



# Detection of bright multiply imaged quasars with Gaia

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**Abstract.** We estimate the fraction of gravitational lens systems among quasars that Gaia will detect. Modeling the population of intervening deflectors by means of a Singular Isothermal Sphere lens model, we find that a fraction of 0.6% of quasars will consist of multiple lensed QSO images. This leads to an expectation of nearly 3000 multiply imaged Quasars to be discovered by Gaia over the whole sky.

**Key words.** gravitational lens statistics – Quasars – Gaia

## 1. Introduction

Gaia is expected to detect about 500,000 QSOs down to the limiting magnitude  $G = 20$  in the Gaia photometric  $G$  band. In this contribution, we answer the following question: among these 500,000 expected QSOs how many should undergo a gravitational lensing event with the formation of multiple images?

We first derive in section 2 an expression for the probability that a QSO, with known redshift and magnitude, undergoes a gravitational lensing event with the formation of multiple images due to the presence of a foreground deflector located near its line-of-sight. Then we generate in section 3 a mock catalogue of QSOs to be detected by Gaia and calculate the gravitational lensing optical depth for each of the 500,000 QSOs. The mean lensing probability through the mock catalogue represents the

fraction of expected lensed sources within the detected population.

## 2. Lensing optical depth

In this section, we derive the analytical expression of the lensing optical depth  $\tau$ , i.e. the probability for a single source to be lensed.

Let us consider a population of deflecting galaxies modeled as Singular Isothermal Spheres (SIS), a spherically symmetric mass distribution which density  $\rho_{SIS}$  is given by

$$\rho_{SIS} = \frac{\sigma^2}{2\pi G r^2}, \quad (1)$$

where  $\sigma$  is the line-of-sight velocity dispersion,  $G$  the Cavendish constant, and  $r$  the radial distance from the center. The SIS reflects quite well the total mass distribution (dark matter halo plus baryonic matter) in both early-type and spiral galaxies. If we assume that Gaia will detect all lensing events which images show a flux ratio smaller than  $R$ , the probability that a

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background QSO at a redshift  $z_s$ , with an apparent magnitude  $b$ , undergoes a gravitational lensing event due to a foreground SIS galaxy with a luminosity  $L$  and a redshift  $z$  near its line-of-sight, is given by

$$\frac{d\tau}{dzdL} = n(L, z) \Sigma_{geom} B \frac{cdt}{dz} \quad (2)$$

where  $n(L, z)$  represents the density of deflectors with a luminosity  $L$  in the comoving reference frame at redshift  $z$ ,  $\Sigma_{geom}$  is the geometric cross section associated with the gravitational lensing event,  $B$  is the amplification bias correction factor and  $\frac{cdt}{dz}$  is an infinitesimal light-distance element in the comoving reference frame. We adopt the standard  $\Lambda$ CDM flat cosmology with  $\Omega_\Lambda = 0.73$  and  $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

The geometrical cross section  $\Sigma_{geom}$  is defined as the area, in a plane at redshift  $z$ , in which the presence of a deflector leads to the formation of multiple images of the background source. In the case of a SIS deflector,  $\Sigma_{geom}$  is given by

$$\Sigma_{geom} = \frac{16\pi^3 \sigma^4}{c^4} \left( \frac{D_{DS} D_{OD}}{D_{OS}} \right)^2 \quad (3)$$

where  $c$  is the speed of light, and where  $D_{DS}$ ,  $D_{OD}$  and  $D_{OS}$  are the different angular-distances between the source ( $S$ ), the deflector ( $D$ ) and the observer ( $O$ ).

A gravitational lensing event leads to an amplification of the background source. As we are interested in the probability for a source with a given apparent magnitude to be gravitationally lensed, we must take into account this brightening of the lensed object. This is done through the amplification bias correction factor  $B$  in Eq. (2).  $B$  depends on the differential number counts of QSOs as a function of their apparent magnitude  $b$ , which is represented by the QSO differential number counts function (DNCF). Boyle et al. (1988) have shown that the QSO DNCF is well modeled by a double power law

$$\frac{dN}{db} \propto \begin{cases} 10^{-\alpha(b-b_{crit})} & \text{if } b < b_{crit} \\ 10^{-\beta(b-b_{crit})} & \text{if } b \geq b_{crit} \end{cases} \quad (4)$$

where  $b$  is the QSO magnitude,  $\alpha$  and  $\beta$  are the slopes of the bright and faint ends, and where  $b_{crit}$  is the so-called critical magnitude. Claeskens (1999) has derived an analytical expression for  $B$  as a function of the DNCF parameters, that we shall use in the present calculations.

In Eq. (2), the comoving spatial density  $n(L, z)$  of deflectors with a luminosity  $L$  at redshift  $z$  might be expressed by means of the Schechter Luminosity Function  $\Phi_{Sch}(L)$ ,

$$n(L, z) = (1+z)^3 \Phi_{Sch}(L) \quad (5)$$

As the early-type galaxies are much more efficient in creating multiple image lens systems, we only consider the early-type galaxy density when calculating the lensing event probability.

The gravitational lensing optical depth  $\tau$  associated with a source is obtained by integrating Eq. (2) over the deflector luminosity and redshift. Assuming a constant spatial comoving density of SIS deflectors and using the Faber-Jackson relation to express  $\Sigma_{geom}$  as a function of the deflector luminosity  $L$ , we obtain

$$\tau = \frac{16\pi^3}{c^4} \Phi_* \sigma_*^4 \Gamma(1 + \alpha + 4\gamma^{-1}) B \int_0^{z_s} (1+z)^3 \frac{cdt}{dz} \left( \frac{D_{od} D_{ds}}{D_{os}} \right)^2 dz \quad (6)$$

where  $\sigma_*$  and  $\gamma$  represent respectively the characteristic velocity dispersion and the exponent of the Faber-Jackson relation, and  $\Gamma$  is the gamma function.

Relation (6) permits to calculate, for a source with known redshift and magnitude, the probability to undergo a gravitational lensing event. It requires the knowledge of the DNCF of the QSO population detected by Gaia, as expressed by Eq. (4). As the DNCF is closely related to the Luminosity Function (LF), we first derive in section 3.1 the LF for the QSOs detected by Gaia and explain how we simulate the mock catalogue in section 3.2. Then in section 3.3 we derive a double power law expression for the DNCF.

### 3. QSO mock catalogue

In this section, we estimate the LF in the  $G$  photometric band for the QSO population to

be detected by Gaia and we use it to simulate a mock catalogue.

### 3.1. QSO LF

Based on the SDSS DR3, Richards et al. (2006) have established the QSO LF in the SDSS  $i$  band, expressed in terms of the absolute magnitude  $M_i$ .

Although the QSO LF is often parametrized by a standard double power-law, the limiting magnitude of the SDSS survey is such that at most redshifts, objects fainter than the characteristic luminosity of the break are not detected. Therefore, the double power law was not justified. The authors have thus determined a LF of the form

$$\Phi_{QSO} = \Phi_{QSO}^* 10^{A_1 [M_i - P_3(\xi)]} \quad (7)$$

where  $P_3(\xi)$  is a third order polynomial in  $\xi = \log\left(\frac{1+z}{1+z_{ref}}\right)$ .

To convert the LF found by Richards et al. (2006) into the Gaia photometric system, we assume that the shape of the LF is expressed as in Eq. (7) where we convert the  $M_i$  into the absolute magnitude  $M_G$ , in the Gaia photometric  $G$  band.

The absolute magnitude conversion is derived, as a function of redshift, using the color transformation relation between the  $g$  SDSS and the  $G$  Gaia photometric systems (see Slezak & Mignard 2007). For the  $(g-i)$  and  $(g-r)$  colors, we use the mean color redshift evolution data points derived by Slezak & Mignard (2007) from the SDSS DR5 data. We fit these with a sixth order polynomial. As we know the  $(g-i)$  and  $(g-G)$  redshift evolution, we calculate the mean shift  $(G-i)$  as a function of the redshift and we convert the absolute magnitude in Eq. (7) using  $M_G = M_i + (G-i)$ .

### 3.2. QSO catalogue generation

For redshifts ranging from 0 to 5, we divide the space in shells with a redshift width of 0.01 in which we randomly generate a QSO population in accordance with the LF obtained in section 3.1, with  $M_G$  ranging between  $-29$  and

$-22$ . The apparent magnitude  $G$  of each source is calculated using

$$G = M_G + 25 + 5 \log(D_l) + K(z) \quad (8)$$

where  $D_l$  is the light-distance, and  $K$  is the K-correction for which we only consider the continuum part of the QSO spectra. Following Richards et al. (2006), the K-correction is calculated by

$$K(z) = -2.5(1 + \alpha_{cont}) \log(1+z), \quad (9)$$

where  $\alpha_{cont} = -0.5$  is the slope of the continuum part of the spectrum. We keep all sources brighter than  $G = 20$ , the limiting magnitude of the survey.

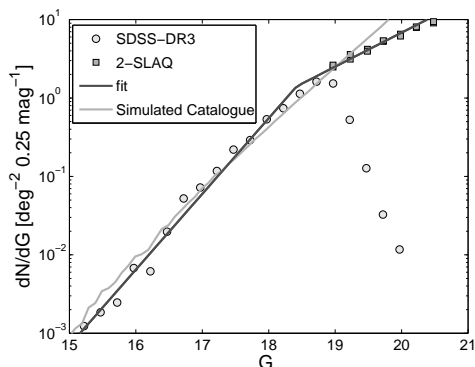
### 3.3. QSO DNCF

Boyle et al. (1988) showed that the DNCF of QSOs is well represented by the dual power law expression (4). Furthermore, we need this dual power law expression to be able to use the analytical expression for the amplification bias  $B$ .

Normally, the DNCF should be derived from the LF of the QSOs. Nevertheless, as mentioned in section 3.1, due to the incompleteness of the SDSS catalogue for faint objects, the LF determined by Richards et al. (2006) is the single power law expression (7), which is accurate enough for a mock QSO catalogue generation purpose, but which is not sufficient for calculating individual lensing probability which are very sensitive to the DNCF parameters  $\alpha, \beta, b_{crit}$ . Consequently, we need to estimate the DNCF by other means.

Richards et al. (2006) have estimated the QSO DNCF in the SDSS  $g$  band on the basis of the SDSS DR 3 QSO Catalogue. This survey is complete down to  $g \simeq 19$ . For the fainter objects, we took data points from the DNCF from the 2-SLAQ survey (also given in Richards et al. 2006).

Assuming that the shape of the DNCF remains unchanged when expressed in terms of Gaia  $G$  magnitudes, we convert the  $g$  magnitudes to  $G$  ones in the DNCF data from Richards et al. (2006), using the photometric system color transformation from Jordi et al.



**Fig. 1.** Differential number counts function derived by Richards et al. (2006) on the basis of the SDSS DR3 and 2-SLAQ data (converted to Gaia  $G$  magnitude). The dark grey line is the two power law DDCF fit of Richards et al. (2006) data. For information, the DDCF of the mock catalogue if also depicted (light grey line).

(2010) and a  $g - i$  mean value for the QSOs in the SDSS DR7. The DDCF data converted to  $G$  magnitudes are shown in Fig. (1) where the SDSS data incompleteness is clearly visible for sources fainter than  $G \sim 19$ .

The SDSS and 2-SLAQ DDCF converted to  $G$  magnitudes are fitted by the two power law expression (4) of the DDCF, taking as initial guess of the critical magnitude the value proposed by Narayan (1989) ( $B_{crit} \approx 19.15$  in the Johnson-Cousin photometric system, converted to  $G$ ). The best fit parameters are

$$\begin{aligned} b_{crit} &= 18.42 \\ \alpha &= 0.9656 \\ \beta &= 0.4338 \end{aligned}$$

The DDCF for the  $G$  magnitude as well as the fitted double power law are shown in Fig. (1). As a comparison, the QSO DDCF derived from the simulated Gaia catalogue is also plotted in Fig. (1) where its single power law behaviour clearly appears, as expected from the LF shape used for the mock catalogue generation.

## 4. Results and conclusions

Using relation (6) and the DDCF parameters derived in section 3.3, we calculate the lensing optical depth associated with all the sources in the mock catalogue. The mean lensing probability  $\tau_{tot}$  over the whole simulated QSO population is found to be

$$\tau_{tot} = 0.0059 \quad (10)$$

$\tau_{tot}$  thus represents the fraction of QSOs in the population detected by Gaia, expected to undergo a gravitational lensing event with the formation of multiple images. As Gaia should detect about 500,000 QSOs, this leads to 2950 expected lens systems in the QSO population.

In this work, there was no consideration of the finite angular resolution of Gaia. Although taking it into account would somehow lower the number of detected lenses, the very large number of lenses expected in the Gaia QSO population should motivate the Gaia community to develop an automatic detection pipeline in order to securely identify most of the gravitationally lensed QSOs.

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