Branch-and-price algorithms for a VRP with time windows and variable departure times

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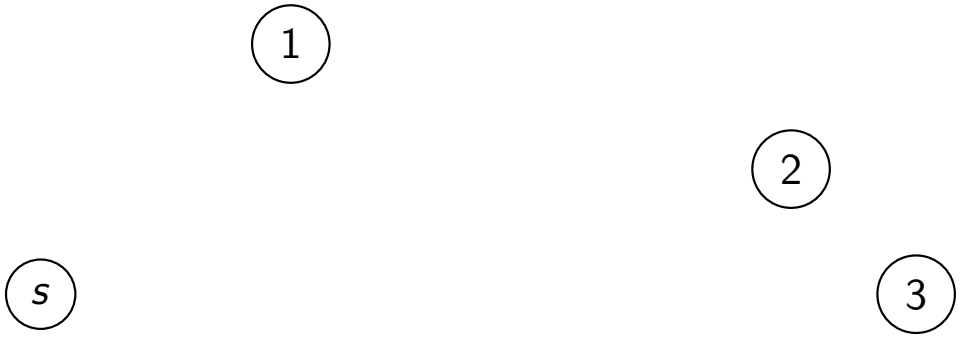
The problem

- Variant of the VRP with time windows
  - Objective is the total route duration.
  - Route start time is a decision variable.
  - Maximum duration defined for each route.
- Solution method: Branch-and-Price (BP)
  - The pricing problem is the Elementary Shortest Path Problem with Resource Constraints (ESPPRC).
- Basic algorithm for the ESPPRC: Feillet's label extension procedure.
- We aim to rigorously compare several advanced algorithms for the ESPPRC within a BP framework.

Standard labeling algorithm

- Each label L_i represents a partial path ending at node i.
- L_i keeps track of the consumption of resources along the partial path.



$$L_s = (C_s = 0, \mathbf{R}_s = \mathbf{0}, \mathbf{E}_s = \mathbf{0})$$

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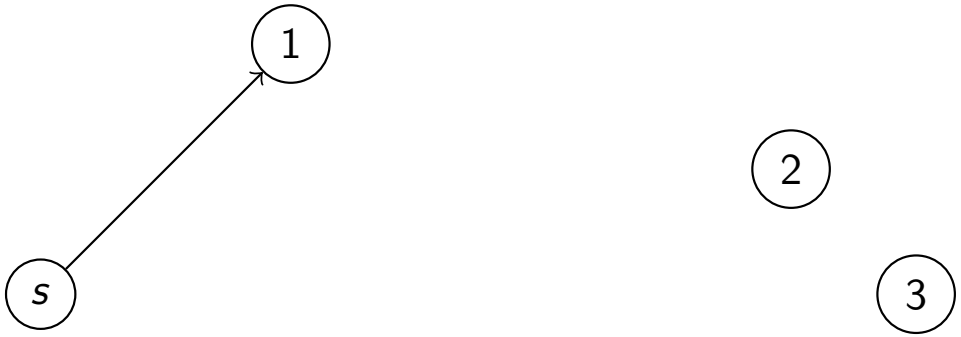

$$L_1 = (C_1, \mathbf{R}_1, \mathbf{E}_1)$$

Standard labeling algorithm

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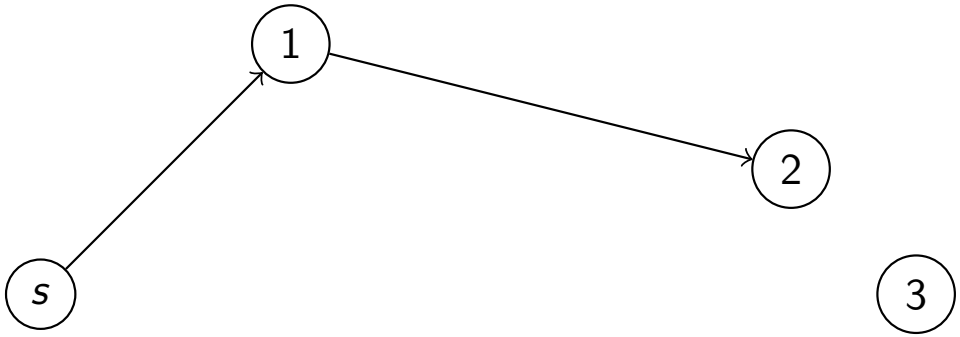

$$L_2 = (C_2, \mathbf{R}_2, \mathbf{E}_2)$$

Standard labeling algorithm

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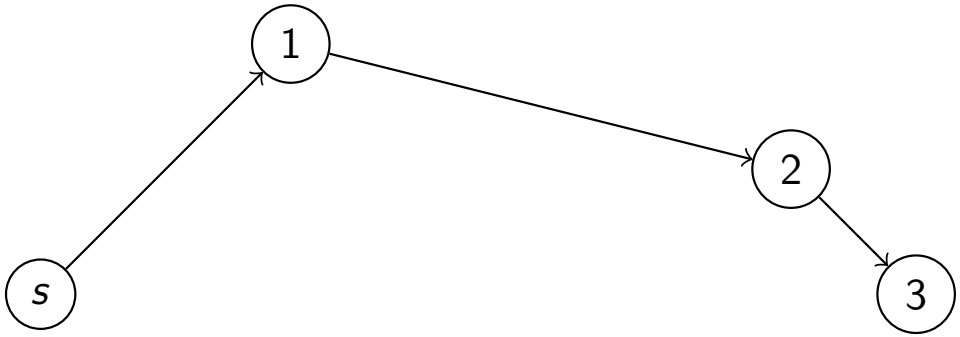

$$L_3 = (C_3, \mathbf{R}_3, \mathbf{E}_3)$$

Standard labeling algorithm

- Each label L_i represents a partial path ending at node i.
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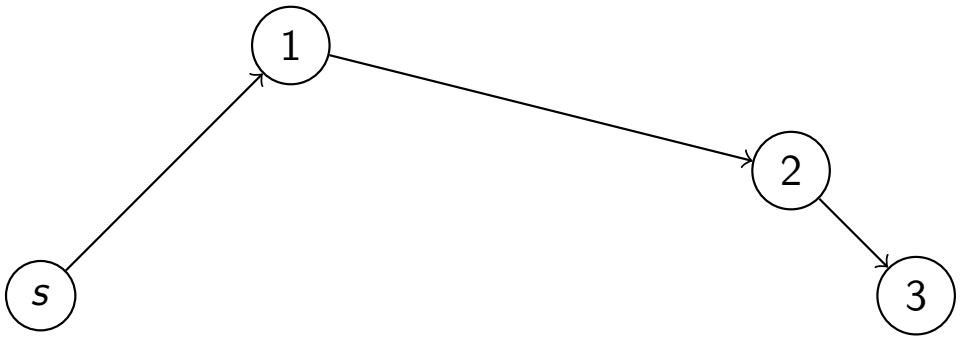

$$L_3 = (C_3, \mathbf{R}_3, e_3^1, e_3^2, \ldots, e_3^n)$$

- Keeping binary resources for every node ensures elementarity.
- This makes label domination more difficult.

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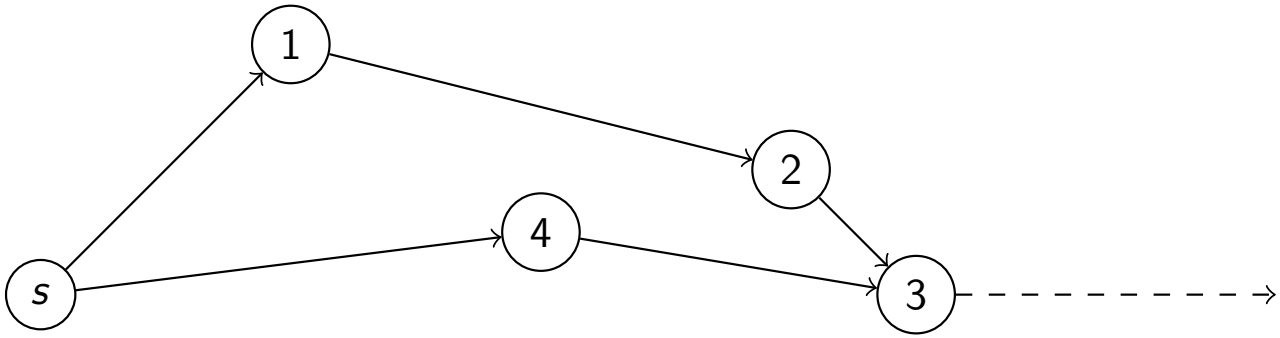

$$L_3 = (C_3, \mathbf{R}_3, e_3^1, e_3^2, \ldots, e_3^n)$$

$$L'_3 = (C'_3, \mathbf{R}'_3, e_3'^1, e_3'^2, \ldots, e_3'^n)$$

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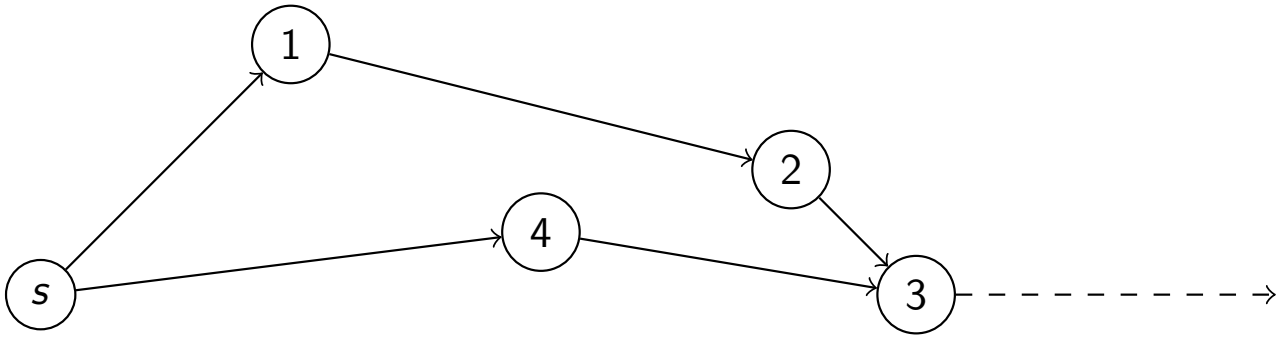

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- Keeping binary resources for every node ensures elementarity.
- This makes label domination more difficult.
- The following algorithms relax the state space with regard to the elementarity resources.

Decremental State Space Relaxation (DSSR)[1]

- Maintain a set Θ of *critical* nodes on which elementarity is enforced.

Algorithm 1 DSSR

1: Initialize Θ
2: **repeat**
3: $\quad P = \text{LabelExtension}(\Theta)$
4: $\quad \Phi = \text{MultipleVisits}(P)$
5: $\quad \Theta = \Theta \cup \Phi$
6: **until** P is elementary

- We manipulate the state space in a *global* way.

[1] Righini and Salani 2008.

Parameters — DSSR

- Critical set initialization:
  - `dssr_init_s`: *none*, *tca*, *hca*, *wtca*, *whca*, *mixed*
  - `dssr_init_n`: $]0, 1[$
- Critical set update rules:
  - `dssr_ins_path_s`: *all-paths*, *one-path*, *in-between*
  - `dssr_ins_path_n`: $]0, 1[$
  - `dssr_ins_node_s`: *all-nodes*, *one-node*, *in-between*
  - `dssr_ins_node_n`: $]0, 1[$

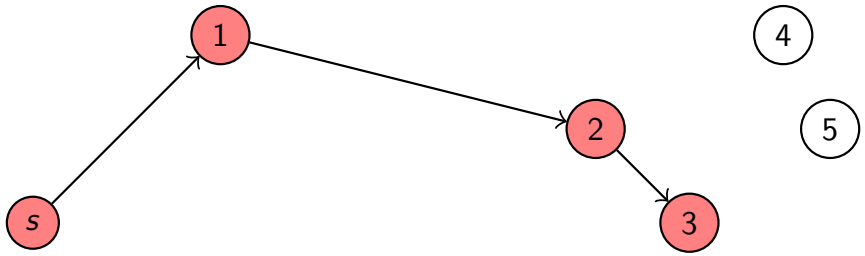
ng-route Relaxation[2]

- Define neighbourhoods N_i, $\forall i$.

[2] Baldacci et al. 2010.

ng-route Relaxation[2]

- Define neighbourhoods N_i, $\forall i$.
- E_i = set of unreachable nodes for path ending at i.

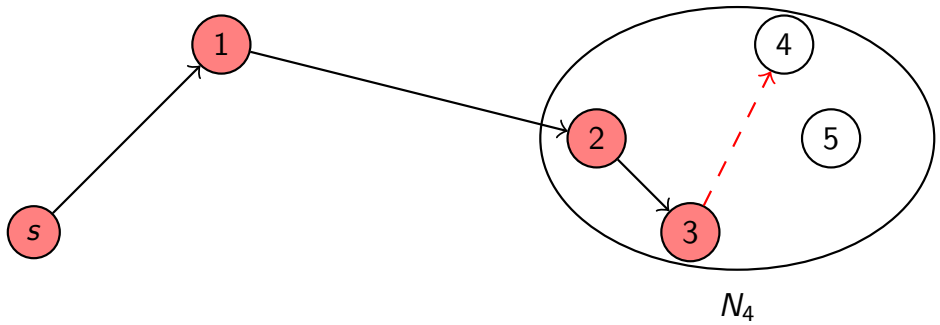


- $\mathbf{E}_3 = \{s, 1, 2, 3\}$

[2] Baldacci et al. 2010.

ng-route Relaxation[2]

- Define neighbourhoods N_i, $\forall i$.
- E_i = set of unreachable nodes for path ending at i.
- When extending to j: $\forall k \in E_i$, insert k in E_j only if $k \in N_j$.



- $\mathbf{E}_3 = \{s, 1, 2, 3\}$
- $j = 4,\ N_4 = \{2, 3, 4, 5\}$

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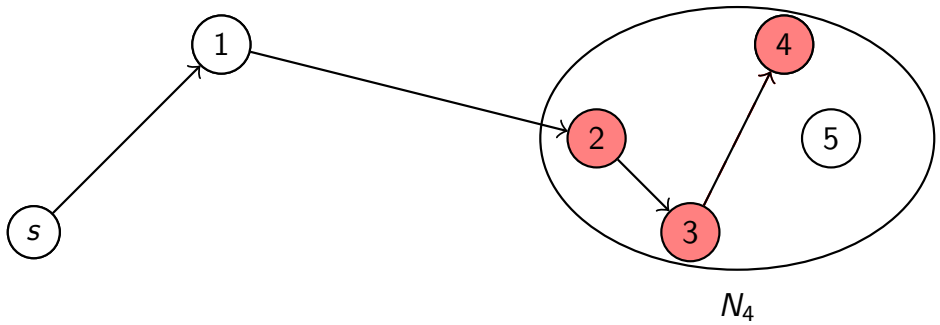

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- $j = 4,\ N_4 = \{2, 3, 4, 5\}$
- $\mathbf{E}_4 = (\mathbf{E}_3 \cap N_4) \cup \{4\} = \{2, 3, 4\}$

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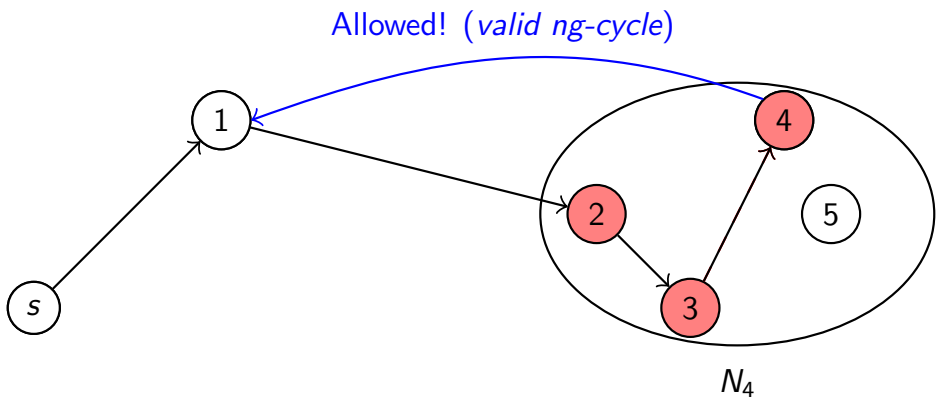

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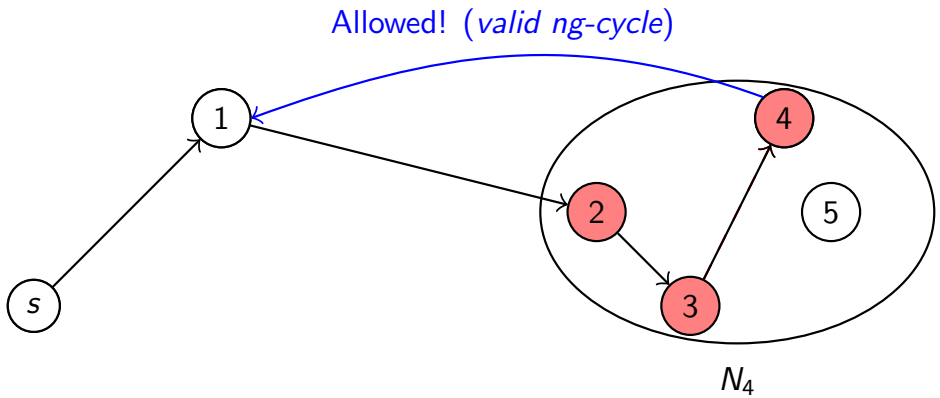
- Resulting path may **not** be elementary.



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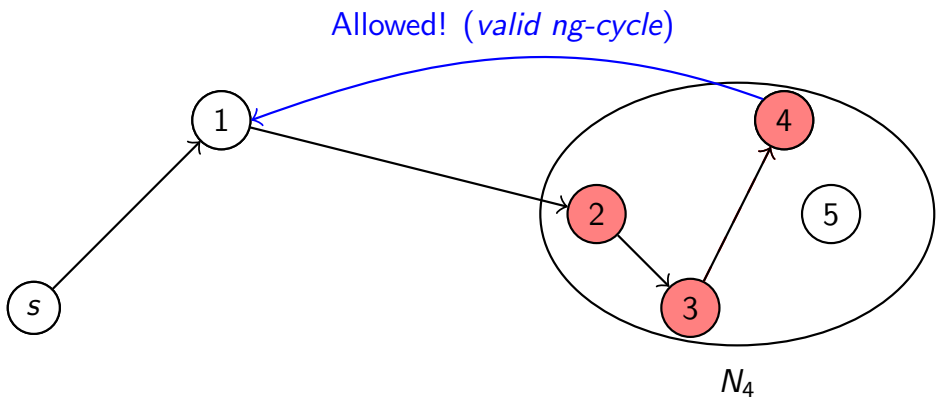
ng-route Relaxation[2]

- Define neighbourhoods N_i, $\forall i$.
- E_i = set of unreachable nodes for path ending at i.
- When extending to j: $\forall k \in E_i$, insert k in E_j only if $k \in N_j$.
- Resulting path may **not** be elementary.
- We manipulate the state space in a *local* way.



- $\mathbf{E}_3 = \{s, 1, 2, 3\}$
- $j = 4,\ N_4 = \{2, 3, 4, 5\}$
- $\mathbf{E}_4 = (\mathbf{E}_3 \cap N_4) \cup \{4\} = \{2, 3, 4\}$

[2] Baldacci et al. 2010.

Parameters — *ng*-route

- Neighborhood size `ng_size`: $]0, 1[$
- Node metric `ng_type`: *travel-time*, *cycle-risk*, *mixed*
- Coefficient for the mixed metric `ng_mix`: $]0, 1[$

ng-route Relaxation and DSSR

- We can combine both algorithms in several ways.
- The straightforward combination uses both neighbourhoods and the set of critical nodes.

Algorithm 2 ng-DSSR-global

1: Initialize Θ, N_i $\forall i$
2: **repeat**
3: P = LabelExtension(Θ, N_i)
4: Φ = MultipleVisits(P, $\{N_i\}_i$) {Only takes nodes in invalid ng-cycles}
5: $\Theta = \Theta \cup \Phi$
6: **until** P is ng-route

ng-route Relaxation and DSSR

- Let us define *applied* neighbourhoods $\hat{N}_i \subseteq N_i \ \forall \ i$ to use throughout label extension[3].
- When best path has invalid cycle C, update applied neighbourhoods of nodes in C.

Algorithm 3 ng-DSSR-local

1: Initialize N_i, $\hat{N}_i$, $\forall i$
2: **repeat**
3: $\quad P =$ LabelExtension$(\{\hat{N}_i\}_i)$
4: $\quad C =$ InvalidCycle$(P, \{N_i\}_i)$
5: $\quad$ Update $(\{\hat{N}_i\}_i, C)$
6: **until** P is ng-route

[3] Dayarian et al. 2015.

ng-route Relaxation and DSSR

- We can derive a different version of DSSR using this local approach[4].
- Let us define local critical sets $\hat{\Theta}_i$, $\forall\ i$.
- When best path has cycle C, update critical sets of nodes in C.

Algorithm 4 DSSR-local

1: Initialize $\hat{\Theta}_i$, $\forall i$
2: **repeat**
3: $P =$ LabelExtension$(\{\hat{\Theta}_i\}_i)$
4: $C =$ Cycle(P)
5: Update $(\{\hat{\Theta}_i\}_i, C)$
6: **until** P is elementary

[4] Martinelli, Pecin, and Poggi 2014.

ng-route Relaxation and DSSR

- ng-DSSR-local (global) outputs ng-routes.
- We can follow-up with DSSR-local (global) to obtain elementary routes.

Algorithm 5 ng-DSSR-local, corrected

1: P = ng-DSSR-local()
2: **if** P is not elementary **then**
3: P = DSSR-local(P)
4: **end if**

Branch-and-Price framework

- After solving the linear relaxation at the root node, solve the integer program with the available columns to obtain an upper bound.
- Branch on the most fractional arc.
- Parameters:
  - Maximum number of generated paths `concat_stop`: $]0, 10000]$
  - Maximum number of paths inserted in the master problem `num_col`: $]0, 1[$
  - Tree navigation strategy `tree_nav`:
    - Depth-first search
    - Breadth-first search
    - Best-first search

Overview

- *ng*-route relax.
- *ng*-DSSR-global
- *ng*-DSSR-local

} *ng*-routes

- DSSR
- Local DSSR
- *ng*-DSSR-global, corrected
- *ng*-DSSR-local, corrected

} Elementary routes

Parameters

Parameter	DSSR	*ng*-r.	G. *ng*-D.	L. *ng*-D.	L. DSSR	G. *ng*-D., corr.	L.*ng*-D., corr.	Range
`tree_trav`	×	×	×	×	×	×	×	{`breadth`, `depth`, `best`}
`n_conc`	×	×	×	×	×	×	×	$]0, 10000]$
`n_col`	×	×	×	×	×	×	×	$]0, 1]$
`dssr_init_s`	×		×		×	×		{`hca`, `tca`, `whca`, `wtca`, `mix`}
`dssr_init_n`	×		×		×	×		$]0, 1[$
`dssr_path_s`	×		×	×	×	×	×	{`1_path`, `in_btw`, `all_paths`}
`dssr_path_n`	×		×	×	×	×	×	$]0, 1[$
`dssr_node_s`	×		×			×		{`1_node`, `in_btw`, `all_nodes`}
`dssr_node_n`	×		×			×		$]0, 1[$
`ng_type`		×	×	×		×	×	{`tt`, `ccr`, `mix`}
`ng_size`		×	×	×		×	×	$]0, 1[$
`ng_mix`		×	×	×		×	×	$]0, 1[$

Table: Parameters choices

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Methodology

- Tune the parameters with the `irace` package[5]
- The tuning set is derived from the Gehring & Homberger instances for the VRPTW.
- Set of 6 instances, one per type (clustered, random, random-clustered) and per size (25 and 50 customers).
- Tuning budget of 5000 experiments for each algorithm, time limit $TL = 3$ hours per experiment.
- Measure of performance:

$$\begin{cases} t & \text{if } t < TL \text{ seconds (3 hours)} \\ TL + \text{best UB} & \text{if best UB has been obtained} \\ \text{big M} & \text{otherwise} \end{cases}$$

- Run the best configuration of each algorithm on the Solomon instances.
- Perform Friedman test + post-hoc Wilcoxon signed rank test on the results.

[5] López-Ibáñez et al. 2016.

Statistical tests on Solomon instances

- The Friedman test reports a p-value $<$ 2.2e-16, denoting statistically significant difference among the data
- Post-hoc test reports that Local DSSR outperforms DSSR, ng-route relaxation, global ng-DSSR, and global corrected ng-DSSR with a p-value $<$ 1e-7
- Local DSSR, local ng-DSSR and its corrected version are statistically equivalent

Solved instances

	DSSR	DSSR-L	NG	NG-DSSR-G	NG-DSSR-L	NG-DSSR-G-C	NG-DSSR-L-C
C1–25	(5, 1, 3)	(6, 3, 0)	(5, 4, 0)	(5, 2, 2)	(5, 4, 0)	(5, 1, 3)	(5, 3, 1)
R1–25	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)
RC1–25	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)
C1–50	(4, 0, 5)	(6, 0, 3)	(6, 0, 3)	(4, 0, 5)	(5, 1, 3)	(5, 0, 4)	(6, 1, 2)
R1–50	(5, 6, 1)	(9, 3, 0)	(7, 5, 0)	(6, 6, 0)	(9, 3, 0)	(5, 7, 0)	(10, 2, 0)
RC1–50	(0, 6, 2)	(1, 7, 0)	(1, 7, 0)	(1, 7, 0)	(1, 7, 0)	(1, 7, 0)	(1, 7, 0)

Table: Solved Solomon instances, 3 hour time limit

$$(n_s, n_p, n_u)$$

- $n_s = \#$ of solved instances
- $n_p = \#$ of partially solved instances (at least the root node)
- $n_u = \#$ of unsolved instances

Additional tests

- We run the best four algorithms on the Solomon instances for 6 hours, on 25, 50, 75 and 100 customers
- The Friedman test reports a p-value = 5.093e-5
- Post-hoc reports that ng is rejected with p-value <0.01 by the other algorithms

Solved instances — 2

	DSSR-L	NG-DSSR-L-C	NG	NG-DSSR-L
C1–25	(7, 2, 0)	(6, 3, 0)	(6, 3, 0)	(7, 2, 0)
R1–25	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)
RC1–25	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)	(8, 0, 0)
C1–50	(6, 1, 2)	(6, 1, 2)	(6, 0, 3)	(6, 1, 2)
R1–50	(9, 3, 0)	(10, 2, 0)	(8, 4, 0)	(9, 3, 0)
RC1–50	(2, 6, 0)	(1, 7, 0)	(1, 7, 0)	(1, 7, 0)
C1–75	(5, 1, 3)	(5, 1, 3)	(5, 0, 4)	(5, 0, 4)
R1–75	(3, 9, 0)	(4, 8, 0)	(4, 8, 0)	(4, 8, 0)
RC1–75	(0, 8, 0)	(0, 8, 0)	(0, 8, 0)	(1, 7, 0)
C1–100	(5, 0, 4)	(5, 0, 4)	(4, 0, 5)	(5, 0, 4)
R1–100	(2, 2, 8)	(3, 1, 8)	(2, 2, 8)	(3, 1, 8)
RC1–100	(0, 2, 6)	(0, 2, 6)	(0, 3, 5)	(0, 5, 3)

Table: Solved Solomon instances, 6 hour time limit

Solved instances — 2

	DSSR-L				NG-DSSR-L-C			
	gap	root gap	time	nodes	gap	root gap	time	nodes
C1-25	0.17%	0.17%	7547.5	51.0	0.13%	0.13%	9236.3	54.8
C1-50	0.01%	0.01%	8333.8	2.1	0.01%	0.01%	7604.8	2.0
C1-75	0.07%	0.07%	11974.3	1.6	0.10%	0.10%	10592.8	1.2
C1-100	0.0%	0.0%	10653.5	0.6	0.00%	0.00%	10130.8	0.6
R1-25	0.42%	0.56%	12.1	7.8	0.42%	0.56%	23.1	14.5
R1-50	0.84%	0.94%	6546.3	167.8	0.76%	1.03%	6600.6	539.2
R1-75	1.02%	1.03%	16213.6	77.2	0.92%	1.05%	16041.1	257.2
R1-100	0.42%	0.50%	19520.2	93.4	0.39%	0.48%	18881.6	93.8
RC1-25	1.15%	1.15%	11.2	25.5	1.15%	1.15%	24.3	13.5
RC1-50	5.67%	5.70%	18819.6	8677.4	5.60%	5.71%	19051.6	8616.4
RC1-75	3.12%	3.14%	21600	1587.3	3.03%	3.10%	21600	2717.1
RC1-100	1.27%	1.78%	21600	261.6	1.72%	1.72%	21600	366.1

Table: Performance on Solomon instances, 6 hour time limit — 1

Solved instances — 2

	NG				NG-DSSR-L			
	gap	root gap	time	nodes	gap	root gap	time	nodes
C1-25	0.17%	0.17%	9125.1	189.3	0.43%	0.47%	8845.6	112.8
C1-50	0.01%	0.01%	7820.6	1.6	0.01%	0.01%	7817.9	1.9
C1-75	0.08%	0.08%	11607.5	1.4	0.08%	0.09%	11262.3	1.4
C1-100	0.00%	0.00%	12608.4	0.4	0.00%	0.00%	10545.3	0.8
R1-25	0.60%	0.78%	33.3	33.0	0.48%	0.69%	13.1	9.2
R1-50	1.00%	1.21%	8317.2	978.5	0.84%	1.04%	6823.3	333.6
R1-75	0.98%	1.08%	15062.0	196.5	1.01%	1.03%	15375.1	176.8
R1-100	0.38%	0.44%	19211.7	59.6	0.39%	0.56%	17823.0	120.5
RC1-25	1.97%	3.77%	1968.1	786.5	1.19%	1.19%	96.8	17.5
RC1-50	6.33%	6.72%	19907.2	10364.8	5.87%	6.14%	18992.2	10211.4
RC1-75	3.50%	3.65%	21600	3591.0	3.26%	3.27%	21101.7	3030.5
RC1-100	1.97%	1.97%	21600	470.8	2.05%	2.38%	21600	302.9

Table: Performance on Solomon instances, 6 hour time limit — 2

Parameter configurations

	n_col_root	conc_stop_root	n_col	conc_stop	tree_nav
DSSR-L					
Best	82%	2820	31%	9247	best-first
Avg	42%	3998	35%	8902	best-first (5)
Range	[10%,81%]	[2764,8481]	[22%,63%]	[7308,9867]	
NG-DSSR-L-C					
Best	27%	7910	42%	9845	depth-first
Avg	45%	5949	37%	8688	depth-first (6)
Range	[26%,98%]	[1003,8320]	[36%,42%]	[5234,9923]	
NG-DSSR-L					
Best	4%	8752	14%	3079	depth-first
Avg	10%	6900	24%	3540	depth-first (6)
Range	[1%,35%]	[5476,8752]	[11%,63%]	[1100,4519]	

Table: BP parameters

Parameter configurations

	init_str_root	init_n_root	ins_path_str_root	ins_path_n_root
DSSR-L				
Best	none	n/a	in-between	25%
Avg	none (6)		in-between (6)	38%
Range				[24%,47%]
NG-DSSR-L-C				
Best	n/a	n/a	one-path	n/a
Avg			one-path (6)	
Range				
NG-DSSR-L				
Best	n/a	n/a	all-paths	n/a
Avg			all-paths (5)	
Range				

Table: DSSR parameters, root

Parameter configurations

	init_str	init_n	ins_path_str	ins_path_n
DSSR-L				
Best	none	n/a	in-between	39%
Avg	none (6)		in-between (6)	57%
Range				[38%,80%]
NG-DSSR-L-CORR				
Best	n/a	n/a	in-between	98%
Avg			in-between (6)	81%
Range				[72%,97%]
NG-DSSR-L				
Best	n/a	n/a	in-between	35%
Avg			in-between (6)	31%
Range				[26%,34%]

Table: DSSR parameters

Parameter configurations

	ng_type_root	ng_size_root	ng_type	ng_size
NG-DSSR-L CORR				
Best	travel-time	6%	travel-time	23%
Avg	travel-time (5)	9%	travel-time (6)	24%
Range		[4%,21%]		[16%,45%]
NG-DSSR-L				
Best	travel-time	17%	travel-time	88%
Avg	travel-time (6)	17%	travel-time (5)	81%
Range		[14%,24%]		[50%,97%]

Table: NG parameters

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Conclusions

- It is a good idea to use a strong methodology for comparing algorithm performance.
- "Local" approach for treating elementarity resources works.
- It makes sense to parametrize differently the algorithm in the root node.
- Best parameter values might be different than the ones suggested in the literature.

Current work

- Perform an instance size—dependant tuning of the best algorithms and compare it with the current ones.
- Perform the same experiments on the classic VRPTW.
- Adapt the algorithms to the multitrip VRPTW.

Thanks for your attention.