# Nuclear Reaction Mechanisms: from Compound Nucleus to Multiple Scattering [†] [‡]

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## Abstract

The mechanisms of nuclear reactions from very low incident energy up to about a few GeV are reviewed. The energy range is divided into three domains corresponding roughly to three different regimes: the compound nucleus, the pre-equilibrium processes and the multiple scattering. The emphasis is put on the concepts at the basis of the understanding of these regimes. The main tools to evaluate cross-sections as well as their input data are briefly presented, but no detail, no comparison with experimental data are provided or discussed. Section 1 is a short reminder of nuclear reaction theory. Section 2 is devoted to the compound nucleus mechanism. Section 3 discusses the high energy regime and the validity of classical multiple scattering approach to this regime. Section 4 deals with the intermediate energy regime, where pre-equilibrium processes are at work. An attempt is made to clarify the various models that have been proposed for this type of reactions. Finally, a brief summary is contained in Section 5.

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## 1. GENERAL FEATURES

We recall some basic features of nuclear reaction theory. We refer to Ref. [1] for more detail. We consider two-body channels. Let $c$ be an indice (or collection of indices) which labels the various channels (including the momentum vectors of the particles) and $\chi_c$ the free wave function in this channel (a product of two plane waves, each normalized to a momentum delta function). For illustrative purpose we will consider nucleon channels, but most of the considerations made below can be extended to other channels. Let $\psi_c^+$ be the scattering wave function with incoming plane wave in channel $c$ and containing only outgoing waves in all exit channels. One can write

$$\psi_c^+ = \chi_c + \frac{1}{E - H_0 + i\varepsilon} V \chi_c, \tag{1.1}$$

where $H = H_0 + V$ is the total hamiltonian, the same for all channels (these formulae can be generalized if the separation of the hamiltonian depends upon the channels, which occurs when the partition of the system is different in different channels). One can define the $\mathcal{S}$-matrix by

$$\mathcal{S}_{c'c} = \langle \psi_{c'}^- | \psi_c^+ \rangle. \tag{1.2}$$

The $\mathcal{S}$-matrix is related to the $T$-matrix by

$$\mathcal{S}_{c'c} = \delta_{c'c} - i2\pi\delta\left(E_c - E_{c'}\right) T_{c'c} \tag{1.3}$$

with

$$T_{c'c} = \langle \chi_{c'} \left| T\left(E_c\right) \right| \chi_c \rangle \tag{1.4}$$

where the operator $T$ can be defined most generally by

$$T(E) = V + V \frac{1}{E - H + i\varepsilon} V \tag{1.5}$$

The kinematical variables are related by (see Fig. 1)

$$E = \frac{\hbar^2 k_c^2}{2m_c} + E_c^T = \frac{\hbar^2 k_{c'}^2}{2m_{c'}} + E_{c'}^T, \tag{1.6}$$

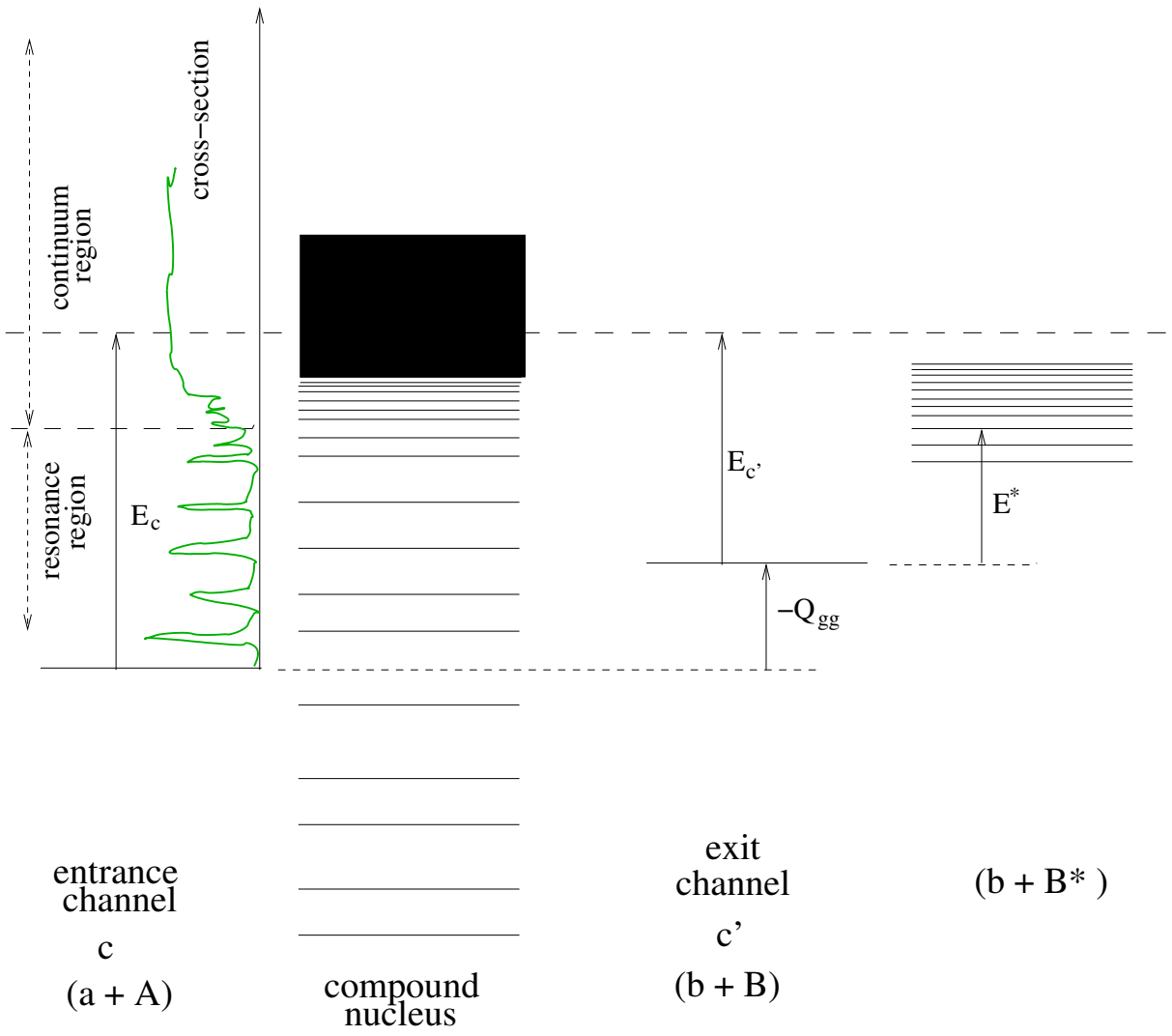


FIG. 1: Energetics of compound nucleus reaction and illustration of the resonance and continuum domains.

where $E_c^T$, $m_c$ and $\vec{k}_c$ are the target energy, reduced mass and relative momentum in channel $c$, respectively, or

$$E_c = \frac{\hbar^2 k_c^2}{2m_c} = \frac{\hbar^2 k_{c'}^2}{2m_{c'}} - Q = E_{c'} - Q, \tag{1.7}$$

where $Q$ is the reaction threshold. In all generality, the elementary cross-section (for a reaction implying the target in state $|i\rangle$ and leaving it in state $|f\rangle$) is related to the $T$-matrix by

$$d\sigma_{cc'} = \frac{(2\pi)^4}{\hbar v_c}\delta\left(\frac{\hbar^2 k_c^2}{2m_c} + E_c^T - \frac{\hbar^2 k_{c'}^2}{2m_{c'}} - E_{c'}^T\right)\left|\langle f\vec{k'}_c|\tilde{T}|i\vec{k}_c\rangle\right|^2 d\omega, \tag{1.8}$$

where $d\omega$ is the density of final states and $v_c$ is the relative velocity in the entrance channel. In Eq. (1.8), the tilda indicates that the matrix elements of $T$ are calculated with the

relative motion wave functions only, taken as plane waves. The differential cross-section, for instance, is given by expression (1.8) with

$$d\omega = \frac{k_{c'}^2 dk_{c'}}{(2\pi)^3} d\Omega, \tag{1.9}$$

which gives (after integration on $k_{c'}$)

$$\frac{d\sigma_{cc'}}{d\Omega} = \frac{2\pi}{\hbar^4} m_c m_{c'} \frac{k_{c'}}{k_c} \left| \langle f\vec{k}_{c'} | \tilde{T} | i\vec{k}_c \rangle \right|^2 = \frac{m_c}{m_{c'}} \frac{k_{c'}}{k_c} \left| f_{cc'}(\Omega) \right|^2, \tag{1.10}$$

where $f_{cc'}$ is the scattering amplitude. For elastic scattering, the scattering amplitude assumes the following partial wave expansion

$$f_{cc}(\theta) = \frac{1}{2ik} \sum_{\ell} (2\ell + 1) \left( e^{i2\delta_\ell} - 1 \right) P_\ell(cos\ \theta). \tag{1.11}$$

It is of pedagogical interest to consider the case where $T$ does not depend upon the angle (no angular momentum), corresponding also to the sum in Eq. (1.11) reduced to the first term. This academic case is realized for elastic and inelastic neutron scattering at low energy on some particular nuclei. One obtains for the integrated cross-sections ($\lambda\!\!\!^{-}_c = 1/k_c$)

$$\sigma_{cc'} = \pi \lambda\!\!\!^{-2}_c \left| S_{cc'} \right|^2 \tag{1.12}$$

and

$$\sigma_{cc} = \pi \lambda\!\!\!^{-2}_c \left| S_{cc} - 1 \right|^2. \tag{1.13}$$

In this case $c$ reduces to a single indice. The scattering matrix $S_{cc'}$ is unitary, which is a consequence of the conservation of flux (probability):

$$\sum_{c'} \left| S_{cc'} \right|^2 = 1. \tag{1.14}$$

Combining (1.12)-(1.14) yields for the total cross-section

$$\sigma_c^T = \sigma_{cc} + \sum_{c' \neq c} \sigma_{cc'} = \sigma_c^{EL} + \sigma_c^R = 2\pi \lambda\!\!\!^{-2}_c (1 - Re\ S_{cc}). \tag{1.15}$$

In Eqs. (1.12)-(1.14), the matrix $S_{cc'}$ is the so-called on-shell $S$-matrix. It is related to the $\mathcal{S}$-matrix (1.3) by the relation

$$\mathcal{S}_{cc'} = \delta_{cc'} - i2\pi \frac{\hbar^2}{m_c k_c} \delta\left(E_c - E_{c'}\right) S_{cc'}. \tag{1.16}$$

Some of the expressions can be generalized for more than two bodies in the final state. In the following, we will consider the possibility of having many nucleons in the final state. Depending upon the physical context, these channels can be described by repeated applications of two-body channel formula (sequential emmission). They may also be described by emission in a two-body channel with the target in unbound states. For this purpose, we will need to sum on numerous channels $c'$. At sufficiently high energy, the threshold energies are so close that one consider that they form a continuum (see Fig. 1). Therefore, one can focus on the exit energy $E_{c'} = \frac{\hbar^2 k'^2_c}{2m_{c'}}$ of the particle. One has

$$\frac{d^2\sigma}{d^3\vec{k}'_c} = \frac{2\pi}{\hbar v_c} \sum_n \delta(E_{c'} - E_c - E_{no}) \left| \langle \vec{k}'_c n | \tilde{T} | \vec{k}_c 0 \rangle \right|^2, \tag{1.17}$$

where $n$ is the target state in the exit channels and $E_{no}$ its excitation energy (assuming the target in its ground state initially), or

$$\frac{d^2\sigma}{dE_{c'} d\Omega'} = \frac{2\pi E_{c'}}{\hbar v_c} \sum_n \delta(E_{c'} - E_c - E_{no}) \left| \langle \vec{k}'_c n | \tilde{T} | \vec{k}_c 0 \rangle \right|^2. \tag{1.18}$$

## 2. THE COMPOUND-NUCLEUS MODEL

### A Introduction

Many reactions at low energy take place by the capture of the incident projectile to form a compound system in an excited state which subsequently decays by particle emission. At very low energy, the most striking feature of these reactions is the presence of sharp and pronounced peaks in the energy dependence of the cross-section. This strongly suggests that the reaction proceeds through a single state (a resonance) of the compound nucleus. The very existence of a small width $\Delta E$ indicates that a relatively long time $\tau$ can be associated with this process through the Heisenberg uncertainty principle: $\tau \approx \hbar/\Delta E$. The latter is substantially larger than the time of passage $t_{pass} \approx 2R/v_{inc}$. This strongly indicates that a quantum-mechanical treatment is necessary. The observation of elastic (coherent) scattering in which the target acts coherently is also pointing to the need of a quantum-mechanical treatment.

Going higher in energy, one observes that the resonances start to overlap, which means that the reaction process is proceeding faster and that a statistical treatment can be used. However both regimes can be characterized by the Bohr hypothesis, formulated below. Still higher in energy, deviations are progressively appearing, which indicate that the reaction can proceed more directly and not through states of the compound system.

### B The Bohr hypothesis

In view of the long time associated with compound nuclear reactions, Bohr [2] proposed that reactions begin with the capture of the projectile by the target nucleons and the sharing of the energy among all nucleons. Subsequently, long after the capture of the projectile, particles are emitted by some statistical process, leading to some final state of the residual nucleus. As a consequence, the decay process is independent of the formation process. This is the famous loss of memory of the compound nucleus. Actually, this memory may not be completely lost because some quantities are conserved and are thus the same in the initial and final channels: angular momentum, parity and probability flux.

This hypothesis has observable consequences which can be tested experimentally. First, ratios of cross-sections to different final states should be independent of the way the compound nucleus is formed. Second, the particles are expected to be emitted isotropically. However, conservation of angular momentum complicates the analysis and replaces this expectation by the one of a symmetric emission about $90^\circ$. Third, for sufficiently high incident energy, the energy spectrum of emitted particles should be thermal-like.

### C The Breit-Wigner formula

We disregard angular momentum for the sake of simplicity. In the resonance region (see Fig. 1), the presence of isolated resonances can be related to the poles $\lambda$ of the $S$-matrix (1.2) through the Breit-Wigner formula

$$S_{cc'} = e^{i(\delta_c+\delta_{c'})}\left(\delta_{cc'} - i\sum_\lambda \frac{\omega_\lambda}{E - E_\lambda + i\Gamma_\lambda/2}\right), \tag{2.1}$$

where the $\delta_c$'s are slowly varying quantities. This formula is also valid in the so-called continuum region, where the widths of the resonances ($\Gamma_\lambda$) are larger than their average spacing ($D$). The Bohr hypothesis implies that

$$\omega_\lambda = \Gamma_{\lambda c}^{1/2} \Gamma_{\lambda c'}^{1/2}, \tag{2.2}$$

where the $\Gamma_{\lambda c}^{1/2}$ 's are real (positive or negative) quantities. Close to an isolated resonance, the $c \to c'$ cross-section writes, according to Eq. (1.12)

$$\sigma_{cc'} \;=\; \pi \lambda\!\!\!^{-2}_c \frac{\Gamma_{\lambda c} \Gamma_{\lambda c'}}{(E \;-\; E_\lambda)^2 \;+\; \frac{\Gamma_\lambda^2}{4}} \tag{2.3}$$

and expresses indeed that the cross-section can be seen as the product of the cross-section for forming the compound nucleus state from channel $c$, i.e. the compound nucleus cross-section $\sigma_c^{CN}$, times the probability $P_{c'}$ for decaying into channel $c'$, which should be proportional to $\Gamma_{\lambda c'}$. Since

$$\sum_{c'} \Gamma_{\lambda c'} = \Gamma_\lambda, \tag{2.4}$$

where the summation is restricted to the open channels, one has

$$P_{c'} = \frac{\Gamma_{\lambda c'}}{\Gamma_\lambda}. \tag{2.5}$$

Therefore the compound nucleus cross-section assumes, in general, the following form in the resonance region

$$\sigma_c^{CN} \;=\; \pi \lambda\!\!\!^{-2}_c \sum_\lambda \frac{\Gamma_{\lambda c} \Gamma_\lambda}{(E \;-\; E_\lambda)^2 \;+\; \frac{\Gamma_\lambda^2}{4}}, \tag{2.6}$$

which can be decomposed into compound elastic and reaction cross-sections (around each $E_\lambda$)

$$\sigma_c^{CN} = \sigma_c^{CE} + \sigma_c^R, \quad \sigma_c^R = \sigma_c^{CN} \frac{\Gamma_\lambda - \Gamma_{\lambda c}}{\Gamma_\lambda}, \quad \sigma_c^{CE} = \sigma_c^{CN} \frac{\Gamma_{\lambda c}}{\Gamma_\lambda}. \tag{2.7}$$

The prediction of the cross-sections in the resonance region is a huge task, comparable to, but much harder than the one of predicting bound states of a stable nucleus. Actually, it is the continuation of this problem to open channels. We refer to Refs. [3–5] for a description of the theories aiming at this issue. However, if one is interested in a statistical treatment, simpler theories are available and cross-sections can even be related to known simple models. This is described in the next section.

Before closing this section, let us mention that the simple factorization of Eqs. (2.1-2.3) is lost if resonances are overlapping, or if the background term in Eq. (2.1) is markedly different from $\delta_{cc'}$ or also if resonances with different angular momentum have to be considered. Indeed in the latter case, Eq. (2.1) becomes

$$S_{cc'} = \sum_J \sum_{\ell=|J-s|}^{J+s} \sum_{\ell'=|J-s'|}^{J+s'} (\ell 0 s m_s | J m_s)\, (\ell' 0 s' m'_s | J m_{s'}) \left( S^{0J}_{c\ell s, c'\ell' s'} - i \sum_\lambda \frac{\Gamma^{J\ 1/2}_{\lambda c} \Gamma^{J\ 1/2}_{\lambda c'}}{E - E_\lambda + i\Gamma^J_\lambda/2} \right). \tag{2.8}$$

We considered an incident particle of spin $s$ and emission of a particle of spin $s'$ and we have taken account of the possible partial waves contributing to the resonances.

### D The Hauser-Feshbach theory

Let us consider the resonance region and introduce average cross-sections. The way of averaging is largely irrelevant. What matters is that the averaging interval $I$ is much larger than the average separation $D$ between resonances. The average cross-sections are thus smooth functions of energy.

For simplicity, we disregard spin and angular momentum. We have in general

$$\langle \sigma_{cc'} \rangle \ = \ \frac{1}{I} \int_{E\ -\ I/2}^{E\ +\ I/2} \pi \lambda\!\!\!^{-2}_c \left| \sum_\lambda \frac{\Gamma^{1/2}_{\lambda c} \Gamma^{1/2}_{\lambda c'}}{E\ -\ E_\lambda\ +\ i\Gamma_\lambda/2} \right|^2 dE. \tag{2.9}$$

We consider first the resonance region. Eq. (2.9) becomes

$$\langle \sigma_{cc'} \rangle = \frac{1}{I} \int_{E-I/2}^{E+I/2} \pi \lambda\!\!\!^{-2}_c \sum_\lambda \frac{\Gamma_{\lambda c} \Gamma_{\lambda c'}}{(E - E_\lambda)^2 + \frac{\Gamma^2_\lambda}{4}} dE, \tag{2.10}$$

which yields

$$\langle \sigma_{cc'} \rangle = \pi \lambda\!\!\!^{-2}_c \frac{2\pi}{D} \overline{\left( \frac{\Gamma_{\lambda c} \Gamma_{\lambda c'}}{\Gamma_\lambda} \right)}. \tag{2.11}$$

The bar indicates an average over the resonances contained in the interval $I$. This can formally be rewritten as [6]

$$\langle \sigma_{cc'} \rangle \ = \ \pi \lambda\!\!\!^{-2}_c \ \frac{\left( \frac{2\pi}{D} \overline{\Gamma}_{\lambda c} \right) \left( \frac{2\pi}{D} \overline{\Gamma}_{\lambda c'} \right)}{\frac{2\pi}{D} \overline{\Gamma}_\lambda} \ F_{cc'} \tag{2.12}$$

or

$$\langle \sigma_{cc'} \rangle = \sigma^{HF}_{cc'} F_{cc'}, \quad \sigma^{HF}_{cc'} = \pi \lambda\!\!\!^{-2}_c \frac{T_c T_{c'}}{T}, \tag{2.13}$$

with

$$T_c = 2\pi \frac{\overline{\Gamma_{\lambda c}}}{D}. \tag{2.14}$$

The partial widths can be considered as stochastic variables; they fluctuate from resonance to resonance without obvious correlation, obeying some overall distribution functions. This seems reasonable in view of the complexity of the compound nucleus formation process. The same reason induces to believe that $\Gamma_{\lambda c}$, $\Gamma_{\lambda c'}$ and $\Gamma_\lambda$ are independent stochastic variables. This gives $F_{cc'} = 1$, if in addition $\Gamma_\lambda$ does not fluctuate. In actual cases, $F_{cc'} \neq 1$, because $\Gamma_\lambda$ fluctuates and because the partial and total widths are not totally independent for various reasons. The most obvious one comes from relation (2.4), which ultimately derives from unitarity (conservation of probability).

It has been shown that the partial widths generally follow a Porter-Thomas distribution [7]

$$p\left(\Gamma_{\lambda c}\right) d\Gamma_{\lambda c} = \frac{1}{\sqrt{2\pi \Gamma_{\lambda c} \overline{\Gamma_{\lambda c}}}} exp\left(-\frac{\Gamma_{\lambda c}^2}{2\overline{\Gamma_{\lambda c}}^2}\right) d\Gamma_{\lambda c}, \tag{2.15}$$

where $\overline{\Gamma_{\lambda c}}$ is the average value of $\Gamma_{\lambda c}$ in the energy interval under consideration. As a consequence, the quantities $F_{cc'}$ can be evaluated from these probability distributions. We do not display the final result, which is formally complicated.

On the other hand, the quantities $T_c$ can be related to some average elastic cross-section in channel $c$. Indeed, one has, from Eq. (2.1), taking account of $\langle f(E) \rangle = f(E + i\frac{I}{2})$ for analytic functions in the upper complex $E$ plane (see Ref. [4] for a demonstration),

$$\langle S_{cc} \rangle \;=\; e^{i2\delta_c} \left(1 \;-\; \pi \; \frac{\overline{\Gamma_{\lambda c}}}{D}\right) \tag{2.16}$$

On the other hand, one can write, from Eq. (1.12)

$$\begin{aligned} \left\langle \sigma_c^{EL} \right\rangle \;&=\; \pi \lambda\!\!\!^{-2}_c \left\langle |1 \;-\; S_{cc}|^2 \right\rangle \\ &=\; \pi \lambda\!\!\!^{-2}_c |1 \;-\; \langle S_{cc} \rangle|^2 \;+\; \pi \lambda\!\!\!^{-2}_c \left( \left\langle |S_{cc}|^2 \right\rangle \;-\; |\langle S_{cc} \rangle|^2 \right) \\ &=\; \sigma_c^{opt} \;+\; \langle \sigma_c^{CE} \rangle, \end{aligned} \tag{2.17}$$

where the last line introduces the definitions of $\sigma_c^{opt}$ and $\langle\sigma_c^{CE}\rangle$, and

$$\left\langle\sigma_c^R\right\rangle \;=\; \pi\lambda\!\!\!^{-2}_c \sum_{c' \,\neq\, c} \left\langle|S_{cc'}|^2\right\rangle \;=\; \pi\lambda\!\!\!^{-2}_c \left\langle 1 \;-\; |S_{cc}|^2\right\rangle \;= \pi\lambda\!\!\!^{-2}_c \left(1 \;-\; |\langle S_{cc}\rangle|^2\right) \;-\; \langle\sigma_c^{CE}\rangle \quad (2.18)$$

or

$$\langle\sigma_c^R\rangle = \sigma_R^{opt} - \langle\sigma_c^{CE}\rangle. \quad (2.19)$$

The quantity $\sigma_c^{CE}$ can also be rewritten as

$$\sigma_c^{CE} = \pi\lambda\!\!\!^{-2}_c \langle|S_{cc} - \langle S_{cc}\rangle|^2\rangle, \quad (2.20)$$

and, owing to Eqs. (2.1) and (2.2), assumes the same form as Eq. (2.12). Therefore

$$\sigma_c^{CE} = \pi\lambda\!\!\!^{-2}_c \left(\frac{2\pi\Gamma_{\lambda c}}{D}\right)^2 \frac{F_{cc}}{2\pi\frac{\Gamma_\lambda}{D}}. \quad (2.21)$$

From Eqs. (2.16), (2.19) and (2.21), one readily obtains

$$\left\langle\sigma_c^R\right\rangle = \pi\lambda\!\!\!^{-2}_c 2\pi\frac{\overline{\Gamma_{\lambda c}}}{D} - \frac{\pi^2\Gamma_{\lambda c}^2}{D^2}\left(1 + \frac{F_{cc}}{\pi\overline{\Gamma_\lambda}/D}\right) \approx \pi\lambda\!\!\!^{-2}_c 2\pi\frac{\overline{\Gamma_{\lambda c}}}{D}. \quad (2.22)$$

The interesting point here is that there exists a simple model for calculating $\langle S_{cc}\rangle$, namely the optical model, which aims to describe the average elastic scattering by a potential model. This potential should be absorptive, because $|\langle S_{cc}\rangle|^2 \leq \langle|S_{cc}|^2\rangle \leq 1$. The absorptive part is responsible for the loss of flux from the incident channel which is transferred to the reaction channels through the compound nucleus. In general, the second term in Eq. (2.22) is negligible.

In the continuum regime, one can show that Eq. (2.11) is still correct, because the interference terms in Eq. (2.9) vanish due to the fluctuating sign of the partial width amplitudes. Eq. (2.13) still holds, but $T_c$ is now given by

$$T_c = 1 - exp\left(-2\pi\frac{\overline{\Gamma_{\lambda c}}}{D}\right), \quad (2.23)$$

generalizing Eq. (2.14), which retains the leading term when $\overline{\Gamma_{\lambda c}}/D$ is small compared to unity. Eq. (2.22) becomes

$$\sigma_R^{opt}\left(\approx \langle\sigma_c^R\rangle\right) = \pi\lambda\!\!\!^{-2}_c T_c. \quad (2.24)$$

It is easy to see from Eqs. (2.7), (2.11) and (2.22) that

$$\sigma_R^{opt} = \langle \sigma_c^{CN} \rangle. \tag{2.25}$$

The quantity $T_c$ (always smaller than unity) is called the transmission coefficient (in channel $c$): it is the probability for the incoming flux to feed the compound nucleus (on the average).

When angular momentum and spins are taken into account, the Hauser-Feshbach cross-section (2.13) becomes

$$\sigma_{cc'}^{HF} = \pi \lambda\!\!\!^{-2}_c \sum_{\ell} \sum_{\ell'} \sum_{J} \frac{(2J+1)}{(2s+1)(2s'+1)} \frac{T_{c'\ell'}^J T_{c\ell}^J}{\sum\limits_{c"\ell"} T_{c"\ell"}^J}, \tag{2.26}$$

where $J$ should satisfy $|\ell - s| \leq J \leq \ell + s$, $|\ell' - s'| \leq J \leq \ell' + s'$. The transmission coefficients are related to the average reaction cross-section by $\sigma_R^{opt} = \pi \lambda\!\!\!^{-2}_c \sum\limits_{\ell} T_{c\ell}^{\ell}$ (for $s = 0$).

One can easily generalize the Hauser-Feshbach formula by considering a summation on several selected channels. It is interesting to consider the case of exit channels generating the same particle, an $\alpha$-particle for instance, and a residual nuclei with excited states so close that they can be considered as forming a continuum. Let us call $E_c$ the incident kinetic energy and $E_{c'}$ the outgoing particle kinetic energy. One has

$$E_c = -Q_{gg} + E_{c'} + E_{c'}^*, \tag{2.27}$$

where $Q_{gg}$ is the $Q$-value of the reaction leaving the target in the ground state and $E_{c'}^*$ the excitation energy of residual nuclei in the exit channel. Let us call $\omega_{c'}(E_{c'}^*)$ the density of states of the residual nucleus, or in other words the density of exit channels. The cross-section for generating a residual energy in the interval $[E_{c'}^*, E_{c'}^* + dE_{c'}^*]$ can be obtained from Eq. (2.13) by introducing a suitable summation

$$d\sigma_{cc'}^{HF}\left(E_{c'}^*\right) = \pi \lambda\!\!\!^{-2}_c \sum_{\ell J} \frac{2J+1}{(2s+1)(2s'+1)} \frac{T_{c\ell}^J \sum\limits_{\ell'} T_{c'\ell'}^J \, \omega_{c'}\left(E_{c'}^*\right) dE_{c'}^*}{\sum\limits_{c"\ell"} \int_0^{E - Q_{gg}} T_{c"\ell"}^J \omega_{c"}\left(E_{c"}^*\right) dE_{c"}^*} \tag{2.28}$$

The necessity to evaluate $T_{c'\ell'}^J$ for a huge number of channels and associated excitation energies reduce the applicability of this theory at low energy only.

In summary, average cross-sections can be calculated by first using an optical model for the (average) elastic scattering. This amounts to solve the Schrödinger equation for partial

waves ($\ell$)

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+\frac{\ell(\ell+1)\hbar^2}{2mr^2}+V(r)-iW(r)\right)u_\ell(r)=Eu_\ell(r). \tag{2.29}$$

The asymptotic behaviour of the latter generates a complex phase shift $\delta_\ell^{opt}$ and the transmission coefficients (see Eq. (2.21)) through

$$T_\ell=1-\left|e^{i2\delta_\ell^{opt}}\right|^2=1-e^{-4Im\delta_\ell^{opt}}. \tag{2.30}$$

The cross-sections are then evaluated with the help of the Hauser-Feshbach formula corrected for width fluctuations.

## E The Weisskopf-Ewing theory

The Weisskopf-Ewing theory [8] is the prototype of evaporation theories for small systems. For our purpose, it describes the properties of the particles emitted by an excited compound nucleus. But it can be formulated as a theory for emitting a particle with given kinetic energy after bombardment of a target by a projectile. We retain notation (2.27) of the last paragraph. We use the generalization of Eq. (2.7) that we rewrite as

$$d\sigma_{cc'} \;=\; \sigma_c^{CN}\;\frac{\overline{\Gamma_{\lambda c'}\left(E^*_{c'}\right)}\;\omega_{c'}\left(E^*_{c'}\right)dE^*_{c'}}{\sum\limits_{c"}\int_0^{E\;+\;Q_{gg}}\overline{\Gamma_{\lambda c"}\left(E^*_{c"}\right)}\omega_{c"}\left(E^*_{c"}\right)dE^*_{c"}}, \tag{2.31}$$

where the bar means average over resonances ($\lambda$). We now use the reciprocity theorem that relates the cross-section for the formation of the compound nucleus by an incident particle of energy $E_{c'}$ on a residual nucleus of excitation energy $E^*_{c'}$ (channel $c'$) to the decay width in this channel (Eq. (2.22))

$$\sigma_{c'}^{CN}(E_{c'})=\pi\lambda\!\!\!^{-2}_{c'}2\pi\frac{\overline{\Gamma}_{\lambda c'}(E^*_{c'})}{D(E)} \tag{2.32}$$

where $D(E)$ is the level spacing in the compound nucleus and $\lambda\!\!\!^{-2}_{c'}=\hbar^2/(2m_{c'}E_{c'})$. One has, combining the last two equations

$$d\sigma_{cc'}(E^*_{c'}) \;=\; \sigma_c^{CN}(E_{c'})\;\frac{E_{c'}\sigma_{c'}^{CN}(E_{c'})\omega_{c'}(E^*_{c'})dE^*_{c'}}{\sum\limits_{c"}\int_0^{E\;+\;Q_{gg}}E_{c"}\sigma_{c"}^{CN}(E^*_{c"})\omega_{c"}\left(E^*_{c"}\right)dE^*_{c"}}. \tag{2.33}$$

First of all, we compare the Hauser-Feshbach and Weisskopf-Ewing theories (Eqs. (2.28) and (2.33)). Essentially the summation on the $T_\ell$ in Hauser-Feshbach theory is replaced by $\sigma_c^{CN}$. This is almost equivalent, owing to the relation following Eq. (2.26). However, this is not exactly true as the density of states $\omega_{c'}$ depends upon $J$ (we did not explicitate this dependence in order to not multiply the indices). Furthermore, Eq. (2.33) can be simplified by a simple approximation on the compound nucleus cross-sections. This is done in the second formulation of the theory described below. Let us finally mention that Eq. (2.33) can be formulated as the cross-section for emission of a particle with energy $E_{c'}$. It suffices to replace $E^*_{c'}$ by $E_{c'}$ in Eq. (2.33) using relation (2.27).

The Weisskopf-Ewing theory for evaporation can be derived in a very general approach. We will outline this derivation. We change slightly our notation. We start with a compound nucleus $A$ in a volume $V$ with excitation energy $E^*$ and consider the probability that it emits a particle $a$ of mass $m_a$ with kinetic energy between $[E', E' + dE']$ leaving the residual nucleus $B$ with excitation energy $E^*_B$. One has of course

$$E^* = E^*_B + S + E', \tag{2.34}$$

where $S$ is the separation energy. The probability per unit time, which is nothing but the partial width in channel $c'(= B + a)$, is given by

$$d\Gamma_a = \frac{2\pi}{\hbar} \left|\langle A|T|Ba\rangle\right|^2 \omega\left(E^*_B\right) \frac{V k'^2 dk'}{(2\pi)^3}, \tag{2.35}$$

with $E' = \hbar^2 k'^2/2m_a$. This can be rewritten using the cross-section for the inverse process $a + B \to A$. The latter is given by the probability per unit time of forming a compound nucleus divided by the average flux (density $1/V$ times the velocity $\hbar k'/m_a$), i.e.:

$$\sigma^{CN}(aB \to A) = \frac{2\pi}{\hbar} \frac{\left|\langle Ba|T|A\rangle\right|^2 \omega_A\left(E^*_A\right)}{\hbar k'/m_a V}. \tag{2.36}$$

Note that expressions (2.35) and (2.36) are exact (in the limit of separated evaporation steps), and that cross-section (2.36) refers to an excited nucleus $B$. Using (2.35) and (2.36) and the micro-reversibility ($\langle A|T|Ba\rangle = \langle Ba|T|A\rangle^*$), one has

$$d\Gamma_a = \sigma^{CN}(aB \to A) \frac{2m_a}{(2\pi)^3 \hbar^2} \frac{\omega\left(E^*_B\right)}{\omega\left(E^*_A\right)} E' dE'. \tag{2.37}$$

Now, the density of states is basically given by

$$\omega\left(E^*\right) = pe^{2\sqrt{aE^*}}, \tag{2.38}$$

where the prefactor $p$ is not explicitated for the moment (details are given in Appendix A), and the excitation energy can be related to temperature $T$ by

$$E^* = aT^2. \tag{2.39}$$

A little algebra gives

$$d\Gamma_a = \sigma^{CN}(aB \to A)\frac{2m_a}{(2\pi)^3\hbar^2}e^{-\frac{S+E'}{T}}E'dE'. \tag{2.40}$$

This shows that the spectrum of the emitted particles looks like a thermal spectrum (not exactly a Boltzmann spectrum), sometimes called a surface evaporation spectrum.

Integrating Eq. (2.40) on $E'$ gives the total width for the particular channels ($\alpha$-particle emission for instance) which have been selected. One obtains

$$\begin{aligned}\Gamma_a &= \sigma^{CN}(aB \to A)\frac{2m_a}{(2\pi)^3\hbar^2}\int_0^{E^*-S} e^{-\frac{S+E'}{T}}E'dE' \\ &\simeq \sigma^{CN}(aB \to A)\frac{2m_aT^2}{(2\pi)^3\hbar^2}e^{-\frac{S}{T}}.\end{aligned} \tag{2.41}$$

This relation shows that the emission is favoured for large $m_a$ and small separation energy. For charged particles, the main additional feature is the limitation of the integration in Eq. (2.41) to the region above the Coulomb barrier $B$, which leads to

$$\Gamma_a \approx \sigma^{CN}(aB \to A)\frac{2m_aT^2}{(2\pi)^3\hbar^2}e^{-\frac{S+B}{T}}\left(1+\frac{B}{T}\right). \tag{2.42}$$

Evaporation calculations proceed as follows: the various widths are evaluated (not with Eq. (2.41) but with the integration of Eq. (2.37)). A random number is compared to the relative widths $\left(\frac{\Gamma_a}{\sum\limits_a \Gamma_a}\right)$ and a channel is selected accordingly. The energy of the emitted particle is chosen at random according to distribution (2.37). The excitation energy of the remnant is deduced from Eq. (2.34). This simple procedure allows for generating further evaporation steps, until the excitation energy falls below the lowest emission threshold.

This multistep evaporation is evidently much harder to calculate with the Hauser-Feshbach formalism.

The parameters entering Weisskopf-Ewing theory are the capture cross-sections (in principle for excited targets) and the level density, basically the level density parameter $a$. The fact that only relative widths are required makes the first parameter less crucial than the second one.

Competition with fission is usually introduced through the transition state method [9, 10]. It is assumed that the transition probability is entirely given by the properties of the barrier appearing in the potential energy curve drawn against the coordinate describing the scission of $A$ into two fragments $B'$ and $B$". The probability of fission is assumed as the number of states at the barrier which can be reached per initial states. Let us call $p$ and $q$ the momentum and position associated with the motion along the fission coordinate. The probability of fission per unit time is then given by

$$\frac{d\Gamma_f}{\hbar} = \frac{\omega\left(E_B^*\right) dE_B^* (dpdq/2\pi\hbar)/dt}{\omega\left(E_A^*\right) dE_A^*}, \tag{2.43}$$

where $E_B^*$, $E_A^*$ (as well as $dE_B^*$ and $dE_A^*$) are related through

$$E_A^* = E_B^* + \frac{p^2}{2M} + \tilde{B} = E_B^* + \varepsilon + \tilde{B}, \tag{2.44}$$

where $M$ is the mass associated with the motion to fission, $\varepsilon$ is the kinetic energy in this motion and where $\tilde{B}$ is the height of the fission barrier (relative to the ground state of the compound nucleus). Using Eq. (2.43) and $dq/dt = p/M$ and $pdp = Md\varepsilon$, one readily obtains

$$d\Gamma_f = \frac{1}{2\pi}\frac{\omega\left(E_B^*\right)}{\omega\left(E_A^*\right)} dE_B^*. \tag{2.45}$$

If $E_A^*$ is sufficiently larger than $E_B^*$, integration of Eq. (2.45) gives

$$\Gamma_f = \frac{T}{2\pi} e^{-\tilde{B}/T}, \tag{2.46}$$

with the help of Eqs. (2.38)-(2.39).

The critical parameters are the fission barrier $\tilde{B}$ and the level density parameter for the deformed complex at the barrier, which could be different from the one for the spherical

original nucleus. Note that this model assumes a transmission coefficient equal to unity. This needs to be corrected: it is usually done by introducing friction in the motion to fission. It is most often formulated in the Kramers formalism [11, 12], that we will not discuss here.

## 3. HIGH ENERGY REACTIONS

### A Introduction

Here we have in mind the GeV range which, to fix the ideas, may be considered to extend from $\sim$ 250 MeV to a few GeV (the limit to the high energy side is set by the possible manifestations of the subnucleon degrees of freedom). The basic idea here, first formulated by Serber, is that the de Broglie wavelength $\lambda\!\!\!^{-}_B$ of the incident particle is small and therefore the latter "probes" the nucleons individually inside the nucleus. The reaction mechanism can be viewed as a succession of separated binary collisions. For this to be correct however, it is necessary that the wave packet of the incident particle striking a target nucleon recovers its asymptotic form before it encounters another nucleon. This occurs after a distance which is not well known, but generally considered to be of the order of the range $r_s$ of the nuclear forces (in fact, it cannot be smaller!). Therefore the following condition is required for the separation of successive collisions:

$$\lambda\!\!\!^{-}_B << r_s << d, \tag{3.1}$$

where $d$ is the average distance between neighbouring nucleons. This is barely fulfilled in the nuclear case. Observation of spectra, where the so-called quasi-elastic scattering (see below) is well isolated, tells that the hypothesis of separated collisions is a good starting point. However, condition (3.1) is probably much less fulfilled for secondary collisions and thus some care should be exercised.

## B A simple model

It is interesting to look at a simplified Glauber model [13]. The incident particle is assumed to travel along a straight line (Glauber aspect) in a nucleus characterized by a density $\rho(r)$. For pedagogical reason, we consider here a uniform density in a sphere of radius $R$:

$$\rho(r) = \rho_0 \theta(R - r). \tag{3.2}$$

It is an easy exercise to calculate the total reaction cross-section. Let us consider an impact parameter $b$. The incident particle enters the nucleus at $z = -z_0 = -\sqrt{R^2 - b^2}$ and leaves it at $z = z_0$ (see Fig. 2). The probability that it has not interacted at position $z(|z| < z_0)$ is

$$P_{surv}(z) = e^{-\sigma_{NN}^{tot} \rho_0 (z + z_0)}. \tag{3.3}$$

Therefore the total reaction cross-section is simply

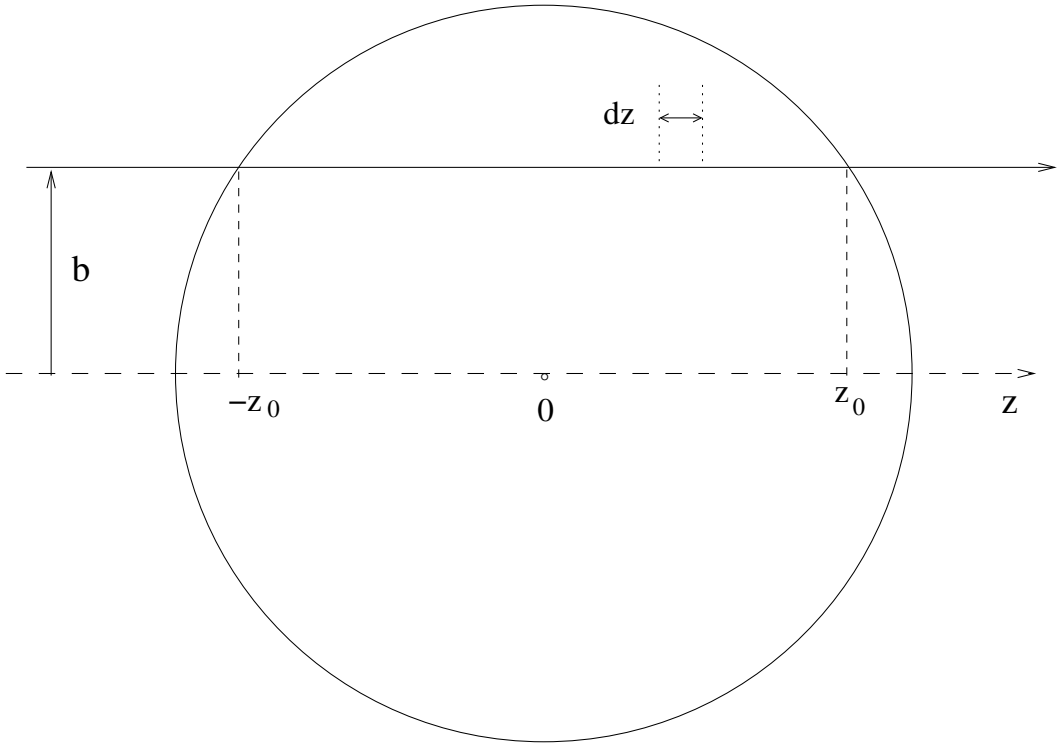


FIG. 2: Kinematic variables relevant to the eikonal approximation.

$$\sigma_{tot} = \int_0^R 2\pi b db \int_{-z_0}^{z_0} P_{surv}(z) \rho_0 \sigma_{NN}^{tot} dz, \tag{3.4}$$

as $\rho_0 \sigma_{NN}^{tot} dz$ is the probability to interact in a path element $dz$. A little algebra yields

$$\sigma_{tot} = \pi R^2 \left[ 1 - \frac{2}{x^2} \left( 1 - e^{-x} \right) + \frac{2}{x} e^{-x} \right], \tag{3.5}$$

with $x = 2\sigma_{NN}^{tot} \rho_0 R$. For nuclei, $\sigma_{NN}^{tot} \rho_0 = 0.6$ fm$^{-1}$. For $R = 5$ fm, one has roughly

$$\sigma_{tot} \approx \pi R^2 (1 - 0.05) \approx \sigma_{geom}. \tag{3.6}$$

Taking account of the diffusivity, one obtains $\sigma_{tot} \approx 1.2 \sigma_{geom}$, which agrees quite well with experiment.

It is also interesting to split this cross-section into cross-sections for events in which the incident particle undergoes $n = 1, 2, 3, ...$ collisions. It is easy to show (although we will not do it here) that, for a given impact parameter $b$, the probability for making $n (\geq 0)$ collisions is given by

$$P_n(b) = \frac{L^n}{n!} e^{-L}, \tag{3.7}$$

with $L = 2\rho_0 \sigma_{NN}^{tot} z_0$ for uniform density, and $L = \int_{-\infty}^{+\infty} \rho \left( \vec{r} = \left( \vec{b}, z \right) \right) \sigma_{NN}^{tot} dz$ for a diffuse density. Let us look at the $n = 1$ cross-section for a sharp surface nucleus. We calculate this quantity explicitly, requiring that the incident particule does not interact before and after this single collision:

$$\begin{aligned} \sigma_1 &= \int_0^R 2\pi b db \int_{-z_0}^{z_0} e^{-\sigma_{NN}^{tot} \rho_0 (z + z_0)} \rho_0 \sigma_{NN}^{tot} e^{-\sigma_{NN}^{tot} \rho_0 (z_0 - z)} dz \\ &= \int_0^R 2\pi b db 2\sigma_{NN}^{tot} \rho_0 \sqrt{R^2 - b^2} e^{-2\rho_0 \sigma_{NN}^{tot} \sqrt{R^2 - b^2}} \\ &= 4\pi R^2 \left[ \frac{1}{x^2} \left( 1 - e^{-x} \right) - \frac{1}{x} e^{-x} - \frac{1}{2} e^{-x} \right], \end{aligned} \tag{3.8}$$

with $x$ given below Eq. (3.5). It is interesting to look at the two limits: $x << 1$ and $x >> 1$. For the first one, one has

$$\sigma_1 \approx 4\pi R^2 \frac{x}{6} = \frac{4\pi R^2 \rho_0 \sigma_{NN}^{tot} R}{3} = A \sigma_{NN}^{tot}. \tag{3.9}$$

In this (weak interaction) limit, all nucleons contribute additively to the cross-section. In the strong interaction limit, one has (neglecting the exponentials in Eq. (3.8))

$$\sigma_1 \approx \frac{4\pi R^2}{x^2} = A_{eff} \sigma_{NN}^{tot}, \tag{3.10}$$

with

$$A_{eff} = \frac{\pi}{\rho_0^2 \left(\sigma_{NN}^{tot}\right)^3}. \tag{3.11}$$

Here, only $A_{eff}$ "effective" nucleons "contribute" to $\sigma_1$. This number can also be written as $A_{eff} = \pi \left(\lambda/\overline{d}\right)^3$, where $\lambda \left(= 1/\rho\sigma_{NN}^{tot}\right)$ is the mean free path and $\overline{d}$ is the average distance between neighbouring nucleons. It is to be remarked that this approximation is valid for very large $A$, but shows nonetheless that $A_{eff} << A$. This is easy to understand since the incoming particle has to escape interaction before and after the single one under consideration. A precise calculation shows that $A_{eff} \simeq 8$ for $^{208}Pb$. Later we will consider the quasi-elastic case corresponding to a single elastic $NN$ scattering. The calculation is the same: it suffices to replace $\sigma_{NN}^{tot}$ by $\sigma_{NN}^{e\ell}$ outside the exponential in the first line of Eq. (3.8). One obviously ends with

$$\sigma_1^{QE} = A_{eff}\sigma_{NN}^{e\ell}, \tag{3.12}$$

with the same value of $A_{eff}$.

### C Simplified quantum picture

A simple quantum approach is provided by the Glauber formalism [13] in the eikonal approximation. The latter is based on four assumptions:

1. $k_c R = \dfrac{R}{\lambda\!\!\!^{-}_c} >> 1$
2. incident particle-nucleon scattering at small angles (momentum transfer essentially perpendicular to the incident direction $\vec{e}_z$)
3. $\delta_\ell(k_c)$ is a smooth function of $\ell$ and $k_c$, where $\ell$ is the angular momentum ($\ell = k_c b$) and $\delta_\ell$ is the incident particle-nucleon scattering phase shift
4. frozen nucleus approximation (no Fermi motion)

It is then possible to relate the total scattering amplitude to individual amplitudes. We simply quote the result (which is demonstrated in Appendix B) for the scattering of the

incident particle on a target in state $i$ suffering a momentum transfer $\vec{q} = \vec{k}_{c'} - \vec{k}_c$ and leaving the target in a state $f$

$$\begin{aligned} f_{fi}(\vec{q}) &= \frac{ik}{2\pi} \int d^2b e^{i\vec{q}.\vec{b}} \int d^3r_1 ... d^3r_A \; \phi_f^*(\vec{r}_1, ..., \vec{r}_A) \; \times \\ &\left[ 1 - \prod_{j=1}^{A} \left( 1 - \frac{1}{2\pi ik} \int d^2\vec{q}\,' e^{-i\vec{q}\,'.(\vec{b} - \vec{s}_j)} f_j(\vec{q}\,') \right) \right] \phi_i(\vec{r}_1, ..., \vec{r}_A), \end{aligned} \tag{3.13}$$

where $\vec{s}_j$ is the transverse position of nucleon $j$, $\vec{s}_j = \vec{r}_j - (\vec{r}_j.\vec{e}_z)\,\vec{e}_z$, and $f_j(\vec{q}')$ is the (elementary) scattering amplitude on nucleon $j$. One can expand the product in Eq. (3.13) by using the formula

$$\prod_{j=1}^{A} (1 - a_j) = 1 - \sum_{j=1}^{A} a_j + \frac{1}{2} \sum_{j=1}^{A} \sum_{\substack{k = 1 \\ k \neq j}}^{A} a_j a_k + ... \tag{3.14}$$

The first term gives the single-scattering amplitude

$$\begin{aligned} f_{fi}^{(1)}(\vec{q}) &= \frac{1}{(2\pi)^2} \int d^2b e^{i\vec{q}.\vec{b}} \int d^3r_1 ... d^3r_A \; \phi_f^*(\vec{r}_1, ..., \vec{r}_A) \; \times \\ &\sum_{j=1}^{A} \int d^2\vec{q}\,' e^{-i\vec{q}\,'.(\vec{b} - \vec{s}_j)} f_j(\vec{q}\,') \, \phi_i(\vec{r}_1, ..., \vec{r}_A). \end{aligned} \tag{3.15}$$

Using

$$\frac{1}{(2\pi)^2} \int d^2b e^{i(\vec{q} - \vec{q}\,').\vec{b}} = \delta(\vec{q} - \vec{q}\,') \tag{3.16}$$

and considering $f_j(\vec{q})$ as independent of $j$ $(\equiv f(\vec{q}))$, one obtains

$$f_{fi}^{(1)}(\vec{q}) = \int d^2\vec{s} e^{i\vec{q}.\vec{s}} f(\vec{q}) \int d^3r_1 ... d^3r_A \; \phi_f^*(\vec{r}_1, ..., \vec{r}_1) \sum_{j=1}^{A} \delta(\vec{s} - \vec{s}_j) \, \phi_i(\vec{r}_1, ..., \vec{r}_A). \tag{3.17}$$

The last integral is nothing but the transition matrix density $\rho_{fi}(\vec{s})$ (for transverse $\vec{s}$). One finally gets

$$f_{fi}^{(1)}(\vec{q}) = f(\vec{q}) \, \tilde{\rho}_{fi}(\vec{q}) = f(\vec{q}) \int d^2\vec{s} e^{i\vec{q}.\vec{s}} \rho_{fi}(\vec{s}). \tag{3.18}$$

For the elastic scattering

$$f_{ii}^{(1)}(\vec{q} = 0) = A \; f(\vec{q} = 0). \tag{3.19}$$

For inelastic scattering, the integral in Eq. (3.18) is much smaller than $A$ and plays the same role as $A_{eff}$ in the simple approach mentioned above.

We simply plug the result for double scattering

$$f_{fi}^{(2)}\left(\vec{q}\right)=\frac{1}{2\pi ik}\int d^{2}\vec{q}\,'f\left(\vec{q}-\vec{q}\,'\right)f\left(\vec{q}\,'\right)\int d^{2}\vec{s}d^{2}\vec{s}\,'e^{i\vec{q}.\vec{s}}e^{i\left(\vec{q}-\vec{q}\,'\right).\vec{s}\,'}\rho_{fi}^{(2)}\left(\vec{s},\vec{s}\,'\right), \tag{3.20}$$

where $\rho_{fi}^{(2)}$ is the two-body transition function:

$$\rho_{fi}^{(2)}\left(\vec{s},\vec{s}\,'\right)\;=\;\frac{1}{2}\int d^{3}r_{1}...d^{3}r_{A}\phi_{f}^{*}\left(\vec{r}_{1},...,\vec{r}_{A}\right)\sum_{j\,\neq\,k}\delta\left(\vec{s}\;-\;\vec{s}_{j}\right)\delta\left(\vec{s}\,'\;-\;\vec{s}_{k}\right)\phi_{i}\left(\vec{r}_{1},...,\vec{r}_{A}\right). \tag{3.21}$$

This amplitude accounts for interference (integration on $\vec{q}\,'$) from all possible ways to generate a momentum transfer $\vec{q}$ and also depends upon correlations inside the target. When calculating the cross-section (at this order)

$$\frac{d\sigma_{fi}}{d\Omega}\left(\vec{q}\right)=\left|f_{fi}^{(1)}\left(\vec{q}\right)+f_{fi}^{(2)}\left(\vec{q}\right)\right|^{2}, \tag{3.22}$$

another interference, between single and double scattering contributions, may appear.

For the elastic scattering, the interference term in Eq. (3.22) does show up, as expected, and is responsible for a destructive interference at $\vec{q}$ values for which $f^{(1)}$ and $f^{(2)}$ are approximately equal in absolute value. For inelastic scattering, even when $\rho^{(2)}\simeq\rho^{(1)}\rho^{(1)}$ (no correlation) the many terms inside expression (3.21) wash out the relative phases between $f\left(\vec{q}\right)$ and $f\left(\vec{q}-\vec{q}\,'\right)$ and quantum interferences do not show up neither in (3.21) nor in (3.22). The remaining effect of the addition of $f^{(2)}$ is a broadening of the angular distribution, as expected.

### D Fermi motion and Pauli blocking effects

Roughly speaking, the Fermi structure of the target does not play a very important role in the scattering of the incident particle, unless the incident particle undergoes a very small momentum transfer. Indeed, if the target can be viewed as a Fermi gas in a potential well, the struck nucleon cannot always receive a small momentum transfer because this is blocked by the Fermi sea. So the collision is forbidden.

This effect is most easily discussed for inclusive reactions, i.e. for reactions leading to a momentum transfer $\vec{q}$ for the incident particle irrespective of the final state in which the

target is left. According to formula (1.17), this inclusive cross-section may be written as

$$\frac{d\sigma}{d^3\vec{q}} = \frac{2\pi}{\hbar v_c} \sum_f \delta\left(E' - E + E_{f0}\right) \left|\langle \vec{k}' f | \tilde{T} | \vec{k} 0 \rangle\right|^2, \tag{3.23}$$

where we have slightly simplified our notation: $E_{f0} = E_f - E_0$ is the excitation energy gained by the target, $E$ and $\vec{k}$ ($E'$ and $\vec{k}'$) are the energy and momentum of the incident particle in the entrance (exit) channel, respectively and $T$ is the full $T$-matrix. Note that $f$ is not necessarily a bound state.

We introduce the single scattering approximation

$$\tilde{T} = \sum_{j=1}^{A} \tilde{t}_{pj}, \tag{3.24}$$

where $\tilde{t}_{pj}$ is the $t$-matrix for the scattering of the incident particle by nucleon $j$. We also introduce the Fermi gas model for the target: $|0\rangle$ is the ground state of the Fermi sea, with all states filled below the Fermi level, $|f\rangle$ is a one particle-hole state $|f\rangle = a^+_{\vec{k}_2} a_{\vec{k}_1} |0\rangle$. Only these states can be connected to $|0\rangle$ in the matrix element (3.23) since $T$ is a two-body operator. Momentum conservation imposes $\vec{k} = \vec{k}' + \vec{k}_1 - \vec{k}_2$ or $\vec{k}_2 = \vec{k}_1 + \vec{q}$. One obtains

$$\begin{aligned} &\sum_f \delta\left(E' - E + E_{f0}\right) \left|\langle \vec{k}' f | \tilde{T} | \vec{k} i \rangle\right|^2 \\ &= \sum_{\vec{k}_1} \delta\left(E' - E + \varepsilon_{\vec{k}_1 - \vec{q}} - \varepsilon_{\vec{k}_1}\right) \left|\langle \vec{k}', \vec{k}_1 - \vec{q} \left| \tilde{t}_{pj} \right| \vec{k}, \vec{k}_1 \rangle\right|^2 n_{\vec{k}_1} \left(1 - n_{\vec{k}_1 - \vec{q}}\right), \end{aligned} \tag{3.25}$$

where $n_{\vec{k}}$ is the occupation number of state $\vec{k}$: $n_{\vec{k}} = 1$, if $k < k_F$ (the Fermi momentum) and $n_{\vec{k}} = 0$ if $k > k_F$, and where $\varepsilon_{\vec{(k)}}$ ($= \varepsilon_k$) is the single-particle energy. Assuming the $t$-matrix to be a smooth function of $\vec{k}_1$ (which is rather justified), one obtains

$$\frac{d\sigma}{d^3\vec{k}} = \frac{2\pi}{\hbar v_{inc}} \left|\langle \vec{k} + \vec{q}, -\vec{q} | \tilde{t} | \vec{k}, \vec{0} \rangle\right|^2 AS\left(\vec{q}, E' - E\right), \tag{3.26}$$

with

$$S\left(\vec{q}, \omega\right) = \frac{\int \frac{d^3k}{(2\pi)^3} \delta\left(\omega + \varepsilon_{\vec{k}_2 - \vec{q}} - \varepsilon_{\vec{k}}\right) n_{\vec{k}} \left(1 - n_{\vec{k} - \vec{q}}\right)}{\int \frac{d^3k}{(2\pi)^3} n_{\vec{k}}}. \tag{3.27}$$

It is not difficult to see that $S$ depends on $\vec{q}$ through its modulus only, owing to the rotational invariance of the Fermi sea ($n_{\vec{k}} = n_k$). The matrix element in Eq. (3.26) can be related to the nucleon-nucleon cross-section. Cross-section (3.26) can be given in terms of $E'$ and

the scattering direction $\Omega'$. One has, requiring absence of interactions before and after the single scattering under consideration:

$$\frac{d\sigma}{dE'd\Omega'} = A_{eff}\left(\frac{d\sigma}{d\Omega}(\vec{q})\right)_{NN} S(q, E'-E), \tag{3.28}$$

with $q \simeq 2k\ sin\frac{\theta'}{2}$. This formula is rather transparent: once again, one obtains the additivity of the target nucleons, limited to the "effective" nucleons, and the factor $S$ accounts for Pauli blocking effects. It also allows for a spreading of the so-called quasi-elastic peak due to Fermi motion. Indeed in the absence of Fermi motion (Fermi momentum $k_F$=0), $S(q, E'-E)$ reduces to $\delta\left(E'-E-\frac{q^2}{2M}\right)$, which is of course the ordinary recoil for a nucleon-nucleon scattering. For $k_F \neq 0$, $S$ presents a maximum for $E'-E = q^2/2M$ but is different from zero, for values of $E'-E$ and $q$ not satisfying this condition. This leads to a broadening of the quasi-elastic peak. The intensity of this peak is rapidly decreasing when $\theta'$ increases because $S$ decreases with $q$, but mainly because the $NN$ cross-section is forward-peaked. Similar considerations can be made for quasi-charge exchange scattering, when an incident proton produces, in a single scattering, a neutron (or vice-versa) in the forward direction with roughly the beam velocity. Comparison of Eq. (3.28) with experiment shows a satisfactory agreement for the quasi-elastic peak, well separated from the rest of the spectra. There is however a systematic discrepancy on the location of the peak, which, in the experiment, lies at a lower energy, by $\sim$ 20-30 MeV, than in the predictions. Extension of the above formalism, explains partly the quasi-inelastic scattering, corresponding to a single $NN \rightarrow N\Delta$ scattering, with the nucleon detected at forward angles.

Double scattering has also been studied in this formalism. The developments are heavy. They do not indicate manifest interference effects, but they lead, as expected, to a (relatively small) widening of the angular distribution.

### E Classical multiple scattering

All the considerations above point out the absence of obvious quantum motion effects (no interference, no coherent effects), except of course for the necessary introduction of the Pauli

blocking. In the reaction process, quantum effects seem to be confined to the small region (and small time interval) when two nucleons closely interact. Apparently, the description of the space-time evolution of the system in this small domain may be by-passed, but quantum effects should nevertheless be retained through the elementary cross-section and its quantum probabilistic interpretation: the outgoing direction (and the outgoing channel, if one considers possible inelastic channels in elementary collisions) is not uniquely determined, but follows some probability law. These arguments strongly support the relevance of a classical multiple scattering approach supplemented by the two quantum "residual" ingredients: Pauli blocking and stochasticity of the outgoing state of an elementary collision.

Classical multiple scattering is by far much simpler that its quantum counterpart. There are two standard formulations of this theory, corresponding to two limits: (i) the mean free path $\lambda << D$, the size of the system; (ii) $\lambda \approx D$. The first one is well documented, as it corresponds to the diffusion of a particle inside another medium (like also neutrons in a nuclear reactor). Diffusion equations (heat equation is the prototype) or Fokker-Planck equations are justified. They however apply to a whole population of "incident" particles or to an average over many "incident" particles. They also imply averaging of the distributions over a distance which is larger than $\lambda$. In case (ii), the averaging over space dimensions should be avoided. The standard approach is based on numerical simulation. The nuclear case falls in this category, although $\lambda$ is smaller than, though comparable with the size of the nuclei. If $\lambda >> D$ (the Knudsen regime), single scattering is dominating and simple approaches, as the ones sketched above, are appropriate.

For nuclear collisions, the most standard simulation tool is embodied in the intranuclear cascade (INC) model, to be shortly described below. This approach, which looks like an empirical procedure, rests in fact on some theoretical foundation (not described here). It has been shown that this model can be considered as a limit of the full quantum nuclear process [14], when the following conditions are fulfilled:

$$\overline{\lambda\!\!\!^{-}_B} << \overline{v_{rel} t_{coll}} << \overline{d}, \tag{3.29}$$

where $\overline{\lambda\!\!\!^{-}_B}$ is the average de Broglie wavelength for the relative motion of the colliding

nucleons, $v_{rel}$ is their relative velocity, $t_{coll}$ is the collision time and $\overline{d}$ is the average distance between neighbouring nucleons. This is roughly equivalent to

$$\lambda\!\!\!^{-}_B << r_s << \overline{d}, \tag{3.30}$$

which ressembles condition (3.1), but for all pairs of colliding nucleons. These conditions are once again barely fulfilled in the nuclear case, especially at the end of the reaction process, when the average energy of the colliding particles are substantially lower than for the incident particle.

The theoretical approaches that are mentioned above show that the INC model amounts to solve the nuclear Boltzmann equation. They also show that there are three reminescences of quantum effects in this equation: the Pauli blocking effect, the stochastic meaning of elementary cross-sections and wave function effects occuring where the average nuclear field is varying rapidly. In very rough terms, the latter refers to the existence of quantum reflexion and transmission of particles hitting the nuclear surface.

### F The intranuclear cascade for nucleon-induced reactions

There are basically two versions of the INC model [15] (see Fig. 3): (i) a "space-like" version, in which the incident particle propagates inside a continuum representing the target and providing a mean free path. The path length is taken at random. At the end of this path the incident particle makes a collision with a nucleon promoted from the continuum. The two outgoing particles are then propagated independently and make further collisions, and so on. The particles are not followed in time. (ii) a "time-like" version, in which the nucleons inside the nucleus are given initially position and velocity. All particles are followed simultaneously in time and make collisions if their minimum distance of approach is small enough.

We now describe the Liège INC model [16], which pertains to the second category. The main features are the following:

- At time $t = 0$, the target nucleons are positioned at random in a sphere of radius $R$ and their momenta are generated randomly in a sphere of radius $p_F$. Neutrons and

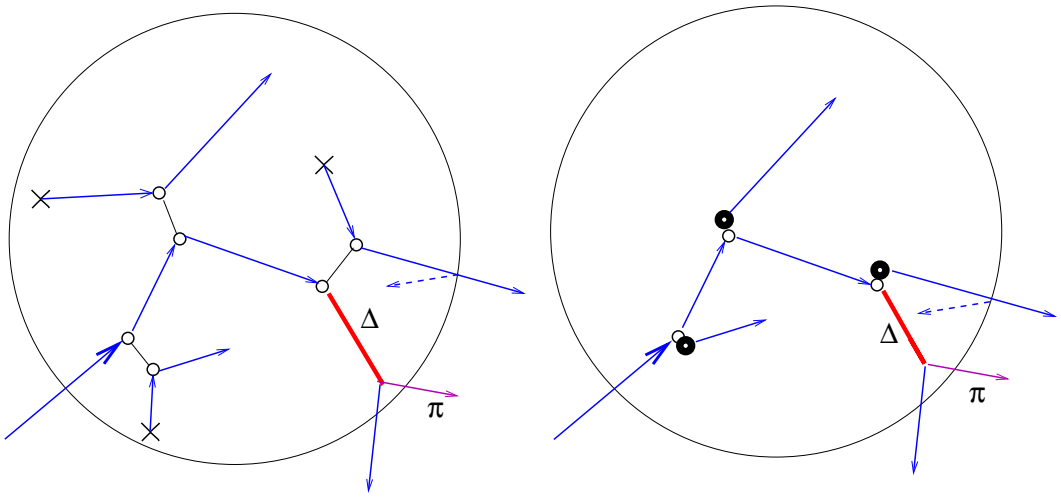


FIG. 3: Schematic illustration of "time-like" (left) and "space-like" (right) intranuclear cascade model.

protons are distinguished according to their isospin. $R$ and $p_F$ are given standard values: $R = 1.12\ \mathrm{A}^{1/3}$ fm and $p_F = 270$ MeV/c.

- Relativistic kinematics is used everywhere.
- All target particles are sitting in a (fixed and constant) attractive potential well of radius $R$ and depth $V_0$.
- The incident particle (of incident energy $T_{lab}$) is provided with an impact parameter $b$, generated randomly in a disk of radius $R$. It is positioned at the nuclear surface within the potential well. Its kinetic energy is therefore $T_{lab} + V_0$ at $t = 0$.
- All particles are set in motion and are assumed to follow straight line trajectories until two of them achieve their minimum distance of approach or until one of them hits the nuclear surface. The time at which this occurs can be predicted and the particles are therefore propagated in a single step.
- After this step, one has to distinguish between these two cases:

1. A particle hits the surface. If the energy of the particle is below the emission threshold, i.e. if its kinetic energy $T_i < V_0$, the particle is reflected on the surface. If $T_i > V_0$, the particle is allowed to be emitted according to the probability law

$$T = \frac{4\sqrt{T_i\,(T_i - V_0)}}{2T_i - V_0 + 2\sqrt{T_i\,(T_i - V_0)}} e^{-2G}, \tag{3.31}$$

where $G$ is the usual Gamow factor ($G = 0$ for neutrons). In the latter, the actual charge inside the nuclear volume is used. If the test for transmission fails, the particle is reflected. Transmission is realized by assuming the same straight line trajectory as before (no refraction) and the particle escapes from the potential well with a final kinetic energy $T_{fin} = T_i - V_0$. Its momentum is changed accordingly. The escaping particle is given a tag, which forbids any further interaction.

2. Two particles achieve their minimum distance of approach. Let us call $\vec{d}_{min}$ the corresponding relative coordinate, $\sqrt{s}$ the total center of mass energy of the two particles and $\vec{\beta}_{cm}$ the velocity of their center of mass with respect to the lab frame. Then

$$b_{min}^2 = d_{min}^2 + \frac{\vec{d}_{min}.\vec{\beta}_{cm}}{1 - \beta_{cm}^2} \tag{3.32}$$

is the squared impact parameter in the c.m. system. If $\pi b_{min}^2 > \sigma_{tot}\left(\sqrt{s}\right)$, $\sigma_{tot}\left(\sqrt{s}\right)$ being the total particle-particle cross-section at c.m. energy $\sqrt{s}$, the motion of the particles is not modified. If $\pi b_{min}^2 < \sigma_{tot}\left(\sqrt{s}\right)$, the particles are forced to scatter. Final channels are determined stochastically according to partial cross-sections, relatively to the total cross-section. The energy of the outgoing particles is determined by energy-momentum conservation. The outgoing polar angle in the c.m. frame is determined stochastically following a probability law consistent with prescribed angular distributions. Collisions are avoided when the Pauli principle is not fulfilled, as discussed below.

- After these possible changes, straight line motion is resumed until a new possible collision or reflection can occur. The process is followed up and terminated according

to a criterion explicitated below.

- The following possible reactions are considered:

$$\begin{aligned} NN &\Leftrightarrow NN, NN \Leftrightarrow N\Delta \\ N\Delta &\Leftrightarrow N\Delta \\ \Delta\Delta &\Leftrightarrow \Delta\Delta \\ \Delta &\Leftrightarrow \pi N. \end{aligned} \tag{3.33}$$

  All the utilized cross-sections and angular distributions are taken from experiment, as far as possible, and are based on the extensive analysis of existing data of Ref. [17] and on recent measurements of $np$ elastic and inelastic scattering in the 0.4-1.6 GeV range [18].

- Pauli principle is implemented as follows. Let us consider two particles at their minimum relative distance and let $i$ and $j$ denote the two particles predicted to be created in the final state. If the two particles are nucleons, the phase space occupation numbers $f_i$ and $f_j$ are evaluated by counting particles of the same kind in a reference volume, consisting of the direct product of a sphere in $r$-space (of radius $r_0$) and of a sphere in $p$-space (of radius $p_0$). The quantities $r_0$ and $p_0$ are taken as $r_0 = 2$ fm and $p_0 = 200$ MeV/c. If particles are close to the surface, only the overlap between the sphere of reference and the target volume is considered. The collision is realized (as explained above) stochastically with a probability

$$P = (1 - f_i)\,(1 - f_j)\,. \tag{3.34}$$

  Pauli principle is not applied to $\Delta$-resonances: only one blocking factor is retained in $N\Delta$ channels. On the other hand, Pauli blocking is enforced in the final state of the $\Delta$-decay.

- The interaction process is stopped at time $t = t_{stop}$, determined by the average behaviour of some quantities. The crucial one is the target excitation energy $E^*$. It is defined through the energy conservation law (which holds at any time):

$$T_{lab} = \sum_{j=1}^{N_{ej}} \overline{T}_j + \sum_{\ell=1}^{N_\pi} \varepsilon_\ell + E^* + S, \tag{3.35}$$

where the first sum runs over the kinetic energy of the (baryonic) ejectiles (the bar indicates the possible inclusion of the $\Delta - N$ mass difference for $\Delta$'s), the second sum runs over the total energy of the pions and where $S$ is the separation energy, here

$$S = (N_{ej} - 1)(V_0 - T_F), \tag{3.36}$$

where $T_F$ is the Fermi energy. The average (over events) excitation energy displays a typical behaviour: it first rises, then decreases quickly, corresponding to the ejection of fast particles, and further decays at a much slower rate, akin to an evaporation process; $t_{stop}$ is taken as the time corresponding to this change of slope.

- At the end of the cascade ($t = t_{stop}$), possibly remaining $\Delta$'s are forced to decay and final quantities are recorded.
- Physical quantities are evaluated by ensemble averages over events.
- In the most recent version, a diffuse surface is introduced.

## 4. PRE-EQUILIBRIUM REACTIONS

### A Introduction

The cross-sections of many nuclear reactions, once the incident energy exceeds $\sim 40$ MeV, are frequently much larger than would be expected from the statistical compound nucleus theory. In many cases, the excess cross-section is forward-peaked and can be attributed to a one-step direct interaction. The most famous model for the description of this mechanism is the Distorded Wave Born Approximation (DWBA), especially relevant if one looks to the reaction leading to selected low-lying states of the residue. In other cases, this explanation is inadequate and it is necessary to consider the possibility of multi-step processes that take place after the first direct collision but long before reaching statistical equilibrium. These pre-equilibrium processes, as they are called, are conveniently divided into two types, the multi-step compound reactions and the multi-step direct reactions. In a multi-step compound reaction, all the particles are bound for at least one stage of the process, whereas

in a multi-step direct reaction at least one particle is always in the continuum. In multi-step compound (MSC) reactions, phases of competing processes interfer so that energy-averaged cross-sections are roughly symmetric around 90°. Multi-step direct (MSD) reactions take place rapidly and have forward-peaked cross-sections. A general formalism has been set up by Feshbach, Kerman and Koonin. This formalism is outlined below, but before it is instructive to look at simplified quasi-classical approaches.

### B The Harp-Miller-Berne model

In this model [19], which is basically an excited Fermi gas model, one considers the occupation number for single-particle levels in the average nuclear potential well. At $t = 0$, their distribution is given by

$$\begin{aligned} n_i &= 1, \quad \text{for } k_i \leq k_F \\ &= 1, \quad \text{for } k_i = k_c \\ &= 0, \quad \text{otherwise,} \end{aligned} \tag{4.1}$$

the second line corresponding to the incident particle. The evolution of this distribution is assumed to follow the master equation:

$$\frac{dn_i}{dt} = \sum_j \sum_k \sum_\ell \omega_{ijk\ell} \left\{ n_k n_\ell \left(1 - n_j\right) \left(1 - n_i\right) - n_j n_i \left(1 - n_k\right) \left(1 - n_\ell\right) \right\} - \lambda_{esc} n_i. \tag{4.2}$$

The first term corresponds to collisions which can feed or deplete $n_i$ and the second term accounts for the possible escape of particle from state $i$, provided the latter lies above the potential well. The transition probability could in principle be calculated numerically $\omega_{ijk\ell} = \frac{2\pi}{\hbar} \left| \langle ij \left| T_{NN} \right| k\ell \rangle \right|^2$, with single-particle wave functions and the $NN$ $T$-matrix, but is most of the time roughly related to the nucleon-nucleon cross-section

$$\omega_{ijk\ell} \approx \sigma_{NN} \rho_0 \langle v \rangle, \tag{4.3}$$

where $\langle v \rangle$ is the average relative velocity. The escape probability $\lambda_{esc}$ can be calculated from transmission factors, but is usually taken as a parameter. Even when the energy of the incident particle is not very high, there is a tremendeous number of single-particle states and these equations are not easy to solve. Therefore, this model has been abandoned for the following one.

## C The Griffin exciton model

The basic idea [20] here is that, by successive collisions, the system evolves from $1p$ (one particle) states to $2p-1h$, $3p-2h$, $4p-3h$,... states, or from 1 exciton to 3, 5, 7,... excitons. Ericson [21] has evaluated the density of $p$ particle-$h$ hole states (with $n=p+h$), at excitation energy $E$:

$$\rho_n(E) = \frac{g^n E^{n-1}}{p!h!(n-1)!}, \tag{4.4}$$

where $g$ is the average density of single-particle states. This expression disregards antisymmetrization. Corrections, which are rapidly negligible when $E$ increases have been derived by Williams [22]. Griffin postulated that the probability for nucleon emission per unit time in the range $[\varepsilon, \varepsilon + d\varepsilon]$ by a $n$-exciton state is given by (basically the same argument as in the transition state model, see Section 2.E, is used)

$$P_n(\varepsilon)d\varepsilon \propto \varepsilon \frac{\rho_{n-1}(U)}{\rho_n(E)} d\varepsilon. \tag{4.5}$$

In this equation, $U$ is the excitation of residual nucleus, i.e. $U = E - \varepsilon$. The total probability for emission of nucleons is given by

$$P(\varepsilon)d\varepsilon = \sum_{[n=n_0]}^{\overline{n}} P_n(\varepsilon)d\varepsilon = \frac{m\varepsilon}{\pi^2\hbar^3} \sum_{[n=n_0]}^{\overline{n}} \frac{\rho_{n-1}(U)}{\rho_n(E)} \tau_n d\varepsilon, \tag{4.6}$$

where $\overline{n}$ is the "equilibrium number" of excitons, i.e. the value of $n$, for which the density is the largest, according to Griffin. This value is basically $\overline{n} \sim \sqrt{2gE}$. In Eq. (4.6), the summation is restricted to odd values of $n$. The quantities $\tau_n$ are the lifetimes of the $n$-exciton states. Initially, they were considered as free parameters. Later on, a so-called Hybrid Exciton model was devised, which yields the following formula for the probability $P(\varepsilon)$:

$$\begin{aligned} P(\varepsilon)d\varepsilon &= \sum_{[n=n_0]}^{\overline{n}} \left[\frac{m\varepsilon}{\pi^2\hbar^3}\frac{\rho_{n-1}(U)}{\rho_n(E)}d\varepsilon\right]\left[\frac{\lambda_c(\varepsilon)}{\lambda_c(\varepsilon)+\lambda_+^{n+2}(\varepsilon)}\right] D_n \\ &= \sum_{[n=n_0]}^{\overline{n}} P^{(n)}(\varepsilon)d\varepsilon. \end{aligned} \tag{4.7}$$

the second set of brackets gives the fraction of those particles that are emitted in the continuum rather than generating a transition from the $n$ to the $n+2$ exciton states. The

quantity $\lambda_+^{n+2}(\varepsilon)$ is the mean free path for producing this transition. It is generally either taken as the mean free path for nuclear collisions (possibly corrected for Pauli blocking): $\lambda_+^{n+2}(\varepsilon) \approx 1/\rho\sigma^{NN}$ or related to the imaginary part of the optical potential. The quantity $\lambda_c(\varepsilon)$ is the mean free path for collisions leading to the continuum. In Eq. (4.7), the quantity $D_n$ is the probability for surviving particle emission from simpler states:

$$D_n = \prod_{[n'=n_0]}^{n-2} \int P^{(n')}(\varepsilon) d\varepsilon, \; D_1 = 1. \tag{4.8}$$

A time-dependent version exists, which allows to study the relaxation of the nucleus toward an equilibrium state and the evaporation-like evolution of the latter. It writes

$$\frac{dP_n(t)}{dt} = P_{n-2}(t)\lambda_+^{n-2} + P_{n+2}(t)\lambda_-^{n+2} - P_n\left(\lambda_+^n + \lambda_-^n\right) - \lambda_{esc}^n P_n, \tag{4.9}$$

where, again, $\lambda_\pm^n$ gives the probability that the $n$ exciton state disappears by collisions leading to $n \pm 2$ exciton states and where $\lambda_{esc}^n$ is the probability for emission from the $n$ exciton state. This allows the emission of fast particles in the early stages of the process.

Up to this point, the exciton model is just able to predict the shape of energy spectrum of the emitted particles. It can predict neither cross-sections nor angular distributions. On the other hand, it is quite able to predict non-thermal energy distributions. Cross-sections can be obtained by multiplication with some total reaction cross-section, taken from elsewhere or from experiment.

Some attempts have been made to include geometrical effects. The idea is to take account of the varying conditions along the (linear) trajectory of the incident particle for different impact parameters. This model is named the Geometry Dependent Hybrid model. It yields:

$$\frac{d\sigma}{d\varepsilon} = \int_0^\infty 2\pi b db P_b(\varepsilon), \tag{4.10}$$

where $P_b(\varepsilon)$ is nothing but the quantity in Eq. (4.7), but evaluated with the conditions (density, path length) typical of the impact parameter $b$ and not with average conditions as before.

Some procedure has been devised to describe angular distribution, namely by introducing a factor which shortens the summation in Eq. (4.6) for peripheral collisions, and by trying to

retain the preferential emission at forward angles, dictated by nucleon-nucleon cross-sections. This gives

$$\frac{d^2\sigma}{d\varepsilon d\Omega} = \sigma_{react} \sum_{[n=n_0]}^{\overline{n}} P^{(n)}(\varepsilon)\, d\varepsilon \sum_L \frac{2L+1}{4\pi} f_L(n) P_L(cos\ \theta)\,, \tag{4.11}$$

where in $n = 3$ (one collision), the summation on $L$ reproduces the angular distribution of the $NN$ cross-section in the nuclear medium. Somewhat ad hoc procedures are used for larger $n$, which of course corresponds to a flattening, since the emission is then expected to be more and more isotropic. Systematics parametrizations of angular distribution and $L$-dependence have been devised by Kalbach [24]. The same author constructed also a phenomenological procedure to account for composite emission, generalizing Eq. (4.7). In fact, this is the only satisfying approach to pre-equilibrium emission of composites.

### D The Feshbach-Kerman-Koonin theory

This theory [25] can be viewed as a microscopic version of the exciton model, where the probability for exciton transitions is calculated on a microscopic basis. Furthermore, it makes a distinction between multiple compound and multiple direct processes.

The basic idea is, as in the exciton model, the successive excitation of particle-hole pairs. One can say that the entrance state is a one-particle state which can lead to $2p-1h$, $3p-2h$,...$(n+1)p-nh$ states. One distinguishes between a $P$-chain, which involves excitation of $p-h$ states with at least one particle in a continuum single-particle state of the nuclear well and the $Q$-chain, which involves excitation of $p-h$ states with all nucleons in bound states in this nuclear potential well. The system can propagate in the $P$ and $Q$-chains, with at each step $n$ the possibility to escape, i.e. to generate a final channel with an emitted nucleon. Several approximations are introduced:

1. no communication between the chains. This is a rather crude approximation, which is however compensated by the possibility of escape from the $Q$-chain. The theory is thus split into one for multi-compound processes ($Q$-chain) and another for multi-direct ($P$-chain) processes. It seems that the first one is appropriate at low energy (10-40 MeV) and the second one is more applicable at higher energy.

2. the chaining hypothesis, namely that transitions only take place from the $n^{th}$ to the $(n\pm1)^{th}$ state. This corresponds to the underlying picture of excitation or de-excitation of particle-hole pairs by two-body interactions.

3. the never-come-back hypothesis, namely that transitions only take place from the $n^{th}$ to the $(n+1)^{th}$ state. This is also physically plausible, since the transition probability is proportional to the final state density, which increases dramatically with $n$, up to some value (see above). In fact, this hypothesis has been checked by calculations without it which indeed showed that the main flux is directed toward increasing complexity.

We first explain the theory for the multi-step compound process. The total width of the $n$-state, i.e. the total probability by unit time from this state, can be divided as

$$\Gamma_n \;=\; \Gamma_n^{\downarrow} \;+\; \Gamma_n^{\uparrow}, \tag{4.12}$$

where the spreading width $\Gamma_n^{\downarrow}$ corresponds to the transition $n \rightarrow n+1$ and the escape width $\Gamma_n^{\uparrow}$ is related to the emission of a particle from the $n^{th}$ stage.

The total emission cross-section is thus given by a product of three factors

$$\sigma_{MSC} = \pi\lambda\!\!\!^{-2}_c \frac{2\pi\Gamma_1}{D_1} \sum_{n=1}^{r} \left( \prod_{k=1}^{n-1} \frac{\Gamma_k^{\downarrow}}{\Gamma_k} \right) \frac{\Gamma_n^{\uparrow}}{\Gamma_n}, \tag{4.13}$$

where the first factor is the probability to form the compound system (in $2p-1h$ states ($n=1$), note the appearence of $D_1$, the density of 2p-1h states!), as given by an optical-model-like expression, the parentheses correspond to the probability of proceeding until step $n$ without emission and the last fraction is the probability of emission from step $n$. In principle, the summation should run until infinity, but the authors propose to stop at the "equilibrium stage" $r$, somewhat arbitarily defined as the step for which $\Gamma_r/\Gamma_{r+1}$ becomes smaller than 10 percent. The reader is invited to compare Eqs. (4.7) and (4.13) and convince himself that the two approaches carry essentially the same physics.

We now briefly sketch the method for evaluating the various widths. We consider the emission of a particle with energy $\varepsilon$. One has

$$E_c = \varepsilon + S + U, \tag{4.14}$$

where $S$ is the binding energy of the emitted particle and $U$ is the excitation energy of the residual nucleus. The perturbative Fermi golden rule is used

$$\Gamma_n^{\uparrow} = \frac{2\pi}{\hbar}\overline{|\langle n|V|n+1\rangle|^2}\rho_n(U), \tag{4.15}$$

where the bar indicates in principle the average over the states $|n\rangle$ and $|n+1\rangle$, and where $\rho_n(U)$ is the density of $(n+1)p-nh$ states. In the Ericson formulation

$$\rho_n(E) = \frac{g(gE)^{2n}}{(n+1)!n!(2n)!}, \tag{4.16}$$

with $g \sim \dfrac{3A}{4\pi^2}$ MeV$^{-1}$ being the average single-particle state density. The average matrix element is usually taken as resulting from a $\delta$-interaction force

$$\overline{|\langle n|V|n+1\rangle|^2} = V_0\overline{\left|\int_0^\infty u_{j_1}u_{j_2}u_{j_3}u_{j_4}\frac{dr}{r^2}\right|^2}^{j_i}, \tag{4.17}$$

where an average over representative single-particle states $j_i$ is to be taken. Furthermore bound state wave functions are taken constant on the nuclear volume and only the scattering wave function of the emitted particle is used entirely. A similar expression is used for the spreading width $\Gamma^{\downarrow}$, with this time the matrix element involving only bound state wave functions.

Note that formula (4.10) can be modified to predict angular distributions. Roughly speaking one has

$$\frac{d\sigma_{MSC}}{d\Omega d\varepsilon} = \pi\lambda\!\!\!^{-2}_c\sum_J\frac{2\pi\Gamma_1}{D_1}\sum_{n=1}^{r}\left(\prod_{k=1}^{n-1}\frac{\Gamma_k^{\downarrow}}{\Gamma_k}\right)\sum_\ell C_{J\ell s}P_\ell(cos\ \theta)\frac{\Gamma_{n,s\ell}^{\uparrow}}{\Gamma_{n,s\ell}}, \tag{4.18}$$

where the $C$'s are angular momentum coupling coefficients. Obviously, angular momentum coupling should be introduced in the expressions for the spreading widths as well.

We now turn to the multi-step direct formulation. It can formally be written as (we retain $U$ instead of $\varepsilon$ as the energy variable)

$$\frac{d^2\sigma_{MSD}}{d\Omega dU} = \frac{d^2\sigma_1}{d\Omega dU} + \frac{d^2\sigma_M}{d\Omega dU}, \tag{4.19}$$

the sum over one-step and multi-step processes. The first term is usually taken as

$$\frac{d^2\sigma_1}{d\Omega dU} = \rho_r(U)\langle\frac{d\sigma}{d\Omega}\rangle, \tag{4.20}$$

where $\rho_r(U)$ is the density of states of the residual nucleus after the first interaction and where $\langle\frac{d\sigma}{d\Omega}\rangle$ is the average nucleon-nucleon scattering cross-section, possibly corrected for Pauli blocking. The multi-step contribution is

$$\begin{aligned}\frac{d^2\sigma_M}{d\Omega dU} = \sum_N \int \frac{d^3k_N}{(2\pi)^3} \cdots \int \frac{d^3k_1}{(2\pi)^3} \frac{d^2W_{N,N-1}\left(\vec{k}_N, \vec{k}_{N-1}\right)}{dU_N d\Omega_N} \\ \frac{d^2W_{N-1,N-2}}{dU_{N-1}d\Omega_{N-1}}\left(\vec{k}_{N-1}, \vec{k}_{N-2}\right) \ \cdots \ \frac{d^2W_{21}\left(\vec{k}_2, \vec{k}_1\right)}{dU_2 d\Omega_2} \frac{d^2\sigma_1}{dU_1 d\Omega_1}.\end{aligned} \tag{4.21}$$

The transition matrix elements are expressed by the Fermi golden rule

$$\frac{d^2W_{N,N-1}}{dU_N d\Omega_N} = \frac{2\pi}{\hbar}\overline{\left|\int d^3\sigma \chi^+\left(\vec{k}_N\right)\chi^-\left(\vec{k}_{N-1}\right)\langle\psi_f|V(r)|\psi_i\rangle\right|^2}\rho\left(k_N\right)\rho_r(U_N), \tag{4.22}$$

where $\rho(k) = \frac{mk}{(2\pi)^3\hbar^3}$ and where the matrix element can be recognized as the distorted wave Born approximation (DWBA) for the transition $\vec{k}_{N-1}\psi_i \rightarrow \vec{k}_N\psi_f$, $\psi$ referring to the target state. The average is taken over the single-particle bound states involved in the transition. The spectroscopic factors are taken equal to unity.

Using distorted waves instead of plane waves have been criticized since these waves are not arising naturally from the theory (especially absorption). Feshbach has shown that introducing appropriate averaging in Eq. (4.22) qualitatively justifies this procedure (fitting experimental data favours it as well!). One has to keep in mind however that the intensity of the interaction, which is also largely taken as a free parameter is somewhat making these considerations a little bit useless, as the magnitude of the matrix element can then be readjusted.

Calculations with MSD are rather cumbersome. However, clever approximations allow to calculate up to $N \approx$ 7-10 rather routinely.

It is to be remarked that the Feshbach-Kerman-Koonin theory is not a fully quantum mechanical theory. Intensities of emission after various steps are summed up and not the amplitudes. For MSD, quantum-mechanically calculated probabilities are used however.

## 5. CONCLUSION

A review of the description of (nucleon-induced) nuclear reactions below a few GeV incident energy has been given. Emphasis has been put on the basic concepts and on

the manifestation of quantum effects. The latter are important in the compound nucleus reactions, for which a quantum description is mandatory. Compelling arguments indicate that quantum motion effects are not manifesting themselves in the high energy regime and that classical multiple scattering approaches can be used, provided they include quantum aspects like Pauli blocking and the stochastic nature of elementary collisions, transmission and reflection by the nuclear potential boundary. We made an exploration of the various models which have been proposed for the intermediate energy regime. We have drawn the attention on the fact that most of these models have the same basic premise: namely the description of the reaction mechanism by an excited Fermi gas, whose evolution is governed by collisions. From this point of view, these models are very close to the INC model; they differ by the way they evaluate transition probabilities. The methods are often rather crude. They are also sometimes inconsistent, based on different approaches for different transition probabilities. This is to be contrasted with the unity and consistency of the INC model. These considerations lead to speculate on whether the high-energy and intermediate-energy regimes are just simply physically equivalent and could be described by some unified approach. This will probably be a matter of discussion for the next few years. The renewed interest in spallation reactions will certainly intensify such a study.

**For those who want to know more.**

We want to recommand a few books. For formal and less formal reaction theory, the old book by Golberger and Watson [1] is still a good standard. The recent book of Fröbrich and Lipperheide [3] provides an easy reference, less complete however. Let us mention also the well documented book by Feshbach [26]. For the compound nucleus reactions, we recommand the book by Lynn [27]. Concerning high energy reactions, there is no good dedicated book, but we can mention the one by Iljinov, Kazarnovsky and Paryev [28]. We recommand Ref.[16] for a detailed description of the INC model. For the intermediate energy domain, there exists a good reference book by Gadioli and Hodgson [29]. On the transition

from the compound nucleus to pre-equilibrium reactions, the reader may have a look at the excellent theoretical review by Mahaux and Weidenmüller [30].

## ACKNOWLEDGMENTS

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## APPENDIX A: DENSITY OF STATES

Formally, the density of states $\omega(E) = \sum_i \delta\left(E - E_i\right)$ is related to the partition function through the relation $\left(\beta = \frac{1}{kT}\right)$

$$Z(\beta) = \sum_i e^{-\beta E_i} = \int_0^\infty dE e^{-\beta E} \omega(E) \tag{A1}$$

and the inverse Laplace transformation

$$\omega(E) = \int_{-i\infty}^{+i\infty} d\beta e^{\beta E} Z(\beta). \tag{A2}$$

Now $Z(\beta) = e^{-\beta F}$, where $F$ is the free energy. One can write

$$\omega(E) = \int_{-i\infty}^{+i\infty} d\beta e^{\beta E + \phi(\beta)} \tag{A3}$$

with $\phi = -\beta F(\beta)$. The last integral can be calculated by the stationary point method. If $\beta_0$ makes the phase of the exponential stationary, i.e. if $\beta_0$ satisfies

$$E + \frac{d\phi}{d\beta}|_{\beta=\beta_0} = 0, \tag{A4}$$

and if $\xi = \frac{d\phi}{d\beta}|_{\beta=\beta_0}$, the integral of Eq. (A3) is equivalent to

$$\omega(E) = \sqrt{\frac{2\pi}{\xi}} e^{\beta_0 E + \phi(\beta_0)} = \sqrt{\frac{2\pi}{\xi}} e^S. \tag{A5}$$

The quantity $S = \beta_0 \left(E - F\left(\beta_0\right)\right)$ fits to the definition of the entropy: $dS = \beta_0 dE$ and $\beta_0$ thus fits to the definition of the actual temperature for a given $E$.

Now, the excitation energy of a nucleus follows roughly $E = aT^2$. The volume can be considered as fixed and thus $dS = \frac{\delta Q}{T} = \frac{dE}{T} = 2adT$. Since the entropy should vanish at $T = 0$,

$$S = 2aT \tag{A6}$$

and

$$F = E - TS = -aT^2 = -\frac{a}{\beta^2}. \tag{A7}$$

It is then easy to calculate the density of states. One gets, from (A3) and (A7)

$$\omega(E) = \sqrt{\frac{2\pi a^{1/2}}{2E^{3/2}}} exp\left\{2\sqrt{aE}\right\}. \tag{A8}$$

Better approximations yield different pre-factors, the exponential remaining basically the same, with possibly a smooth energy-dependent $a$ parameter.

The evaluation of $a$ can be done most easily in the Fermi gas limit (in the grand canonical ensemble). Then, the grand partition function writes

$$\begin{aligned} \mathcal{Z}(\beta) &= \sum_i e^{-\beta(E_i - \mu_p Z_i - \mu_n N_i)} \\ &= \sum_{\left\{ \begin{array}{l} n_1,\ n_2, ... \\ n_1^{'},\ n_2^{'}, ... \end{array} \right\}} e^{-\beta \sum_s n_s \left(\varepsilon_s^p - \mu_p\right)} e^{-\beta \sum_{s'} n_{s'} \left(\varepsilon_{s'}^n - \mu_n\right)} \end{aligned} \tag{A9}$$

where we have distinguished between neutrons and protons, where $\mu_p$ and $\mu_n$ are the proton and neutron chemical potentials reapectively and where $s$ and $s'$ label the proton and neutron single-particle states, respectively. One has

$$\ell n \mathcal{Z} = \ell n \mathcal{Z}_p + \ell n \mathcal{Z}_n \tag{A10}$$

with

$$\ell n \mathcal{Z}_p = \ell n \sum_{\{n_1, n_2 ...\}} e^{-\beta \sum_s n_s \left(\varepsilon_s^p - \mu_p\right)} = \ell n \prod_s \left( \sum_{n_s} e^{-\beta n_s \left(\varepsilon_s^p - \mu_p\right)} \right) = \sum_s \ell n \left(1 + e^{-\beta \left(\varepsilon_s^p - \mu_p\right)}\right) \tag{A11}$$

since the occupation numbers can be $n_s = 0, 1$ only for fermions. We now make the density of single-particle states continuous for simplicity with density of states $g_i(\varepsilon), i = p, n$. One has, generalizing (A11)

$$\begin{aligned} \ell n \mathcal{Z}_p &= \int_0^\infty d\varepsilon g_p(\varepsilon) \ell n \left[1 + e^{-\beta(\varepsilon - \mu_p)}\right] \\ &= \int_0^{\mu_p} d\varepsilon g(\varepsilon) \ell n \left\{ e^{-\beta(\varepsilon - \mu_p)} \left[1 + e^{\beta(\varepsilon - \mu_p)}\right] \right\} + \int_{\mu_p}^\infty d\varepsilon g_p(\varepsilon) \ell n \left[1 + e^{-\beta(\varepsilon - \mu_p)}\right] \\ &= -\beta \int_0^{\mu_p} g_p(\varepsilon) \left(\varepsilon - \mu_p\right) d\varepsilon + \int_0^{\mu_p} g_p(\varepsilon) \sum_{n=1}^\infty \frac{(-)^n}{n} e^{n\ beta(\varepsilon - \mu_p)} d\varepsilon \\ &\quad - \int_{\mu_p}^\infty g_p(\varepsilon) \sum_{n=1}^\infty \frac{(-)^n}{n} e^{n\beta(\varepsilon - \mu_p)} d\varepsilon. \end{aligned} \tag{A12}$$

The first integral is nothing but the (average)total energy at zero temperature minus $\mu_p$ times the number of protons. The other integrals can be performed easily if $g_i(\varepsilon)$ is approximated by its value at the chemical potential (Fermi energy) $g_i(\mu_i) \equiv g_i$. One has (taking account

of the one-half spin of the nucleons)

$$\ell n \mathcal{Z}_p = -\beta \left( E_p^0 - \mu_p N_p \right) - \frac{\pi^2}{6\beta} g_p, \tag{A13}$$

where $E_p^0$ is the energy at zero temperature. Since $\ell n \mathcal{Z}_p + \ell n \mathcal{Z}_n$ is just $-\beta G = -\beta(F - \mu N)$, one ends with ($g = \frac{1}{2}(g_p + g_n)$)

$$a = \frac{\pi^2}{6} (g_p + g_n) = \frac{\pi^2}{6} g. \tag{A14}$$

In a pure Fermi gas model for large nuclei

$$g(\varepsilon_F) = \frac{3A}{2T_F}, \tag{A15}$$

where $T_F$ is the Fermi kinetic energy ($\approx$38 MeV). One thus finally gets

$$a = \frac{A}{16} \mathrm{MeV}^{-1} \tag{A16}$$

More effects may be taken into account in level-density formula. Let us mention two of them. In Eq. (2.39), only the randomized energy can be related to the temperature. The collective energy should be subtracted from the available energy. This can rather easily be done for rotational energy. Pairing effects are also playing some role, as they contribute to the collective energy, but also because they can drastically change the single-particle level density $g(\varepsilon)$ around the Fermi energy.

We will also consider the density of $p$-particle-$h$ hole states, based on a single-particle spectrum with a constant density of single-particle states $g$. In general, the density of states $\rho_p(E)$ of $p$-particle states at energy $E$ is given by

$$\rho_p(E) = \frac{1}{p} \int_0^E \rho_{p-1}(\varepsilon) \rho_1(E - \varepsilon) d\varepsilon, \tag{A17}$$

since there are $p$ ways of adding a particle. If $\rho_1(E) = \text{constant} = g$, one readily gets

$$\rho_p(\varepsilon) = \frac{g^p E^{p-1}}{p!} \tag{A18}$$

and, of course, a similar expression for $\rho_h(E)$. Finally, the density of $p$-particle-$h$ hole states, with $n = p + h$ is given by

$$\rho_{ph}^n(E) = \int_0^E \rho_p(\varepsilon) \rho_h(E - \varepsilon) d\varepsilon \tag{A19}$$

$$= \frac{g^n}{p!h!} \int_0^E \varepsilon^{p-1}(E-\varepsilon)^{h-1} d\varepsilon \tag{A20}$$

$$= \frac{g^n E^{n-1}}{ph(n-1)!}. \tag{A21}$$

## APPENDIX B: GLAUBER THEORY IN THE EIKONAL APPROXIMATION

Let us first consider potential scattering. The Schrödinger equation for the scattering wave function can be written as

$$\Delta\psi + n^2(\vec{r})k^2\psi = 0, \tag{B1}$$

with $n(\vec{r})k = \sqrt{k^2 - \frac{2m}{\hbar^2}V(\vec{r})}$ and $E = \hbar^2k^2/2m$. The basic assumption of Glauber theory is that for large $k$ and small scattering angle, $\psi$ is essentially a plane wave times a slowly varying function. Writing $\psi = e^{i\phi(r)}$, one has

$$\Delta\psi = ie^{i\phi}\left(\Delta\phi + i\left(\vec{\nabla}\phi\right)^2\right) \approx -e^{i\phi}\left(\vec{\nabla}\phi\right)^2, \tag{B2}$$

neglecting higher order derivatives. Eq. (B1) simply becomes

$$\left(\vec{\nabla}\phi\right)^2 = n^2(\vec{r})k^2 \tag{B3}$$

whose solution is

$$\phi(\vec{r}) = \vec{k}.\vec{r}_0 + k\int_{\vec{r}_0}^{\vec{r}} n(\vec{r}\,')ds\,' \tag{B4}$$

where $\vec{r}_0$ is an arbitrary point, "long" before the potential domain. Choosing the $z$-axis along the incident direction and $\vec{r}$ and $\vec{r}_0$ having the same transverse coordinate, one obtains

$$\phi(\vec{r}) = \vec{k}.\vec{r} + k\int_{z_0}^{z} \left[n\left(\vec{r}\,'_\perp, z'\right) - 1\right] dz'. \tag{B5}$$

The point $\vec{r}_0$ can be taken "at minus infinity" $(\vec{r}_0 = (\vec{r}_\perp, -\infty), \vec{r}_\perp = \vec{r}\,'_\perp)$, without loss of generality. Furthermore, for large $k$,

$$k\left[n(\vec{r}) - 1\right] \simeq \frac{V(\vec{r})}{\hbar^2 k/m} = \frac{V(\vec{r})}{\hbar v}. \tag{B6}$$

This leads to the following representation of the wave function $(\vec{b} = \vec{r}\,'_\perp)$

$$\psi(\vec{r}) \approx exp\left\{i\vec{k}.\vec{r} - \frac{i}{\hbar v}\int_{-\infty}^{z} dz' V(\vec{b}, z')\right\}. \tag{B7}$$

This function cannot be a good approximation everywhere, as it relies too much on ray optics. It can be however a good approximation in the range of the potential. This is sufficient to evaluate the scattering amplitude from the standard formula

$$\begin{aligned} f(\theta) &= -\frac{m}{2\pi\hbar^2}\int d^3r e^{-i\vec{k}.\vec{r}}V(r)\psi(r) \\ &= -\frac{m}{2\pi\hbar^2}\int d^2b\int_{-\infty}^{+\infty}dz e^{i\vec{q}.\vec{b}}V(\vec{b},z)exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z}dz'V(\vec{b},z')\right\}. \end{aligned} \tag{B8}$$

The integral over $z$ can be done analytically, using

$$V(\vec{b},z)exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z}dz'V(\vec{b},z')\right\} = i\hbar v\frac{d}{dz}exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z}dz'V(\vec{b},z')\right\}. \tag{B9}$$

One gets

$$f(\theta) = \frac{ik}{2\pi}\int d^2\vec{b}e^{i\vec{q}.\vec{b}}\left\{1-e^{2i\chi(b)}\right\}, \tag{B10}$$

with the eikonal phase

$$\chi(\vec{b}) = -\frac{1}{2\hbar v}\int_{-\infty}^{+\infty}V(\vec{b},z)dz. \tag{B11}$$

This expression can be extended to multiple scattering, corresponding, in the frozen approximation, to a generalized potential

$$V(\vec{r}) = \sum_j v_j(\vec{r}-\vec{r}_j). \tag{B12}$$

The corresponding eikonal phase is just the sum of the individual phases

$$\chi(\vec{b}) = \sum_j \chi_j(\vec{b}-\vec{s}_j), \tag{B13}$$

where $\vec{s}_j$ is the tranverse position (component of $\vec{r}_j$ perpendicular to the incident direction). One may transform back the individual phases to individual scattering amplitudes by inverting Eq. (B10). Of course, one has also to take the quantum average of position $\vec{r}_j$. We just quote the result, which takes the same form for elastic or inelastic scatterings:

$$\begin{aligned} F_{fi}(\vec{q}) = \frac{ik}{2\pi}\int d^2\vec{b}e^{i\vec{q}.\vec{b}}\int d^3\vec{r}_1...d^3r_A\ \phi_f^*(\vec{r}_1,...,\vec{r}_A) \\ \left\{1-\prod_{j=1}^{A}\left[1-\frac{1}{2\pi ik}\int d^2\vec{q}\,'e^{-i\vec{q}\,'.(\vec{b}-\vec{s}_j)}f_j(\vec{q}\,')\right]\right\}\phi_i(\vec{r}_1,...,\vec{r}_A). \end{aligned} \tag{B14}$$

Expanding the curly brackets allows to decompose the amplitude into single, double,... scattering contributions.

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