comparative analysis is conducted for different configurations of a layered unconfined hillslope aquifer. In the analytical solutions the equivalent hydraulic conductivity is used, whereas in the numerical analysis, using MODFLOW [6], the hydraulic conductivity of each distinct layer is considered. The results obtained with MODFLOW are the reference, considered herein as a more exact solution of the recession flow, to which the results of the analytical approximations are compared.

## 2. Materials

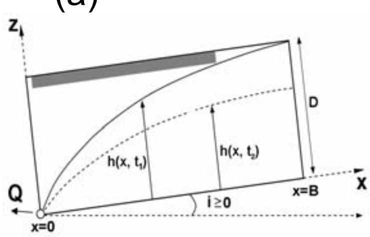
2D numerical models with heterogeneous conditions were constructed in MODFLOW, to simulate recession for different conditions. The validity of those models was tested by comparing for identical situations recession curves or outflow rates from the model output with the result of the analytical solution considering equivalent hydraulic conductivity values (see Eq. 2). The following three different cases are analyzed.

<u>*Case 1*</u>: Horizontal aquifer floor ($i$=0) (Fig. 1a). Two approximations were evaluated, Brutsaert's horizontal floor solution (Eq. 4), and Boussinesq (Eq. 5). Where, $q$ and $q_b$ [$L^2$ $T^{-1}$] are the flowrates per unit width, $K$ [$L$ $T^{-1}$] the hydraulic conductivity, $f$ [-] the drainable porosity (equivalent to the specific yield in a unconfined aquifer), $D$ [L] the initial

hydraulic head at distance *x=B*, *p* a constant introduced to compensate for the approximation (calibration factor), *B* [L] the width of the aquifer and *α'*, generally known as the recession coefficient.

_Case 2:_ Sloping aquifer floor (see Fig. 1a). Here Brutsaert's solution (Eq. 3) was used, with $K'=KpD\cos(i/f)$, $U=K\sin(i/f)$, $a=-U/(2K')$ and $z_n=(2n-1)\,\pi/2$ for nearly horizontal flow or thick aquifers, and $z_n=n\pi$ for steep slopes or shallow aquifers. From a comparative analysis between the analytical and numerical solution of the receding flow of an unconfined homogeneous aquifer [7], a slope angle *i* equal to 1% was selected because of the better match in the recession curves.

(a)



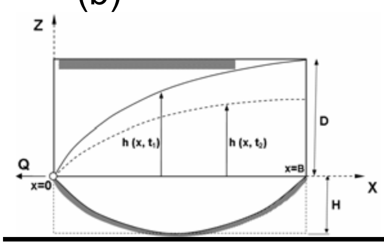

$$q=\frac{2DK'f}{B^3}\sum_{n=1,2,...}^{\infty}\frac{z_n^2\left[\left(2e^{-aB}\cos z_n\right)-1\right]\exp\left[-K'\left(z_n^2/B^2+U^2/4K'^2-K'\right)t\right]}{z_n^2/B^2+U^2/4K'^2+U/2K'B} \quad (3)$$

$$q=\frac{2KpD^2}{B}\sum_{n=1,2,...}^{\infty}\exp\left[-\frac{(2n-1)^2\pi^2KpD}{4B^2f}t\right] \quad (4)$$

$$q_b=\frac{0.862K(D/B)}{(1+\alpha' t)^2} \quad (5)$$

$$\alpha'=\frac{1.115KD}{fB^2} \quad (6)$$

(b)




$$q=\frac{\pi KH}{2B}Ce^{-\alpha t} \quad (7)$$

$$\alpha=\frac{\pi^2KH}{4fB^2} \quad (8)$$

Fig. 1: a) Conceptual drawing of the cross-section of a hillslope aquifer with horizontal/inclined slope, and b) Sketch of Boussinesq's concave conceptual model

_Case 3:_ Concave aquifer floor depth (see Fig. 1b). Equation 7 is applied for this conceptual scheme, where *H* [L] is the depth under the outlet and *C* [-] an arbitrary constant equivalent to *p.* From the comparative analysis between the analytical and numerical solution [7], H=3.2 m was selected from the possible H values because for this value the best agreement was reached between the predicted recession curves.

Soil hydraulic properties, *K* (m $s^{-1}$) and *f* (-), of five representative materials, clay (M1), silt (M2), fine sand (M3), coarse sand (M4), and gravel (M5) were taken from Smith and Wheatcraft [8]. In each scenario of aquifer composition the hydraulic properties, *M*(*K,f*), were kept uniform and constant, and this over the entire flow domain of each distinct layer.

Recession curves of 300 days were computed and compared to the analytical approximations. The outlet of the aquifer was mimicked by a cell with a head of 0.005

m. The initial condition was obtained with a steady-state simulation considering a constant recharge rate, all along the upper boundary of the flow domain. The recession curves were then computed with a transient-state simulation starting from the above described initial condition with a zero recharge rate. The simulated discharge is compared to the discharge calculated with the Boussinesq analytical approximation, from which deviations (Eq. 9) are calculated and expressed in percent:

$$Deviation\ (\%) = 100 * \frac{Q_{sim} - Q_{Bouss}}{Q_{Bouss}} \quad (9)$$

The adopted geometry and discretization (for details see Rocha et al. [7]), are: length of the flow domain (orthogonal to the draining stream) B=400 m, aquifer thickness D=8 m; length parallel to the stream 1 m, cell width 2 m, and four uniform layers with equal thickness (see Fig. 2). A time step of 1 day was used in all the simulations.

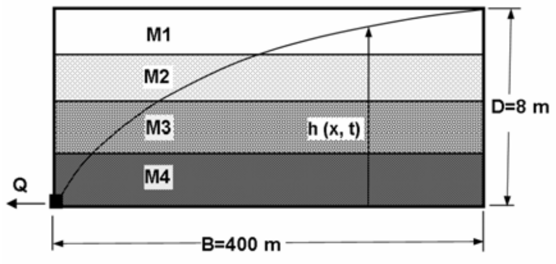



Fig. 2: Schematic presentation of the geometry of the numerical model, consisting of four layers with distinct hydraulic properties *M*; the black square (■) is the outlet with imposed potential

## 3. Results and discussion

The results of the comparative analysis of the recession outflows derived with the analytical approximations with equivalent hydraulic properties and the numerical model are presented in the following and expressed in percent deviation (Eq. 9).

<u>*Case 1*</u> *(Horizontal aquifer floor):* From the possible different layered configurations with five different hydraulic properties, as depicted in the scheme in Fig. 2, 38 possible combinations were analyzed with the constraint of maintaining unconfined conditions within the domain. From those, 12 representative combinations were selected according to the dominant materials present, and three broad heterogeneous groups were considered under the following criteria: *Fine materials*, for clay, silt and fine sand (M1, M2, M3); *intermediate*, for fine and coarse sand (M3, M4), and *coarse* for coarse sand and gravel (M4, M5) (see Fig. 3).

In the left column of Fig. 3 are depicted the flowrate recession curves for isotropic homogeneous aquifer conditions, corresponding to the hydraulic properties M1 (clay), M3 (fine sand) and M4 (coarse sand) versus the recession curves of layered aquifer combinations, using MODFLOW. In the middle and right column of Fig. 3 are shown the percent deviations between the simulated flow and flow derived with Eq. 4 (right column) and Eq. 5 (middle column), for an isotropic homogeneous aquifer and the considered layered aquifer combinations.

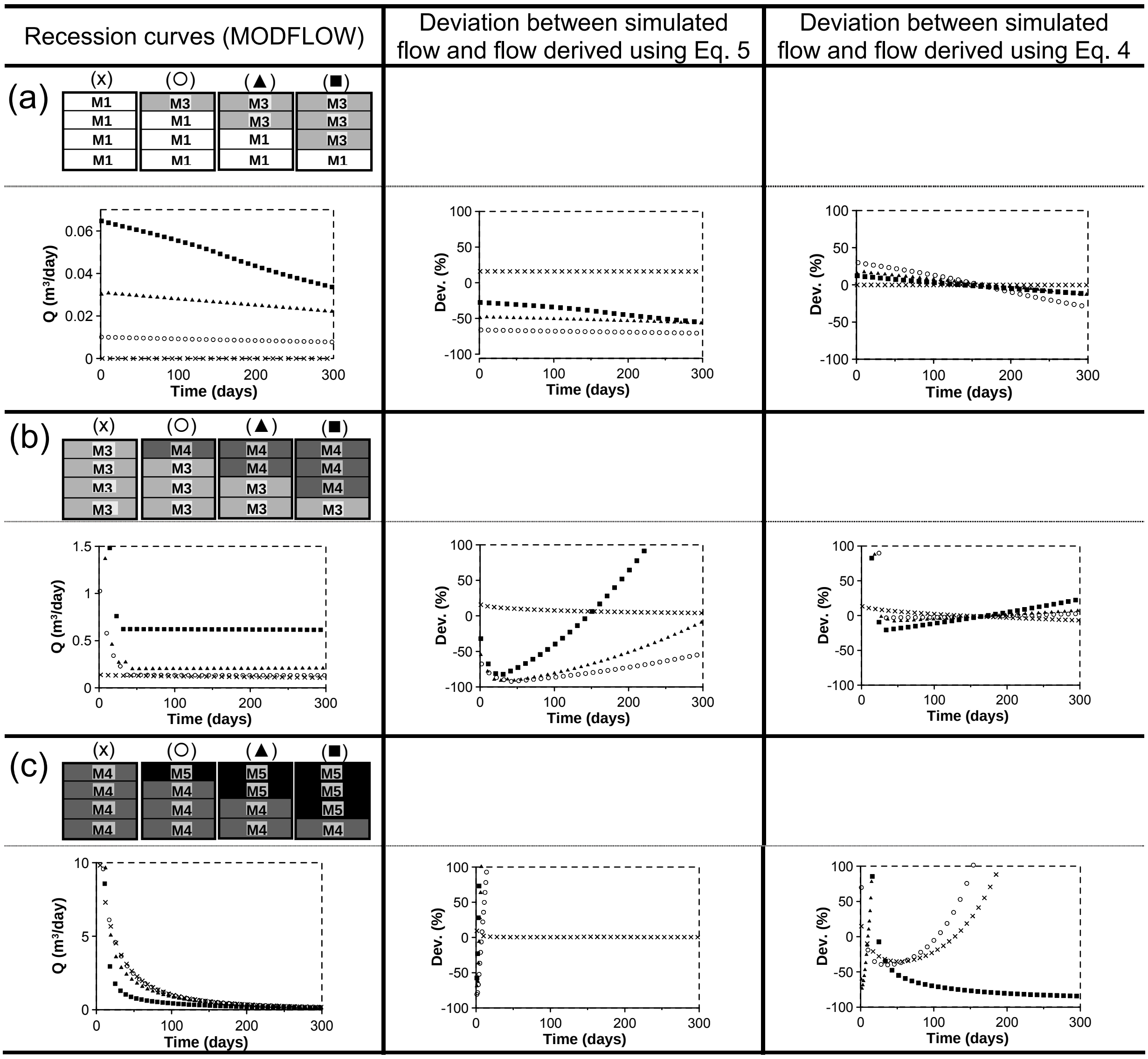



Fig. 3: Recession curves (from MODFLOW) for homogeneous and different heterogeneous combinations, and deviations between those and recessions from Boussinesq's quadratic (Eq. 5) and Brutsaert's exponential (Eq. 4) solution using equivalent hydraulic properties. a) fine, b) intermediate, and c) coarse materials

When the distinctive layers consist of *fine* materials (Fig. 3a) recession curves do not differ much in shape from the homogeneous case (M1), except when the dominant material is fine sand (M3). It is observed that when fine sand becomes dominant that the initial discharges are higher. Unfortunately the results did not allow to asses the time when the influence of fine sand layers ends. As can be seen in Fig. 3a the agreement between the simulated flow and flow derived with Eq. 5 (Boussinesq) is strongly affected when equivalent parameters are used, resulting in deviations in the order of 50%. The effect on the agreement when using Eq. 4 (Brutsaert) is also affected but deviations are less than 30%, however it is important to point out that this particular analytical approximation has the advantage of the calibration factor *p*. Figure 3b shows that in *intermediate* materials the effect of the heterogeneity is clearly observed. The influence of coarse sand can be assessed by the change of slope in the recession. Once again the agreement of simulated flow and flow derived with Eqs. 4 and 5 is clearly affected. Recession curves from heterogeneous domains (Fig. 3c), *coarse* materials, do not differ much from the flow in homogeneous conditions (coarse sand only), being difficult to asses (as in the case of *fine* materials) the time when the influence of the coarser material ends. However, the agreement of simulated flow and flow derived with Eqs. 4 and 5 is visibly affected. These results show that even though the recession curves may not differ much, good agreement found for homogeneous conditions is no longer found when equivalent hydraulic properties are used.

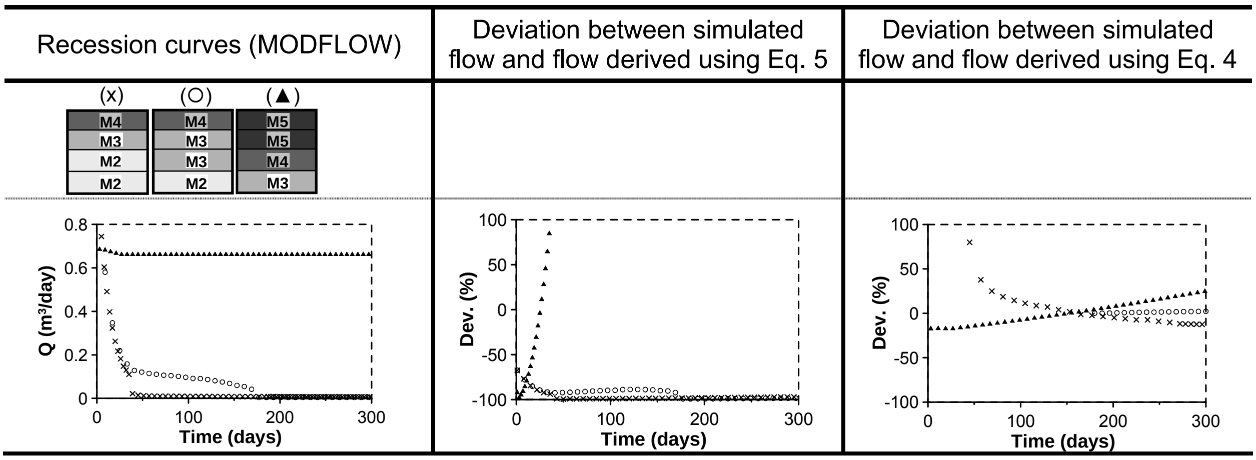



Fig. 4: Recession curves (from MODFLOW) for different heterogeneous combinations (three different materials) and deviations between those and the recessions curves obtained with Boussinesq's quadratic (Eq. 5) and Brutsaert's exponential (Eq. 4) solution, using equivalent hydraulic properties

Figure 4 presents the recession curves and the agreement between the simulated flow and the flow derived with Eqs. 4 and 5 for an aquifer composed of 4 layers of which 3 layers have different hydraulic properties. The observed changes in the recession slopes are also reflected in the curves depicting the deviation between the simulated flow and flow derived with the analytical approximations. Furthermore, it is observed that only the exponential approximation (Eq. 4) shows moderate agreement for the recession periods where the flow is believed to be influenced mainly by fine materials, a finding consistent with the results presented in [7].

<u>Case 2</u> *(Sloping aquifer floor):* From the comparative analysis found in [7] it is observed that recession curves obtained with Eq. 3 deviate less from the numerical ones mainly

for fine materials. Therefore in the analysis only heterogeneous combinations, with hydraulic properties M1 and M3, corresponding to clay and fine sand, were considered. Results of the agreement of simulated flow and flow derived with Eq. 3 do not differ much from the ones obtained for homogeneous conditions. However, fine sand (M3) being dominant present, deviation values increase.

*<u>Case 3</u> (Concave aquifer floor):* Also from [7] it is found that Eq. 7 is mainly valid for fine materials. Given previous only heterogeneous combinations with hydraulic properties M1 and M3 were considered. Deviation results when compared to the homogeneous case are different and particularly important, up to 50%, when fine sand is dominant.

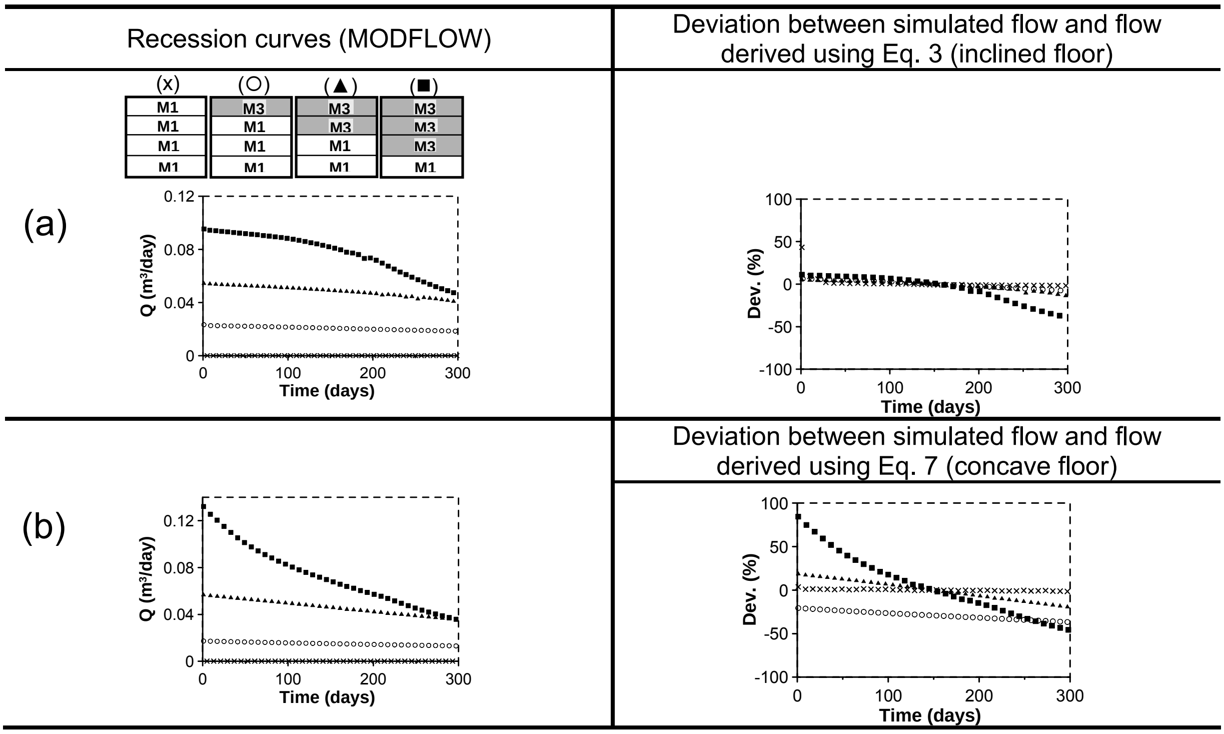



Fig. 5: Recession curves (MODFLOW) for homogeneous and different heterogeneous combinations (mainly fine materials), and deviations between those and recessions derived using Brutsaert (Eq. 3) and Boussinesq (Eq. 7) solution assuming equivalent hydraulic properties. a) inclined floor, 1% slope, and b) concave floor with H=3.2 m

## 4. Conclusions

In the present research the impact of heterogeneity was assessed on the agreement between the analytical approximation and numerical solution of the flow recession in an unconfined hillslope aquifer with different domain configurations. The equivalent hydraulic conductivity concept was used in the analytical approximations to mimic the effect of the layered configuration of the aquifer, and MODFLOW, a numerical tool, was used as reference method. Results showed that in every case analyzed, the agreement observed under homogeneous conditions is negatively affected when taking into account heterogeneity using equivalent hydraulic properties in the analytical approximations. This is particularly true when the quadratic analytical approximation (Eq. 5) is applied. However, it is observed that differences between the analytical approximation and MODFLOW results decrease when the distinct layers in the aquifer consist of fine materials and when the exponential type equations, such as Eq. 3 and Eq. 4 are used. The latter is consistent with results found in [7], where it is stated that exponential type analytical approximations are mainly valid for fine materials. Results also suggest that the use of equivalent parameters in analytical approximations should be carefully applied and used, because even though recession curves might not be that different under heterogeneous conditions, employing slightly different hydraulic property values (particularly *K)* may result in large deviations when compared to numerical model results. Sensitivity to the hydraulic properties is also reported in [7].

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