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Magnetic Susceptibility of Neutron Matter.

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Abstract. – Binding energy and single-particle properties of uniform polarized neutron matter are calculated in the frame of Brückner theory. The magnetic susceptibility of neutron matter is extracted. No ferromagnetic transition is predicted in the density domain investigated.

The knowledge of neutron matter properties is crucial to understand the equilibrium properties, the composition and the evolution of neutron stars, as well as the relationship between their properties and the asymmetric nuclear-matter equation of state. In particular, the magnetic susceptibility of neutron matter may play an important role in neutron star physics. Pulsars, believed to be rapidly rotating neutron stars, possess intense magnetic fields, up to a few 10^{12} G [1] and it is possible that part of this field is sustained by a permanent magnetized phase inside the neutron star, as suggested by permanent relic magnetic fields in binary millisecond pulsars [2]. Moreover, a partly magnetized neutron matter could influence the evolution of a neutron star. It is shown in ref. [3] that the presence of a magnetized phase can change the neutron opacity, which in turn determines the cooling of the star [4].

Previous calculations of the neutron matter magnetic susceptibility [5-9], based on static potentials, seem to indicate a possible ferromagnetic transition at a density, which may be as small as $1.5\rho_0$ or as high as $10\rho_0$. A recent calculation [10], using a relativistic mean-field approach, indicates a transition around $\sim 10\rho_0$. However, it is not clear whether this result originates from relativistic effects or from the σ-ω model for the meson field, which is now sometimes considered as not very realistic.

Here, we want to extend our previous study of neutron matter [11], based on Brückner theory to investigate the problem of magnetic susceptibility at zero temperature and in a range of densities which will be discussed below.

Introducing a polarization parameter $\varepsilon = (N(+) - N(-))/(N(+) + N(-))$, where $N(\pm)$ is the number of spin-up (-down) neutrons, one can define the magnetic susceptibility as

$$\chi = \frac{\mu_n^2 \rho}{(\partial^2(E/A)/\partial\varepsilon^2)_{\varepsilon=0}}, \tag{1}$$

where μ_n is the neutron magnetic moment, ρ the baryon density and $E/A\,(\rho, \varepsilon)$ is the energy per particle of neutron matter at density ρ and polarization ε. Relation (1) results from the usual definition of χ (magnetization divided by the value of the applied magnetic field H) and the determination of the magnetization by minimizing (with respect to ε) the total energy of the system at fixed H. Around $\varepsilon = 0$, the latter has the following form:

$$W = \frac{E}{A}(\rho, 0) + \frac{1}{2} a\varepsilon^2 - \mu_n \varepsilon H, \tag{2}$$

where a is a constant. There are good reasons to believe that the energy of neutron matter E/A (ρ, ε) remains a quadratic function of ε, even if $\varepsilon \to 1$. Actually, this seems to be the case for any kind of polarization of nuclear matter, be it in spin or in isospin direction. In particular, a recent detailed calculation of the binding energy of nuclear matter for any value of the isospin parameter [12] shows a perfect quadratic dependence up to maximum isospin asymmetry. The energy variation needed to spin-polarize nuclear matter being roughly the same as to polarize it in isospin [13], the same functional dependence is expected in the spin asymmetry parameter. Therefore, it is reasonable to believe that the function E/A (ρ, ε) for neutron matter is quadratic in ε. We will adopt this hypothesis as our starting point. Therefore the quantity a can be calculated by comparing E/A $(\rho, \varepsilon = 0)$ (neutron matter) and E/A $(\rho, \varepsilon = 1)$ (totally magnetized neutron matter).

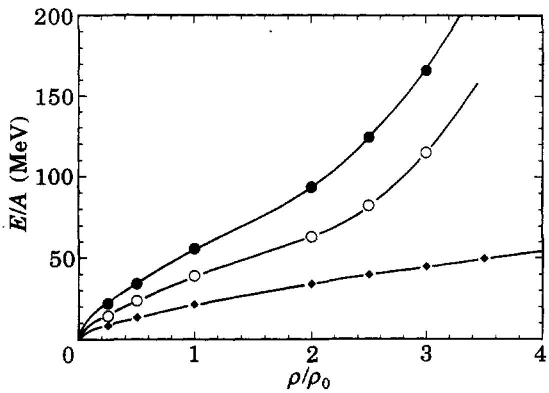

Fig. 1.

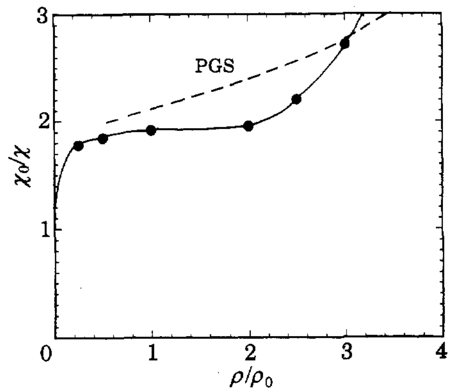

Fig. 2.

Fig. 1. – The full dots represent the energy per baryon of totally polarized neutron matter, calculated in first-order Brückner-matrix approximation, as a function of baryon density ρ ($\rho_0 = 0.17\ \mathrm{fm}^{-3}$). The open dots give the energy per baryon necessary to align all spins in neutron matter and the lozenges give the kinetic-energy contribution to the latter quantity.

Fig. 2. – Quantity χ_0/χ, as calculated in this work (dots). The dashed curve corresponds to the results of ref. [8].

The quantity E/A $(\rho, \varepsilon = 1)$ is shown in fig. 1, as well as the energy necessary to align all spins, *i.e.* $\Delta E = E/A(\rho, \varepsilon = 1) - E/A(\rho, \varepsilon = 0)$, the difference with neutron matter binding energy. These calculations have been performed in lowest-order Brückner theory, using a realistic two-body interaction, namely the Paris potential and an effective three-body interaction, described in ref. [11], similar also to the one used in ref. [14]. By comparison, we also show the kinetic-energy contribution to ΔE, which is nothing but the energy required to align all spins in a free Fermi gas. One can see that for $\rho \lesssim 3\rho_0$, the latter contribution is dominating the one coming from interaction energy.

It is usual to discuss the magnetic susceptibility in comparison to the (nonrelativistic) free Fermi gas one:

$$\chi_0 = \frac{3\mu_n^2 \rho}{2T_F}, \tag{3}$$

where $T_F = \hbar^2 k_F^2 / 2M$ is the Fermi energy. The onset of (liquid phase) ferromagnetism would be signalled by a vanishing χ_0/χ ratio. In our case, the latter is displayed in fig. 2. There is no ferromagnetic transition for ρ smaller than $3\rho_0$ and no sign of its appearance at densities slightly above this value.

To assert our results and to understand the origin of the decreasing susceptibility with density, we looked at the average field felt by a neutron. It is given in fig. 3 for several values of the density. The most important feature is the relatively small magnitude of the mean field (compared for instance to the nuclear-matter case at the same density). It is much smaller than the Fermi energy, which means that the neutrons are weakly interacting. This is an *a posteriori* justification of using Brückner theory at densities as large as $3\rho_0$. Relativistic kinematical corrections will be small since the Fermi energy at the latter density is ~ 180 MeV only. Thus, the real and only assumption of our approach lies in using static potentials, allowing nevertheless for medium-induced short-range (Brückner-type) renormalization, but not for medium renormalization of the meson fields mediating the nucleon-nucleon interaction.

The smallness of the average field results from the small number of participating partial waves, but also from some cancellation, as can be seen from fig. 4, which shows the contributions of the most important partial waves to the interaction energy. These

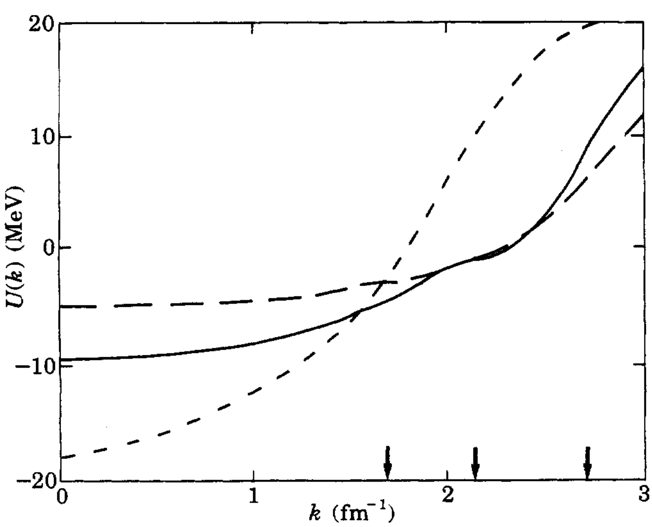

Fig. 3.

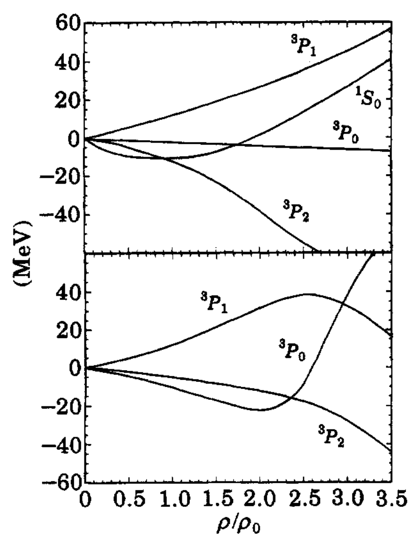

Fig. 4.

Fig. 3. – Average potential felt by a neutron of momentum k in totally polarized neutron matter at baryon density ρ equal to $0.5\rho_0$ (long dashes), ρ_0 (full curve) and $2\rho_0$ (short dashes), respectively. The arrows indicate the respective values of the Fermi momentum.

Fig. 4. – Contributions to the binding energy per baryon of the most important nucleon-nucleon partial waves, as functions of baryon density ρ, in neutron matter (upper part) and totally polarized neutron matter (lower part).

contributions are nothing but the average over the Fermi sphere of half of the contributions of the respective partial waves to the mean field U.

From fig. 4, we see that around ρ_0, the basic contribution to the polarization energy (besides the kinetic-energy part) is due to the disappearance of even partial waves, basically the $^1S_0$ wave. At larger density, it comes from the modification of the $^3P_2$ contribution partially compensated by those of the $^3P_0$ and, to a lesser extent, $^3P_1$ waves. These modifications originate from the fact that a larger domain of relative momentum is involved, but also, and mainly, from the Brückner renormalization of the effective interaction.

In conclusion, in the frame of our approach, a ferromagnetic phase is not expected at, say $\rho \lesssim 4\rho_0$. This result confirms those of ref. [6] (although a smaller range of density is explained in this work) and of ref. [8], as can be seen from fig. 2. In the last work, a variational approach is used, which, in principle, is better suited than ours to the study of dense matter. However, it seems that, as for the neutron matter case [11], the numerical results of both methods (for realistic potentials) are close to each other. Our results are at variance with those of ref. [5] and [7], where too crude interaction models have probably been used: hard spheres for ref. [5] and Scott-Moszkowski in even waves only for ref. [7]. Finally, concerning the difference between the results of ref. [10] and ours, it is hard to determine whether it is due to relativistic effects or to the absence of medium renormalization of the interaction in the simple Hartree-Fock model adopted in ref. [10].

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