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SPIN AND SPIN-ISOSPIN POLARIZED NUCLEAR MATTER*

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Abstract: Properties of $(N = Z)$ nuclear matter are investigated through a Brueckner method using the Paris potential for two special configurations, one when all spins are pointing in the same direction, the other when neutron spins are pointing upward and proton spins downward. The results of the calculations are used to evaluate the energy of the isoscalar giant spin dipole vibrations and the value of the spin-spin part of the nucleon-nucleus optical-model potential.

1. Introduction

The properties of ordinary (spin and isospin) symmetric nuclear matter are fairly well understood both in nonrelativistic [1,2]) and in relativistic approaches [3-5]), although the physical picture of the underlying one-particle fields are quite different. Nor are the fine details of the saturation mechanism the same in both cases. In the nonrelativistic approaches the lack of saturation when only two-body forces are introduced is corrected by the introduction of three-body forces. In the relativistic approaches, the extra saturation is provided by the introduction of a negative-energy component of the Dirac spinors. It is not yet clear whether these two effects are completely disconnected or not. The isospin-asymmetric and isospin-saturated (neutron) matter is equally well described in both approaches, as indicated by recent works [6,7]).

Here, we want to investigate the properties of spin-asymmetric matter. This is the first approach to this system, except for a rather old work (see below). As in our previous work [7]), we will mainly concentrate on the spin saturated matter (all spins pointing in the same direction), since the usual properties are expected to depend linearly or quadratically on the spin asymmetry, as this seems to be the case for isospin asymmetric matter [5]). Not very much attention has been paid to the spin degrees of freedom because of the lack of experimental probes. The spin-asymmetry energy is believed to be related to the spin giant resonances, in close analogy with the connection which exists between the (isospin) asymmetry energy and the energy

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of the electric giant dipole resonances. So far, there are very scarce measurements, if any, of the spin resonances (see below). Another quantity which can be related to the spin degrees of freedom is the spin–spin part of the optical potential, which has been determined, rather badly, in some cases. Another motivation for our work is the possibility that this state of nucleon matter might be described quite differently by relativistic and nonrelativistic approaches, as the Dirac equation is often opposed to the Schrödinger equation when nuclear spin degrees of freedom are at work [8]. Let us notice that relativistic approaches to this topic do not exist up to now.

We thus here extend our previous works [2,7]) by applying our Brueckner approach to this state of nucleon matter. For completeness, we also investigate another configuration in which all neutrons are pointing their spin upward whereas protons are pointing in the opposite direction (with equal number of neutrons and protons). The latter is depicted in fig. 1, which also establishes our vocabulary. (We will also use "phase" instead of "matter".) Systems very similar to our spin-isospin matter have been discussed rather extensively, in connection with pion condensation.

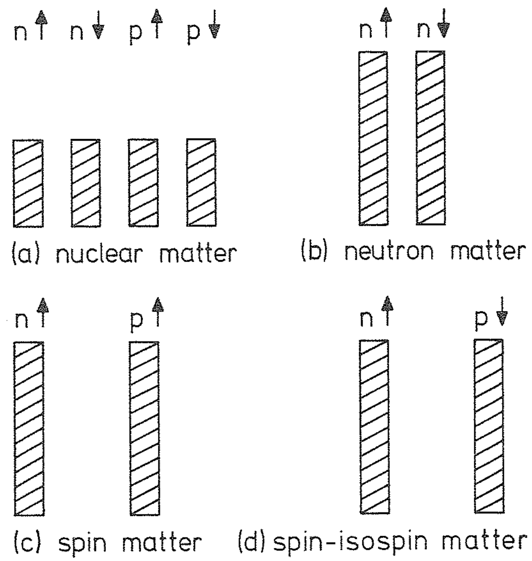

Fig. 1. Schematic representations of nuclear matter (a), of neutron matter (b) and of the two configurations calculated here (with equal nucleon density). See text for detail.

This paper is organized as follows. In sect. 2, we briefly establish our notation and present our results. In sect. 3, we discuss the connection with available experimental indications and the possible modification of the effective interactions when going from one phase to the other. Sect. 4 contains our conclusion.

2. Spin and spin–isospin saturated matter

2.1. THEORETICAL BACKGROUND

We are basically interested in a uniform medium of density ρ, with particles occupying level of momentum k with probability $n(k)$:

$$\rho = \nu \int \frac{d^3k}{(2\pi)^3} n(k) , \tag{2.1}$$

where ν is the degeneracy (4 for nuclear matter, 2 for the two phases studied below). In the Brueckner approach, the central quantity is the g-matrix defined as

$$g = v + v\frac{Q}{e} g \tag{2.2}$$

where v is the two-body potential [see ref. [9]) for details]. Here we used the Paris potential. The binding energy per particle can then be expressed as a perturbation series in terms of the g-matrix. As usual, we will limit ourselves to the first-order expression

$$\frac{U}{A} = \nu \int \frac{d^3k}{(2\pi)^3} n(k) \left\{ \frac{\hbar^2 k^2}{2m} + \tfrac{1}{2} U(k) \right\} , \tag{2.3}$$

where $U(k)$ is the one-particle mean field

$$U(k) = \nu \int \frac{d^3k'}{(2\pi)^3} n(k') \langle kk'|g|\widetilde{kk'}\rangle . \tag{2.4}$$

The latter is a complex quantity above the Fermi level at zero temperature. In the present work, we will restrict ourselves to this case, for which one has $n(k) = 1$ if $k < k_F$, the Fermi momentum and $n(k) = 0$ otherwise.

2.2. RESULTS

The binding energy per particle of the spin and spin–isospin matter is shown in fig. 2, in comparison with the one of two other (nuclear matter and neutron matter) phases. For each of these phases, the conventional three-body forces of refs. [10,11]) is included. As explained in ref. [2]), this is not sufficient to ensure proper saturation in nuclear matter. The latter requires a small phenomenological additional three-body force, as in variational calculations [1]). Since here we are basically interested in differences between various phases, this feature is rather unimportant for our purpose.

The remarkable result is that spin matter behaves very much like neutron matter. Similarly to the latter case, we may define a spin-asymmetry energy $E_s(\rho)$. Let γ be the spin asymmetry for a general (otherwise isospin symmetric, $N = Z$) matter:

$$\gamma = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)} . \tag{2.5}$$

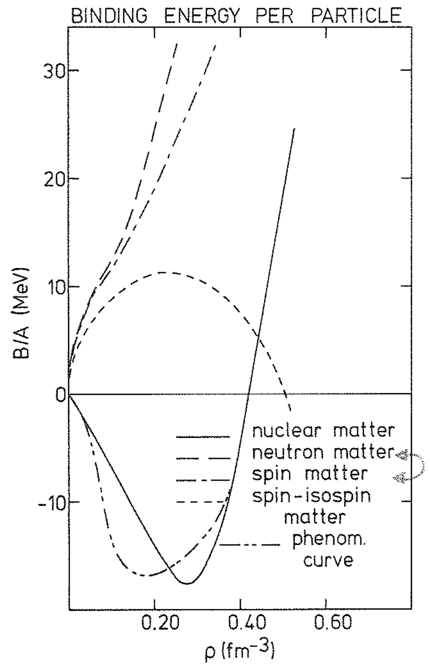

Fig. 2. Binding energy of the four phases of nucleon matter discussed in this work. For nuclear matter, the full curve represents the result of a calculation, using two-body force and usual microscopically calculated three-body forces. The dashed double dotted curve is obtained by adding a purely phenomenological additional three-body force [see ref. [2]) for discussion].

For such a system the energy per particle is expected to follow a quadratic law (as for the isospin-asymmetric matter [6,12]))

$$\frac{U}{A}(\rho, \gamma) = \frac{U}{A}(\rho, 0) + E_s(\rho)\gamma^2 . \tag{2.6}$$

Of course, in this case,

$$E_s(\rho) = \frac{U}{A}(\rho, 1) - \frac{U}{A}(\rho, 0) \tag{2.7}$$

where $\gamma = 1$ corresponds to the spin-saturated matter. The value of $E_s(\rho)$ is given in fig. 3, as well as the contribution which comes from the kinetic energy alone. The ordinary (isospin) asymmetry energy E_T calculated in the same conditions [7]) is shown for comparison.

The binding energy of the spin-isospin matter is given in fig. 2. We recall that this phase is composed by half of neutrons with spin up and by half of protons with spin down. Its behaviour is strikingly different from that of the spin- and neutron matter. We predict that nuclear matter is globally unstable when $\rho \gtrsim 2.5\rho_0$, when the spin-isospin phase has a lower energy. However, if one calculates the

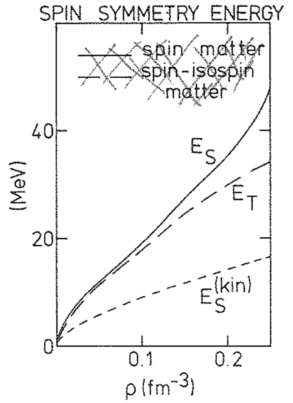

Fig. 3. Spin-symmetry energy $E_S(\rho)$ (eq. (2.6), full curve) of nuclear matter and its kinetic-energy contribution (short dashes). The isospin-symmetry energy $E_T(\rho)$ (long dashes) is shown for comparison.

sound velocity in the latter phase, one observes that it becomes imaginary when $\rho \geqslant 0.25\ \text{fm}^{-3}$. For these densities, the spin–isospin matter could nevertheless exist with a non-uniform density pattern (liquid crystal), owing to anharmonicities, with a lower energy.

2.3. SINGLE-PARTICLE POTENTIAL

For spin-asymmetric ($N = Z$) matter, the single-particle potential $U(k)$ felt by a travelling nucleon (of momentum k) should depend upon the projection s_z $(=\frac{1}{2}\sigma_z)$ of its spin along a given direction and upon the polarisation degree γ (eq. (2.5)) of the matter. In close analogy with the isospin case [7]) and with the Lane model [13]), we write

$$U(k,\gamma) = U_0(k) + \tfrac{1}{4}\sigma_z \gamma U_\sigma(k)\,. \tag{2.8}$$

For the spin–isospin asymmetry, we may define the following polarisation degree as

$$\delta = \frac{N(\mathrm{n}\uparrow + \mathrm{p}\downarrow) - N(\mathrm{n}\downarrow + \mathrm{p}\uparrow)}{N(\mathrm{n}\uparrow + \mathrm{p}\downarrow) + N(\mathrm{n}\downarrow + \mathrm{p}\uparrow)}\,. \tag{2.9}$$

This quantity is equal to zero for nuclear matter and to one for the spin–isospin phase. Let ζ be equal to +1 if the nucleon is either (n↑) or (p↓) and −1 if it is (n↓) or (p↑). In matter with an equal number of neutrons and protons and an equal number of spins pointing upward or downward, the single-particle potential felt by a nucleon can be written as

$$U(k,\delta) = U_0(k) + \tfrac{1}{4}\zeta\delta U_{\sigma\tau}(k)\,. \tag{2.10}$$

In expressions (2.8) and (2.10), U_0 is the single-particle potential corresponding to nuclear matter. The latter and the single-particle potentials for spin matter $U(k, \gamma=1)$ and for spin-isospin matter $U(k, \delta=1)$ are given in fig. 4. Inside the Fermi sea, one has roughly (for $\rho=\rho_0$) $U_\sigma \approx 160$ MeV and $U_{\sigma\tau} \approx 80$ MeV. Our value for U_σ (and U_τ) qualitatively agrees with the prediction of refs. [12,14]), but we obtain a somehow smaller value for $U_{\sigma\tau}$. This may be due to the better description of the tensor force in the Paris potential.

3. Discussion

3.1. SPIN-SYMMETRY ENERGY

In a hydrodynamical model for collective motion [15,16]), the energy E_{SGR} of the spin giant resonance ($\Delta S=1$, $T=0$) in nuclei, which is then viewed as oscillations of nucleons with spin up against nucleons with spin down, can be related to the spin-symmetry energy $E_s(\rho_0)$. Using our prediction for the latter, one has

$$E_{SGR} \approx 74 A^{-1/3}\ \text{MeV}, \tag{3.1}$$

which, of course, is very close to the giant electric dipole ($\Delta S=0$, $\Delta T=1$) resonance. The giant spin vibration seems to have been seen in some pion-induced reactions [17]), but will be studied much better with polarized beams and polarimeter detectors [18]).

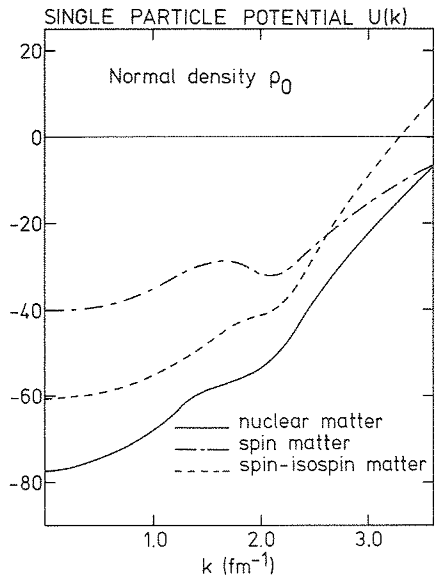

Fig. 4. Single-particle potential $U(k)$ (eq. (2.3)) for three different phases.

3.2. EFFECTIVE INTERACTIONS IN VARIOUS PHASES

The asymmetry energy E_s (as well as E_T) is the sum of two contributions, one coming from the kinetic energy, $E_s^{(\mathrm{kin})}$, and the other coming from the interaction energy, $E_s^{(\mathrm{int})}$. Evidently, if the interaction was both spin- and isospin-independent, one would have $E_{s,T} = E_{s,T}^{(\mathrm{kin})}$. As shown by fig. 3, $E_s^{(\mathrm{kin})}$ is quite smaller than E_s. Therefore, the interactions play an important role in the spin-asymmetry energy. The approximate equality $E_s \approx E_T$, is thus fortuitous and depends from the detail of the nucleon–nucleon interaction.

One may wonder whether the effective interaction is changing with the spin-asymmetry parameter γ or with the isospin counter part $\beta = (N-Z)/A$. The answer to this question is contained in fig. 5, which shows the contribution to the binding energy of various channels. Clearly when going from nuclear matter to spin matter, the effective interaction is drastically changed in two particular channels, namely the $^3S_1$ and the $^3D_2$ channels, at least for $\rho \gtrsim \rho_0$. Up to this value, the effective interaction does not change importantly when going from nuclear matter ($\gamma = 0$) to spin matter ($\gamma = 1$).

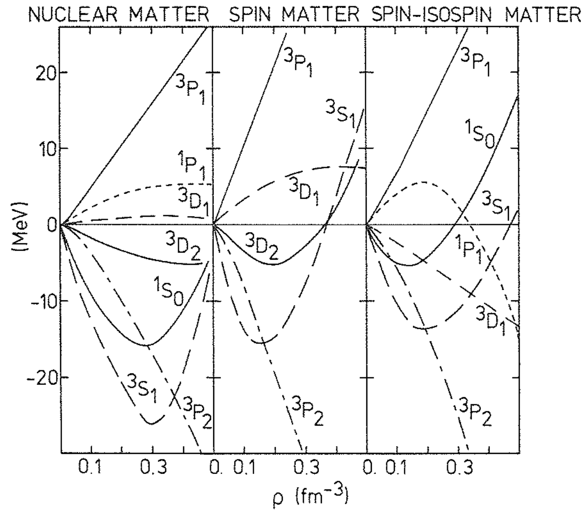

Fig. 5. Contributions to the binding energy (second term in eq. (2.3)) of various channels for three different phases. See text for detail.

When going from nuclear matter to spin-isospin matter, the effective interaction is changing in channels $^1S_0$, $^3S_1$, $^3D_1$ and $^1P_1$. We stress here that the modification of the effective interaction is solely due to Brueckner-type renormalization, which changes because of the composition of the phases and the modification of the Fermi spheres.

The spin-isospin phase we studied here is a kind of pion-condensed phase. It may be considered as a spin-isospin ordered liquid crystal with an infinitely large wavelength. The problem of pion condensation has given rise to a huge amount of work [see refs. [19,20]) for a review]. In most of the cases, it is studied through the pion self-energy inside nuclear matter. The general consensus is that pion condensation should occur at large density (3–$5\rho_0$), especially due to the reduction of the one-pion contribution induced by short-range correlations and due to a modification of the particle–hole interaction as a result of the change of the pion propagator itself [21]). On the other side, the problem has not received very much attention when attacked from the point of view of the liquid crystal configurations of nuclear matter. It is not yet clear whether the two approaches are equivalent. Rather old works [22–24]) have looked to possible spin–isospin orderings. Recently, Blümel and Dietrich [25]) have looked to spin–isospin ordering in single-particle level occupation in finite nuclei. They observed that "pion-condensed" states may occur at not very high energy. Their calculation in infinite matter using effective forces except for pion exchange can be compared to our calculation (for $\kappa = 0$ in their vocabulary). We find that for $\rho = \rho_0$, the excitation of the spin–isospin phase requires $\sim 3.5\ \mathrm{MeV \cdot fm^{-3}}$, whereas they found $\sim 1.6\ \mathrm{MeV \cdot fm^{-3}}$. This presumably comes from the readjusted pion exchange in their cases and more importantly from the lack of medium corrections of the nucleon–nucleon effective forces especially of its short-range part.

We stress that our calculation, as well as the work of ref. [25]), neglects the medium correction of the effective interaction in the pion-exchange channel, due to the modification of the pion propagator itself (only changes due to Brueckner renormalization are taken into account here). Our predictions for the onset of instabilities should then be understood as lower bounds. For $\rho \lesssim \rho_0$, this medium correction is however not important, and our results are nevertheless valuable to study the spin–spin part of the nucleon–nucleus average potential (see next section). They may also be compared with the recent work of ref. [25]), which also neglects these medium corrections interacting spin–isospin ordering at normal nuclear density.

3.3. COMPARISON OF U_σ WITH EXPERIMENT

In the local-density approximation, the mean field experienced by a nucleon inside a nucleus can be written as

$$U = U_0 + U_\tau \frac{\boldsymbol{t} \cdot \boldsymbol{T}}{A} + U_\sigma \frac{\boldsymbol{s} \cdot \boldsymbol{\Sigma}}{A} + U_{\sigma\tau} D, \qquad (3.2)$$

where $\boldsymbol{\Sigma}$ is the expectation value of the target spin operator in the ground state. The quantity D can be written as

$$D = \frac{1}{4A} \left\langle \sum_i \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}_i \boldsymbol{\tau} \cdot \boldsymbol{\tau}_i \right\rangle, \qquad (3.3)$$

where i runs over the nucleons inside the target and where the brackets mean an average over the ground state. This quantity cannot be written simply in terms of separable quantities as the first three terms. In the spin–isospin matter it simply reduces to $\frac{1}{4}\zeta\delta$. It obviously vanishes in isospin-symmetric or spin-symmetric nuclei. As underlined in ref. [14]), the analysis of U_σ is complicated by the fact that a spin unsaturated nucleus has also an isospin different from zero. As long as the spin is concerned, only a valence nucleon in a subshell j is playing a role. Therefore, usual angular momentum algebra [26]) tells that if the spin $\boldsymbol{I}=\boldsymbol{j}$ of the nucleus is not changed, the operator $\boldsymbol{s}\cdot\boldsymbol{s}_i$ can be replaced as

$$\boldsymbol{s}\cdot\boldsymbol{s}_i\to a\boldsymbol{s}\cdot\boldsymbol{I}\,, \tag{3.4}$$

with

$$a=\begin{cases}1/2j & \text{if } j=l+\frac{1}{2}\,,\\ -\dfrac{1}{2(j+1)} & \text{if } j=l-\frac{1}{2}\,.\end{cases} \tag{3.5}$$

Usually the phenomenological analyses are done with a restricted form

$$U=U_0+U_\tau\frac{\boldsymbol{t}\cdot\boldsymbol{T}}{A}+U_{\mathrm{ss}}\frac{\boldsymbol{s}\cdot\boldsymbol{I}}{A}\,. \tag{3.6}$$

Therefore, the quantity U_{ss} should be identified to

$$U_{\mathrm{ss}}=(U_\sigma\pm U_{\sigma\tau})a\,, \tag{3.7}$$

where the plus sign stands when the incoming nucleon and the valence nucleon are both protons or both neutrons and the minus sign otherwise. The experimental determinations of U_{ss} are rather scarce [27-32]). It seems that no effort has been made in the last decade. Therefore, we have only at our disposal rather old and uncertain values of U_{ss}. The uncertainty does not come from the difficulty of the measurements only. The analysis is blurred by the presence of a possible spin tensor term [28]) and by the compound-nucleus contribution [33,34]). The data are summarized in table 1, where we compare with our predictions. The most studied case concerns n+^{59}Co, for which one has a minus sign in eq. (3.7). The determination of U_{ss} appeared to be quite imprecise and its value (and even its sign) seems to depend very much on the energy. Using mean values of U_σ and $U_{\sigma\tau}$ (over the Fermi sea) we obtain a positive and rather large U_{ss}. If we take into account the k-dependence of the potentials U, and if we relate the appropriate value of k to the incoming energy by standard methods [35]), we obtain a smaller value, which results from the variation above the Fermi sea. It seems that in ^{59}Co the additional spin effects (coupling with the surface collective states, compound nucleus contribution, core polarization as studied in ref. [36])) are roughly sufficient to counterbalance the bulk spin–spin effects. In the other cases in table 1, the comparison is more meaningful, since U_σ and $U_{\sigma\tau}$ add to each other. We obtain reasonable agreement in proton scattering on ^{27}Al and ^{59}Co. The last case in table 1 is not very clear. In ref. [32]), the same experimental value is quoted for Yt and Rb. Since the orbits correspond to $j=l-\frac{1}{2}$ and $l+\frac{1}{2}$,

TABLE 1

Value of the quantity U_{ss}, as extracted from experiment for various systems and as predicted in our work*

System	Energy (MeV)	State (proton)	U_{ss} (exp)	U_{ss} (th) (MeV)
$n+{}^{59}Co$	0.3–1.0	$(1f_{7/2})^{-1}$	47–84	10.3
	1.0–8.0		(34–90) [27–29]	
	8–31		11.8 [30]	6.3–4.5*
	0.39–2.88			2.7 (ref. 36))
$p+{}^{27}Al$	18	$(1d_{5/2})^{-1}$	24 [31]	46
				28.8*
$p+{}^{59}Co$	47.5	$(1f_{7/2})^{-1}$	42 [31]	33
				20.5*
$p+{}^{89}Yt$	5	$(2p_{1/2})$	124 [32]	{∓77
$p+{}^{87}Rb$		$(2p_{3/2})^{-1}$		{∓52*

* The figures without an asterisk are obtained by using average values on the Fermi seas, while the asterisk indicates when the k-dependence has been taken into account. The prediction of ref. 36) is shown for comparison. The third column gives the state of the valence nucleon.

respectively, one would expect a change of sign. Information at our disposal does not allow us to judge the fitting procedure of ref. 32).

4. Conclusion

We have studied spin and spin–isospin saturated phases of nuclear matter and compared with our previous calculations of normal and isospin-saturated (neutron) matter. We stress that, for spin matter, there is only one calculation 14) prior to ours, but ours uses a more modern nucleon–nucleon potential, namely the Paris potential, and ensures a better self-consistency of the mean field ["continuous" choice, see ref. 9) for detail]. For the spin–isospin matter, our calculation constitutes a detailed microscopic approach, with the same features as above, of a possible spin–isospin ordering of nuclear matter. However, it neglects the modification of the effective interaction in the pion-exchange contribution, and thus its validity is probably limited to nuclear and subnuclear densities.

We found that spin-polarizing nuclear matter requires at least as much energy as isospin-polarizing it. As a consequence, one expects the $\Delta S=1$ giant resonances to lie more or less at the same energy as the $\Delta T=1$ ones. On the contrary the spin–isospin phase lies not so high in energy. It is globally unstable at density $\gtrsim 2\rho_0$, but it is unstable under density oscillations at densities slightly larger than ρ_0. We also calculated the spin and spin–isospin parts of the mean field. We compared with the phenomenological spin–spin interactions in nucleon–nucleus scattering. Our results agree satisfactorily with experimental data, since the latter are rather poorly determined.

Let us finally conclude with a remark. A great debate in nuclear physics is the necessity of using a relativistic dynamical scheme. It is not yet clear whether the latter is established. Since spin is intimately related to the Dirac equation, it would be interesting to know whether Dirac-Brueckner theory gives different results from ours for spin-saturated matter. The comparison is not yet possible, since the relativistic counterpart of our calculation is still to come.

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