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Numerical Modeling of Lateral-Torsional Buckling of Steel I-Beams under Fire Conditions—Comparison with Eurocode 3

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ABSTRACT: A geometrically and materially non-linear finite element program, i.e., a general model, has been used to determine the lateral-torsional resistance of steel I-beams under fire conditions, according to the same material properties of Eurocode 3, Part 1-2. Two yield strengths, one cross section, one type of load and four different time exposures to the ISO 834 standard fire have been considered. The numerical results have been compared to the results of the simple model presented in Eurocode 3, Part 1-2. When compared with the general model, this simple model leads to a safety level that depends on the slenderness of the beam, being unsafe for intermediate non-dimensional slenderness. A new proposal has been made for a simple model that ensures a conservative result when compared to the general model.

KEY WORDS: lateral-torsional buckling, fire, numerical modeling, new proposal, Eurocode 3, ISO 834.

INTRODUCTION

ALTHOUGH THE PROBLEM of lateral-torsional buckling of steel beams at room temperature is well known [1], the same problem at elevated temperature is not. Among the work done in this field there is the paper from Bailey et al. [2] who use a three-dimensional computer model to investigate the ultimate behavior of uniformly heated unrestrained beams. In this paper, a simple model for fire resistance of lateral-torsional buckling of steel I-beams is presented. It is based on the numerical results of the SAFIR program, a geometrically and materially non-linear code specially established for the analysis of structures submitted to fire [3].

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The capability of this code to model the lateral-torsional buckling of beams has been demonstrated [4] at room temperature by comparisons with the formulas of Eurocode 3, Part 1-1 [5].

Particular attention was paid to the possibility of using the same model as the one proposed in Eurocode 3, Part 1-1 [5], simply modifying material properties depending on the temperature. This is the procedure currently proposed in Eurocode 3, Part 1-2 [6], although its accuracy has never been demonstrated at elevated temperatures. Some points have never been clearly addressed such as the fact that the stress-strain relationship of steel at elevated temperatures does not remain elastic-perfectly plastic (which is the basic hypothesis of the model developed at room temperature) or the influence of thermal stresses, created by higher temperatures, which usually develop at the end of the flanges.

In the numerical analyses, a three-dimensional (3D) beam element has been used. It is based on the following formulations and hypotheses:

- Displacement type element in a total co-rotational description.
- Prismatic element.
- The displacement of the node line is described by the displacements of the three nodes of the element, two nodes at each end supporting seven degrees of freedom, three translations, three rotations and the warping amplitude, plus one node at the mid-length supporting one degree of freedom, the non-linear part of the longitudinal displacement.
- In pure bending the cross section remains plane and perpendicular to the longitudinal axis. This hypothesis due to Bernoulli neglects the shear energy.
- No local buckling is taken into account, which is the reason why the proposal in this paper is valid only for Class 1 and Class 2 sections [5].
- The strains are small (von Kármán hypothesis), i.e.,

$$\frac{1}{2}\frac{\partial u}{\partial x} << 1$$

where u is the longitudinal displacement and x is the longitudinal co-ordinate.
- The angles between the deformed longitudinal axis and the undeformed but translated longitudinal axis are small, i.e.,

$$\sin\varphi \cong \varphi \quad \text{and} \quad \cos\varphi \cong 1$$

where φ is the angle between the arc and the cord of the beam finite element.
- The longitudinal integrations are numerically calculated using Gauss's method.
- The cross section is discretized by means of triangular or quadrilateral fibers. At every longitudinal point of integration, all variables, such as temperature, strain, stress, etc., are uniform in each fiber.

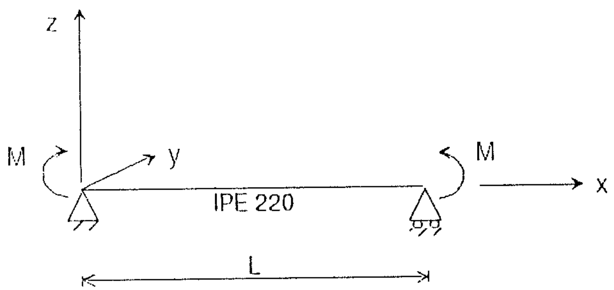


Figure 1. Simply supported beam submitted to moments at the ends.

- The tangent stiffness matrix is evaluated at each iteration of the convergence0 process (pure Newton-Raphson method).
- Residual stresses are considered by means of initial and constant strains [7].
- The material behavior in case of strain unloading is elastic, with the elastic modulus equal to Young's modulus at the origin of the stress-strain curve. In one cross section, some fibers that have yielded may therefore exhibit a decreased tangent modulus because they are still on the loading branch, whereas, at the same time, some other fibers behave elastically. The plastic strain is presumed not to be affected by a change in temperature [8].

A simply supported steel beam has been studied to compare the results between Eurocode 3, Part 1-2 [6] and the SAFIR code under fire conditions. The beam has been submitted to a uniform moment (see Figure 1) and cannot deflect laterally or twist at the supports.

The results of Eurocode 3 and the SAFIR code were compared for the unprotected beam after 10, 15, 20 and 30 min of exposure to the ISO 834 standard fire.

LATERAL-TORSIONAL BUCKLING UNDER FIRE CONDITIONS

Analysis According to Eurocode 3

The temperature of the beam after the desired time has been obtained using the simplified equation of Eurocode 3, Part 1-2 [6]. From this temperature (which is uniform in the beam cross section), the buckling resistance moment, $M_{b,fi,t,Rd}$ at time t has been determined according to (for Class 1 or Class 2 cross sections):

$$M_{b,fi,t,Rd} = \frac{\chi_{LT,fi}}{1.2} w_{pl,y} k_{y,\theta,com} f_y \frac{1}{\gamma_{M,fi}} \tag{1}$$

where:

$\chi_{LT,fi}$ is the reduction factor for lateral-torsional buckling in the fire design situation

$w_{pl,y}$ is the plastic section modulus
$k_{y,0,com}$ is the reduction factor for the yield strength at the maximum temperature in the compression flange $\theta_{a,com}$, reached at time t
$\gamma_{M,fi}$ is the partial safety factor for the fire situation (usually $\gamma_{M,fi} = 1$)

This equation is used if the non-dimensional slenderness $\bar{\lambda}_{LT,\theta,com}$ for the temperature reached at time t, exceeds the value of 0.4.

The constant 1.2 is an empirically determined value and is used as a correction factor which allows for a number of effects.

The reduction factor for lateral-torsional buckling in a fire design situation, $\chi_{LT,fi}$, must be determined in the same way as it is at room temperature, but using the non-dimensional slenderness $\bar{\lambda}_{LT,\theta,com}$ (or $\bar{\lambda}_{LT,fi}$, if the temperature field in the cross section is uniform) given by

$$\bar{\lambda}_{LT,\theta,com} = \bar{\lambda}_{LT,fi} = \bar{\lambda}_{LT}\sqrt{\frac{k_{y,\theta,com}}{k_{E,\theta,com}}} \tag{2}$$

where

$\bar{\lambda}_{LT}$ is the non-dimensional slenderness at room temperature given by [5] (for Class 1 or Class 2 cross sections)

$$\bar{\lambda}_{LT} = \frac{\lambda_{LT}}{\lambda_1} \tag{2a}$$

where

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} \tag{2b}$$

$$\lambda_{LT} = \pi\sqrt{\frac{E w_{pl,y}}{M_{cr}}} \tag{2c}$$

where M_{cr} is the elastic critical moment for lateral-torsional buckling of the beam. Substituting from Equations (2b) and (2c) in Equation (2a)

$$\bar{\lambda}_{LT} = \sqrt{\frac{w_{pl,y} f_y}{M_{cr}}} = \sqrt{\frac{M_{pl}}{M_{cr}}} \tag{2d}$$

where M_{pl} is the plastic moment resistance of the gross cross section;

$k_{E,\theta,com}$ is the reduction factor for the slope of the linear elastic range at the maximum steel temperature reached at time t

According to Equation (2), the factor $\sqrt{k_{y,\theta,com} / k_{E,\theta,com}}$ is the factor that multiplies the non-dimensional slenderness at room temperature in order to yield the non-dimensional slenderness at elevated temperature $\bar{\lambda}_{LT,\theta,com}$. Its variation with temperature is represented in Figure 2 and shows a particular feature of the non-dimensional slenderness. It could be expected that this slenderness would increase constantly with the temperature but, according to the Eurocode 3 material model, this is clearly not the case.

The full line of Figure 3 shows the design curve for lateral-torsional buckling according to Eurocode 3. For all temperatures greater than 20°C this curve is unique and named EC3,fi in that figure. On the vertical axis is the ratio

$$\frac{M_{b,fi,t,Rd}}{M_{fi,\theta,Rd}} \tag{3}$$

where

$M_{b,fi,t,Rd}$ is the design lateral-torsional buckling resistance moment at time t of a laterally unrestrained beam given by Equation (1) and

$M_{fi,\theta,Rd}$ is the design moment resistance of a Class 1 or 2 cross section with a uniform temperature θ_a. It may be determined from:

$$M_{fi,\theta,Rd} = k_{y,\theta} \frac{\gamma_{M0}}{\gamma_{M,fi}} M_{Rd} \tag{4}$$

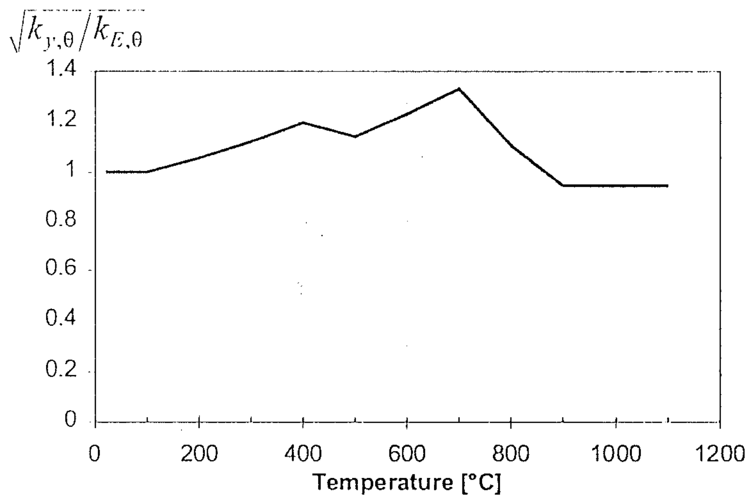


Figure 2. Dependency with temperature of the factor.

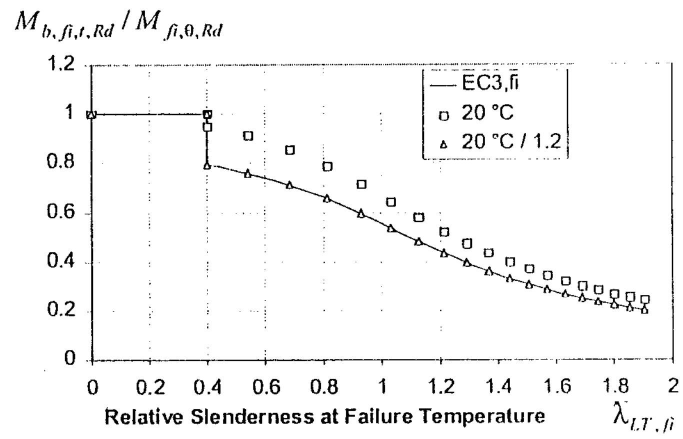


Figure 3. Beam design curve of Eurocode 3 for fire situation (EC3,fi) and at room temperature (20°C).

where $\gamma_{M0} = 1.0$, $\gamma_{M,fi} = 1.0$ and M_{Rd} is the plastic resistance of the gross cross section $M_{pl,Rd}$ for normal temperature, which is given by

$$M_{Rd} = \frac{w_{pl,y} f_y}{\gamma_{M0}} \tag{5}$$

where $\gamma_{M0} = 1.0$.

This figure also shows that the lateral buckling design curve at elevated temperature is different from the curve at 20°C by the empirical factor 1.2 (the curve at elevated temperature, EC3,fi, is the curve at 20°C divided by 1.2).

Therefore, it must be emphasized that, throughout this paper, the ratio $M_{b,fi,t,Rd}/M_{fi,\theta,Rd}$ will be used for the purposes of comparison. It is obtained as the reduction factor for lateral-torsional buckling in the fire design situation $\chi_{LT,fi}$ divided by 1.2, for the Eurocode 3, Part 1-2 results, i.e.,

$$\frac{M_{b,fi,t,Rd}}{M_{fi,\theta,Rd}} = \frac{\chi_{LT,fi}}{1.2} \text{ for the Eurocode 3, Part 1-2 results} \tag{6a}$$

or directly from

$$\frac{M_{SAFIR}}{M_{fi,\theta,Rd}}, \text{ for the SAFIR results} \tag{6b}$$

It must also be mentioned that the class of the analyzed cross-section was checked for all the analyzed temperatures to see if it remained a Class 1 cross section as at room temperature. This was done using the modified value of ε given by [6]:

$$\varepsilon = [(235/f_y)(k_{E,0}/k_{y,0})]^{0.5} \tag{7}$$

It has been concluded that, in this case, the class of the cross section doesn't change with temperature.

Analysis with the SAFIR Code

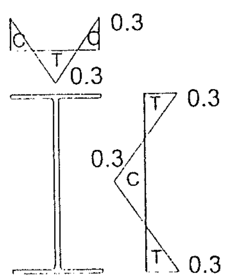
An unprotected IPE 220 section is supposed to be heated on 4 sides by the ISO 834 time temperature curve. The evolution of the temperature field is obtained using a finite element analysis. So the temperature field is not uniform like the one obtained with the simplified equation of Eurocode 3.

For the temperature fields reached at time t = 10, 15, 20 and 30 min, the moment was applied with step increments of 100 Nm.

The numerical simulations were carried out under the following assumptions:

- Beam lateral imperfection: sinusoidal, with a maximum value of L/1000 [9–11].
- Longitudinal integration: two Gauss points.
- The warping function is assumed not to be affected by temperatures but the torsional stiffness is adapted, according to the variation of the steel properties, with temperature.
- Residual stresses: constant across the thickness of the web and of the flanges. Triangular distribution as in Figure 4, with a maximum value of 0.3 × 235 MPa [12], for the Fe 360 steel as well as for the Fe 510 steel.

The beam design curves for all the time instants studied are shown in Figures 5–8 for the Fe 360 and Fe 510 steel. In these figures, $M_{b,fi,t,Rd}$ is the design lateral-torsional buckling resistance moment at time t of a laterally unrestrained



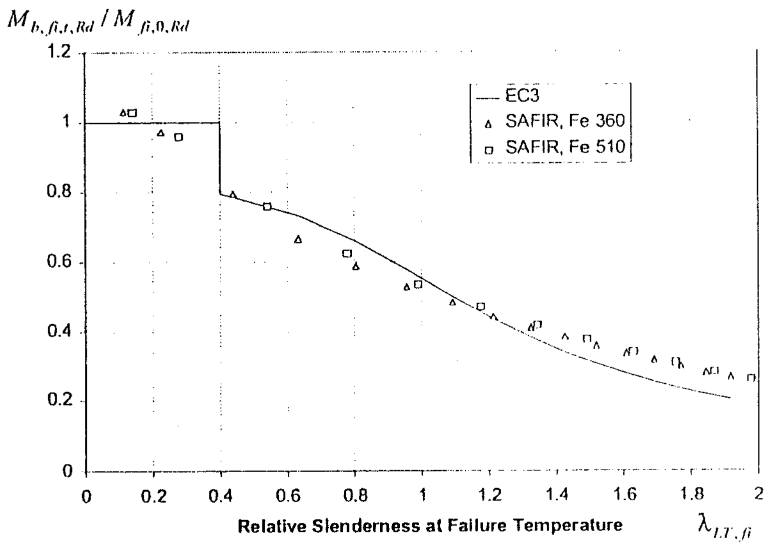


Figure 5. *Beam design curve after 10 minutes. Comparison between the Eurocode 3 and SAFIR, for Fe 360 and Fe 510 steel.*

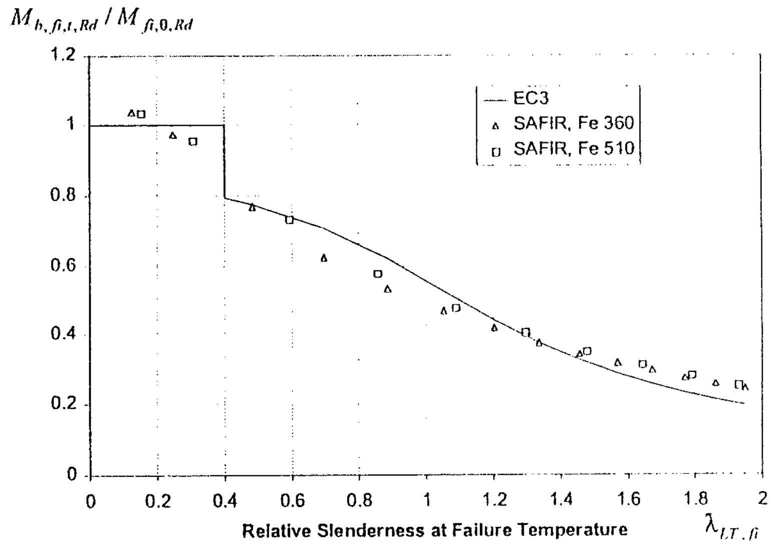


Figure 6. *Beam design curve after 15 minutes. Comparison between the Eurocode 3 and SAFIR, for Fe 360 and Fe 510 steel.*

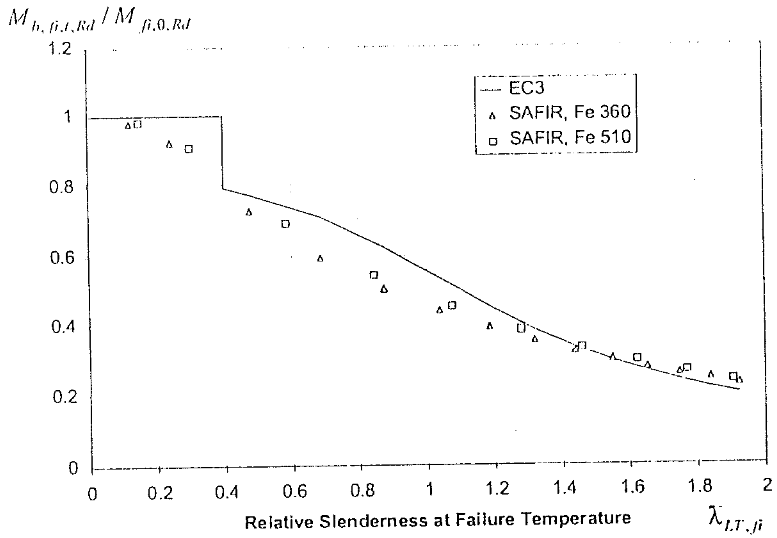


Figure 7. Beam design curve after 20 minutes. Comparison between the Eurocode 3 and SAFIR, for Fe 360 and Fe 510 steel.

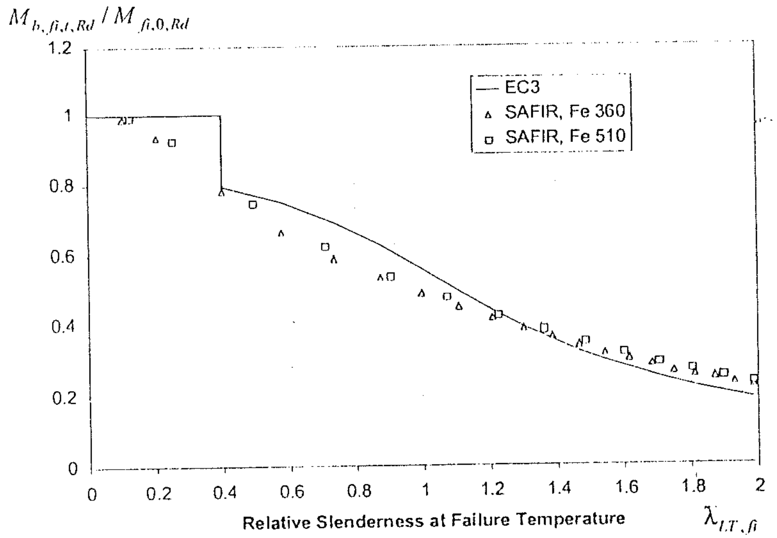


Figure 8. Beam design curve after 30 minutes. Comparison between the Eurocode 3 and SAFIR, for Fe 360 and Fe 510 steel.

beam given by Equation (1) or calculated by the SAFIR code and the design moment resistance $M_{fi,0,Rd}$ of a Class 1 or 2 cross section is given by Equation (4) evaluated for the temperatures obtained with the simplified equation of Eurocode 3, i.e., 554°C, 680°C, 733°C and 827°C for the times of 10, 15, 20 and 30 min respectively. The relative slenderness was calculated at failure temperature according to Equation (2).

Figure 9 shows the beam design curve obtained with the SAFIR results for the times of 10, 15, 20 and 30 min, all plotted at the same chart for the Fe 360 and Fe 510 steel. These curves are not coincident as in the case of the Eurocode 3 curve at elevated temperatures (see, for instance, curve EC3,fi in Figure 3, or curve EC3 in Figures 5 to 8).

From Figures 5 to 9, it can be seen that the numerical values are higher on the vertical axis for Fe 510 than they are for Fe 360. As stated in Reference [11], "This is due to the fact that the residual stresses do not depend on the yield strength. Their relative influence is therefore smaller when the yield strength is increased. This phenomena is not accounted for in the simplified model of Eurocode 3, where the buckling curve does not vary with the yield strength." In fact, the reduction factor for lateral-torsional buckling, χ_{LT}, depends on the yield strength as well as the non-dimensional slenderness, $\overline{\lambda}_{LT}$, but the lateral-torsional buckling curves from Eurocode 3 do not depend on the yield strength as can be seen in Figure 12.

The reason why, in Figures 5, 6 and 9, the ratio $M_{SAFIR}/M_{fi,0,Rd}$, for low values of

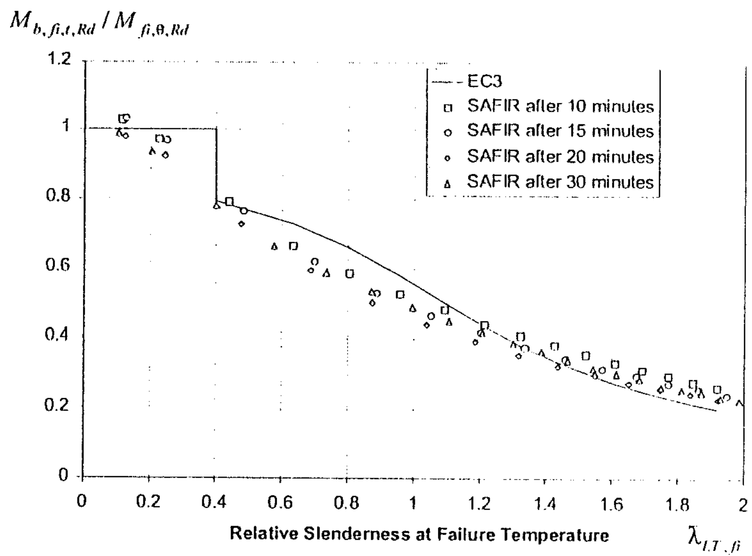


***Figure 9a.** Beam design curve obtained with Eurocode 3 and SAFIR, for Fe 360 (after 10, 15, 20 and 30 minutes).*

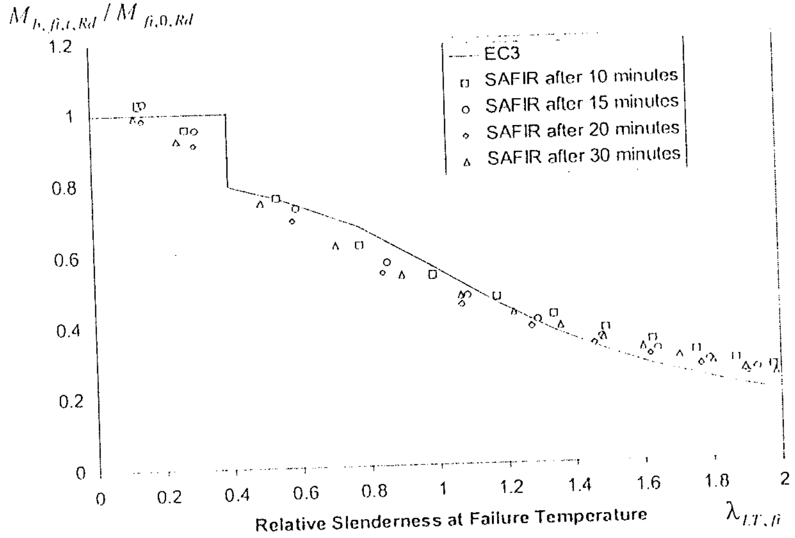


Figure 9b. Beam design curve obtained with Eurocode 3 and SAFIR, for Fe 510 (after 10, 15, 20 and 30 minutes).

the slenderness and for times of 10 and 15 min, is greater than 1, is that the temperature field obtained with SAFIR is not uniform, with temperatures in the flanges lower than the uniform temperature given by the simplified equation of the Eurocode 3 used to calculate $M_{fi,0,Rd}$, because the flanges are thicker than the web. The uniform temperature after 10 min calculated with the Eurocode 3 is 554°C, which is higher than the temperature in the flanges calculated with SAFIR. For longer durations, this effect tends to disappear because the temperature field becomes more and more uniform. After 30 min, the uniform temperature field obtained with the simplified equation of the Eurocode 3 is 827°C. The maximum temperature difference after 30 min, for the SAFIR results, is only 11.9°C while, after 10 min, it is 55.9°C.

NEW PROPOSAL

Adopting the same proposal as Franssen et al. [11], our approach to a new proposal is given below. The lateral-torsional buckling resistance moment is

$$M_{b,fi,t,Rd} = \chi_{LT,fi} w_{pl,y} k_{y,0,com} f_y \frac{1}{\gamma_{M,fi}} \tag{8}$$

$$\chi_{LT,fi} = \frac{1}{\phi_{LT,0,com} + \sqrt{[\phi_{LT,0,com}]^2 - [\bar{\lambda}_{LT,0,com}]^2}} \tag{9}$$

$$\phi_{LT,0,com} = \frac{1}{2}\left[1 + \alpha\bar{\lambda}_{LT,0,com} + (\bar{\lambda}_{LT,0,com})^2\right] \tag{10}$$

and, as in Equation (2)

$$\bar{\lambda}_{LT,0,com} = \bar{\lambda}_{LT}\sqrt{\frac{k_{y,0,com}}{k_{E,0,com}}}$$

where

$\bar{\lambda}_{LT}$ is the non-dimensional slenderness at room temperature, and with $\alpha = \beta\varepsilon$ the imperfection factor

β is the severity factor, to be chosen in order to ensure the appropriate safety level

and $\varepsilon = \sqrt{235/f_y}$, with f_y in MPa as the yield strength

Comparing Equations (1) and (8), we can verify that, with this new proposal, we do not use the empirical constant 1.2 which is used as a correction factor in the proposal of the Eurocode 3.

Equations (9) and (10) are in fact exactly the same as those defined at room temperature in Eurocode 3, Part 1-1 [5], except that the threshold limit of 0.20 for $\bar{\lambda}_{LT}$ does not appear in Equation (10). The fact that the threshold limit does not appear changes the shape of the buckling curve. It differs from that at room temperature. The new curve starts at $\chi_{LT} = 1.0$ for $\bar{\lambda}_{LT} = 0.0$ but it decreases even for very low slenderness, instead of having a horizontal plateau up to $\bar{\lambda}_{LT} = 0.4$ (see Figures 10 and 11).

The lateral-torsional buckling curve now varies with the yield strength due to the parameter ε which appears in the imperfection factor, α of Equation (10). It must be emphasized that the corresponding factor of the Eurocode 3 is constant and takes the value of 0.21 for hot-rolled profiles, leading to the same buckling curve for all steel grades. As can be seen in Figure 12, with this new proposal, the beam design curve for lateral-torsional buckling now depends on the steel grade whereas the proposal of the Eurocode 3 does not. This dependence of the lateral buckling curve with the steel grade can be numerically supported with the results already shown in Figures 10 and 11.

When comparing this simple model with experimental results for the fire resistance of axially-loaded members [13], Franssen et al. determined a severity factor with a value of 0.65. It must be mentioned that the value of 0.65 for

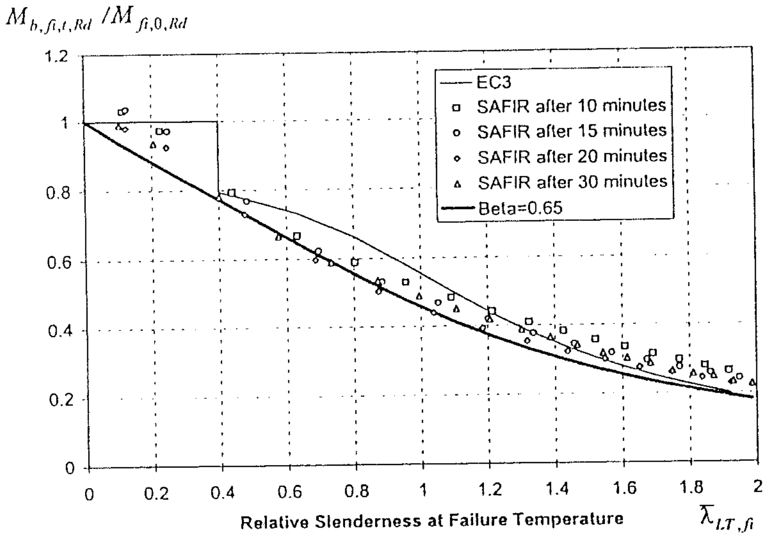


Figure 10. Beam design curve obtained with Eurocode 3, SAFIR (after 10, 15, 20 and 30 minutes) and with the new proposal, with β = 0.65 (Fe 360 steel).

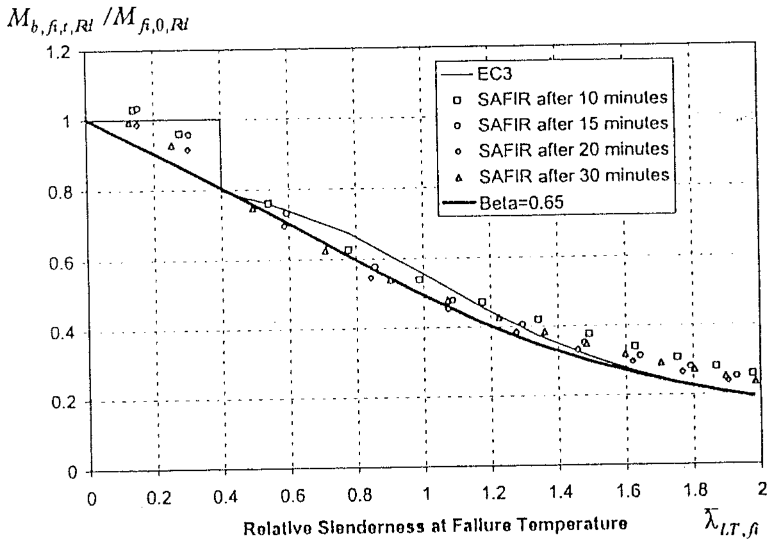


Figure 11. Beam design curve obtained with Eurocode 3, SAFIR (after 10, 15, 20 and 30 minutes) and with the new proposal, with β = 0.65 (Fe 510 steel).

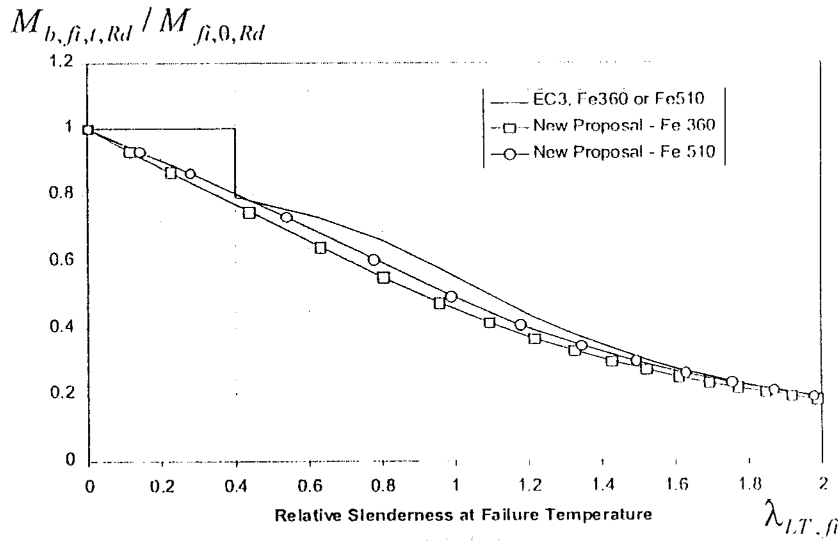


Figure 12. Beam design curves at elevated temperature obtained with the proposal of the Eurocode 3 and with the new proposal with β = 0.65.

the severity factor is the adopted value in the Belgian and French National Application Documents of Eurocode 3, Part 1-2 [14,15].

The same value has been used in Figure 10 for Fe 360 steel and in Figure 11 for Fe 510 steel and it can be seen that the proposal safely covers the numerical results.

CONCLUSIONS

The physical fact that Young's modulus decreases faster than the yield strength when the temperature increases, plus the fact that the stress-strain relationship at elevated temperatures is not the same as at room temperature, produces a modification of the lateral-torsional buckling curve at elevated temperatures. The horizontal plateau valid at 20°C up to a non-dimensional slenderness of 0.4, vanishes at elevated temperatures [11]. The simple models based on the lateral-torsional buckling curve that is valid at room temperature led to a safety level that depends on the slenderness of the beam, the results being unsafe for intermediate length beams. It has been possible to make a new proposal of a lateral-torsional buckling curve for hot-rolled I-section beams submitted to fire, based on the proposal suggested earlier [11] for axially-loaded hot-rolled H-sections submitted to fire. The beam design curve based on the reduction factor for lateral-torsional buckling in fire design situation and the non-dimensional slenderness evaluated at the ultimate temperature now depends on the steel grade, which is not the case in Eurocode 3, Part 1-2.

It has been found that the same severity factor as the one used for the case of axially-loaded columns, i.e., $\beta = 0.65$ [13] could also be used here. This leads to the same philosophy as the one of Eurocode 3, i.e., to use the same formulas for the reduction factor for lateral-torsional buckling and for flexural buckling.

The severity factor β of the proposed simple calculation model has been established analyzing only the behavior of the IPE 220 profile. Further analysis of the numerical results should be done considering different steel I-sections.

It would also be worth having results of well instrumented and carefully carried out experimental tests to verify whether the present proposal can actually reproduce the test results and to fix definitely the value of the severity factor. As there is a low probability for the two structural imperfections, residual stresses and initial imperfection, being simultaneously in a test with the high amplitude assumed here in the numerical simulations, this could lead to the fact that the final adopted severity factor will be less severe than the one proposed in this paper.

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NOMENCLATURE

f_y	yield strength
$k_{y,0,com}$	reduction factor for the yield strength at the maximum temperature in the compression flange $\theta_{a,com}$, reached at time t
$k_{E,0,com}$	reduction factor for the slope of the linear elastic range at the maximum steel temperature in the compresion flange $\theta_{a,com}$ reached at time t
$M_{b,fi,t,Rd}$	buckling resistance moment in the fire design situation
$M_{fi,0,Rd}$	design moment resistance of a Class 1 or 2 cross section with a uniform temperature θ_a
M_{SAFIR}	buckling resistance moment in the fire design situation given by SAFIR
M_{Rd}	plastic moment resistance of the gross cross section, $M_{pl,Rd}$ for normal temperature
t	time
$w_{pl,y}$	plastic section modulus

Greek

α	imperfection factor
β	severity factor
λ_{M0}	partial safety factor (usually $\lambda_{M0} = 1.0$)
$\lambda_{M,fi}$	partial safety factor for the fire situation (usually $\lambda_{M,fi} = 1.0$)
$\bar{\lambda}_{LT}$	non-dimensional slenderness at room temperature
$\bar{\lambda}_{LT,0,com}$	non-dimensional slenderness for the maximum temperature in the compression flange $\theta_{a,com}$
$\bar{\lambda}_{LT,fi}$	non-dimensional slenderness in the fire design situation
$\chi_{LT,fi}$	reduction factor for lateral-torsional buckling in the fire design situation

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