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Edited by
D. Filges, F. Goldenbaum
Forschungszentrum Jülich, Germany
and
Y. Yariv
Soreq – Nuclear Research Centre, Israel

THE LIEGE INTRANUCLEAR CASCADE MODEL

J. Cugnon[1] and P. Henrotte[2]
Université de Liège, Institut de Physique B5, Sart Tilman, B-4000 LIEGE 1, Belgium

Abstract

The features of the Liège intranuclear cascade model are presented. Emphasis is put on the time structure of the model : the fate of all particles is followed as a function of time. The prescription for stopping the cascade process, before setting evaporation, is discussed. Three versions of the numerical model (or code) are shortly presented, two for nucleon-nucleus collisions, differing from each other on the treatment of the nuclear surface, and one for deuteron-nucleus collisions. A discussion of the true parameters of the model is presented. A short account of the predictive power and of the performances of the model is given. Finally, some numerical aspects of the codes are discussed.

1. Introduction

The Liège Intranuclear Cascade (INC) model has been built about twenty years ago to describe heavy ion collisions in the GeV range [1] and has been extended further to antiproton [2], nucleon [3], pion [4] and light ion induced reactions [5]. This evolution is opposite to the one of similar models and is at the origin of some of its features. We will center below on the version of the Liège INC model pertaining to nucleon and light ion-induced reactions. The latter evolved from an early extensive study of the physics of nucleon-induced reactions [5] to an elaborate tool aiming at a good description of many observables in recent ^{3}He and proton nucleus interactions in the GeV range [6-9].

In Section 2, we describe the basic premises of the Liège INC model and the ingredients of its most recent versions. Section 3 gives a short account of the performances of the models. In Section 4, we provide some information on the numerical aspects and the availability of the code.

2. The Liège INC model for nucleon-induced reactions

We first give an explicit description of the Liège INC model embodied in the numerical code denoted as incl2. We give some indications about the other most recent version incl3 and about the version concerning deuteron-induced reactions.

2.1. *The INC code incl2*

All the features of this INC model are entirely described in Ref. [10] and in previous references cited therein. The basic philosophy of the model is to follow the fate of all particles as time evolves. The main features are the following :

[1] e-mail : J.Cugnon@ulg.ac.be
[2] e-mail : P.Henrotte@ulg.ac.be

- At time t = 0, the target nucleons are positioned at random in a sphere of radius R and their momenta are generated randomly in a sphere of radius p_F. Neutrons and protons are distinguished according to their isospin. R and p_F are given in standard values : $R = 1.12\ A^{1/3}$ fm and $p_F = 270$ MeV/c.

- Relativistic kinematics is used everywhere.
- All target particles are sitting in a (fixed and constant) attractive potential well of radius R and depth V_0.

- The incident particle (of incident energy T_{lab}) is provided with an impact parameter b, generated randomly in a disk of radius R. It is positioned at the nuclear surface within the potential well. Its kinetic energy is the refire $T_{lab} + V_0$.

- All particles are set in motion and are assumed to follow straight line trajectories until two of them achieve their minimum distance of approach or until one of them hits the nuclear surface. The time at which this occurs can be predicted and the particles are therefore propagated in a single step.

- After this step, one has to distinguish between these two cases :

(i) *A particle hits the surface.* If the energy of the particle is below the emission threshold, i.e. if its kinetic energy $T_i < V_0$, the particle is reflected on the surface. If $T_i > V_0$, the particle is transmitted randomly with a probability T given by

$$T = \frac{4\sqrt{T_i(T_i - V_0)}}{2T_i - V_0 + 2\sqrt{T_i(T_i - V_0)}}\, e^{-2G} \qquad (1)$$

where G is the usual Gamow factor (G = 0 for neutrons). In the latter, the actual charge inside the nuclear volume is used. If the test for transmission fails, the particle is reflected. Transmission is realized by assuming the same straight line trajectory as before (no refraction) and the particle escapes from the potential well with a final kinetic energy $T_{fin} = T_i - V_0$. The momentum is changed accordingly. The escaping particle is given a tag, which forbids any further interaction.

(ii) *Two particles achieve their minimum distance of approach.* Let us call $\bar{d}_{min}$ the corresponding relative coordinate, $\sqrt{s}$ the energy in the center of mass of the two particles and $\bar{\beta}_{cm}$ the velocity of the center of mass with respect to the lab frame. Then

$$b_{min}^2 = d_{min}^2 + \frac{\bar{d}_{min} \cdot \bar{\beta}_{cm}}{1 - \beta_{cm}^2} \qquad (2)$$

is the squared impact parameter in the c.m. system. If $\pi b_{min}^2 > \sigma_{tot}(\sqrt{s})$, $\sigma_{tot}(\sqrt{s})$ is the total particle-particle cross-section at c.m. energy $\sqrt{s}$), nothing will be changed. If $\pi b_{min}^2 < \sigma_{tot}(\sqrt{s})$, the particles are forced to scatter. Final channels are determined stochastically according to partial cross-sections, relatively to the total cross-section (see below). The outgoing polar angle in the c.m. frame is

determined stochastically following a probability law consistent with prescribed angular distributions (see below). Collisions are avoided when the Pauli principle is not fulfilled, as discussed below.

- After these possible changes, straight line motion is resumed until a new possible collision or reflection can occur. The process is followed up and terminated according to a criterion explicitated below.

- The following possible reactions are considered

$$\begin{aligned} &NN \Leftrightarrow NN\ ,\ NN \Leftrightarrow N\Delta \\ &N\Delta \Leftrightarrow N\Delta \\ &\Delta\Delta \Leftrightarrow \Delta\Delta \\ &\Delta \Leftrightarrow \pi N \end{aligned} \tag{3}$$

All the utilized cross-sections and angular distributions are taken from experiment, as far as possible, and are based on the global analysis of the existing data in Ref. [11]. We just comment on the cross-sections which cannot be measured directly. The $NN \rightarrow N\Delta$ cross-section is taken as the total inelastic NN cross-section, which is certainly correct up to $p_{lab} = 2.0$ GeV/c ($T_{lab} = 1.27$ GeV) and the angular distributions are taken from experiment [12]. The $N\Delta \rightarrow NN$ cross-section is determined by detailed balance [1] although the angular distribution is taken isotropic. The $\pi N \rightarrow \Delta$ cross-section is taken to be equal to the experimental total cross-section up to $p_{lab} = 3$ GeV/c. For unmeasured channels, use of the isobaric symmetry (as also for $NN \rightarrow N\Delta$) is made as extensively as possible. Of course, above the Δ-resonance region, the procedure is incorrect. However, the main consequence is just the description of $\pi N \rightarrow \pi N$ process as a fast two-step process with the correct cross-section.

In $NN \rightarrow N\Delta$ process, a definite mass is ascribed to the Δ. The latter is determined stochastically according to Lorentzian distribution centered on the nominal mean Δ-mass (1232 MeV) with a width of $\Gamma = 110$ MeV, subject to cuts dictated by the available NN energy. The Δ is given a lifetime, in its proper frame, determined by an exponential law, with a mean value equal to $\hbar/\Gamma$. Of course, in $\pi N \rightarrow \Delta$ reactions, the Δ-mass is fixed by energy conservation. The Δ-decay is supposed to be anisotropic : in the $NN \rightarrow N\Delta$ process, the helicity h of the Δ-particle is taken as $h = \cos^2\theta$, where θ is the c.m. polar angle at which the Δ-particle is produced. The pion is supposed to be emitted, in the c.m. of the Δ-resonance, at an angle θ_π, with respect to the direction of motion of the Δ, given by the distribution

$$\frac{d\sigma}{d\Omega} \propto 1 + 3h\cos^2\theta_\pi \tag{4}$$

Collisions undergone by the Δ-resonance between its formation and its decay are assumed to conserve its helicity.

- The Pauli principle is implemented as follows. Let i and j denote the two particles predicted to be created in the final state. If the two particles are nucleons, the phase space occupation f_i (f_j) is evaluated by counting particles of the same kind is a reference volume, consisting of the direct product of a sphere in r-space (of radius r_0) and of a sphere in p-space (of radius p_0). The quantities r_0 and p_0 are taken as $r_0 = 2$ fm and $p_0 = 200$ MeV/c. If particles are close to the surface, only the overlap between the sphere of reference and the target volume is considered. The collision is realized (as explained above) stochastically with a probability

$$P = (1 - f_i)(1 - f_j) \tag{5}$$

Pauli principle is not applied on Δ-resonances: for instance only one blocking factor is retained in NΔ channels. On the other hand, Pauli blocking is enforced in the final state of the Δ-decay.

- The interaction process is stopped at time $t = t_{stop}$, determined by the average behaviour of some quantities. The crucial one is the target excitation energy E*. It is defined through the energy conservation law (which holds at any time)

$$T_{lab} = \sum_{j=1}^{N_{ej}} \bar{T}_j + \sum_{\ell=1}^{N_\pi} \varepsilon_\ell + E^* + S \tag{6}$$

where the first sum runs over the kinetic energy of the (baryonic) ejectiles (including possibly the Δ-N mass difference for Δ's), the second sum runs over the total energy of the pions and where S is the separation energy, here

$$S = N_{ej}(V_0 - T_F), \tag{7}$$

where T_F is the Fermi energy. There are also conservation laws for momentum and baryon number that we write

$$\vec{p}_0 = \sum_{j=1}^{N_{ej}} \vec{p}_j + \sum_{\ell=1}^{N_\pi} \vec{p}_\ell + \vec{p}_{rem} \tag{8}$$

and

$$A + 1 = N_{ej} + A_{rem} \tag{9}$$

Energy conservation is exactly fulfilled by the cascade process. The quantity E* may thus also be defined as

$$E^* = \sum_{j \in A_{rem}} \bar{T}_j - \sum_{i=1}^{A} T_i^0 + N_{ej} T_F - V_0 \tag{10}$$

where T_i^0 stands for the initial kinetic energy and where the first sum refers to the remnant. The average (over events) excitation energy displays a typical behaviour, illustrated in Fig. 1: it first rises, then decreases quickly, corresponding to the ejection of fast particles, and further decays at a much slower rate, akin to an evaporation process. On the other hand, momentum conservation is not fulfilled at any step of the cascade process, because of the reflexion of particles on the static potential wall. Therefore, Eq. (8) can only be considered as a definition of $\vec{p}_{rem}$, assuming that the model provides a good description of the momentum of the outgoing particles.

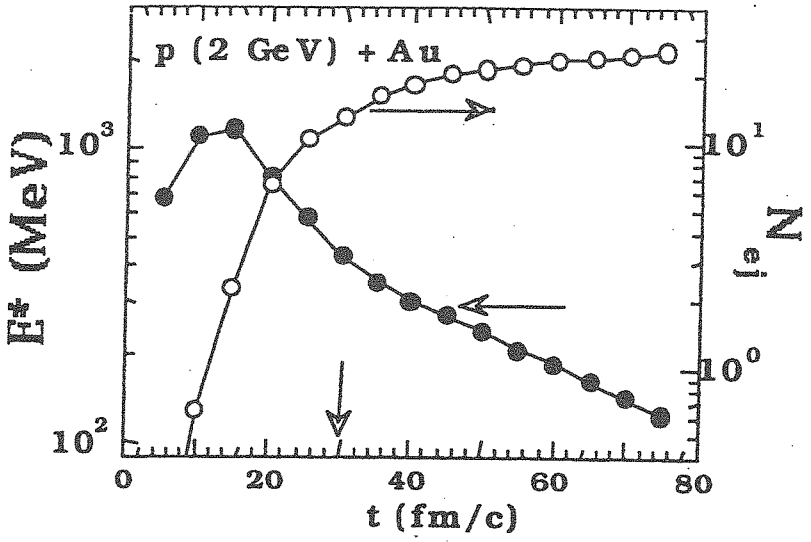

Fig. 1. *Evolution of the excitation energy (full dots, left scale) and of the number of ejectiles (open dots, right scale) in central p + Au collisions at 2 GeV. The vertical arrow indicates the value of the stopping time.*

The change of slope in Fig. 1 is correlated to changes of variation in other quantities, as explained in Ref. [7]. In particular, the momentum distribution inside the target becomes isotropic at the same time. This time determines the stopping time t_{stop}, within some uncertainty (see below). The quantity t_{stop} has been parametrized [13], once for all, for all systems between 100 MeV and 2 GeV. Note that, if E* vanishes before $t = t_{stop}$, the event is stopped, preventing E* from becoming negative, due to the stochastic handling of Pauli principle.

- At the end of the cascade ($t = t_{stop}$), remaining Δ's are forced to decay and final quantities are recorded. Physical quantities are evaluated by ensemble averages over events.

2.2. *The Liège incl3 model*

This model differs from the previous one by the treatment of the surface. In the initial sate, any nucleon is given, as before, a momentum p in agreement with the ideal Fermi gas law. Then, the same nucleon is positioned randomly in a sphere of radius R(p) given by the law

$$\left(\frac{p}{p_F}\right)^3 = \frac{1}{N}\int_0^R \left|\frac{d\rho(r)}{dr}\right| r^3 dr \qquad (11)$$

where ρ(r) is the density distribution (here a Saxon-Woods cut at $r = R_{max} \approx R + 4a$) and where N is a normalization factor equal to the same integral of Eq. (11) extended to $R = R_{max}$. It can easily be shown that this procedure amounts to sampling the following phase-space distribution

$$\frac{dN}{d^3\vec{r}\,d^3\vec{p}} = f(\vec{r},\vec{p}) = \frac{6(R(p)-r)\,\delta(p_F - p)}{\left(\frac{4\pi}{3}\right)^2 R^3(p)\,p_F^3}, \qquad (12)$$

where θ stands for the Heavyside function. It is also easy to verify, by integration over $\vec{r}$ or over $\vec{p}$, that the distributions in $\vec{r}$ and $\vec{p}$ are ρ(r) and the Fermi gas law, respectively. One has to notice however that distribution (12) introduces $\vec{r}$ - $\vec{p}$ correlations in the nuclear surface region which are different from those issued from Hartree-Fock or similar calculations [14].

Furthermore, nucleons are supposed to move in a potential well of depth V_0 and of radius equal to their corresponding R(p). In practice, this means that nucleons always feel a potential of depth V_0, but the boundary of this potential lies farther for a large momentum than for a small momentum. This procedure has the technical advantage of preserving the prediction of the time at which nucleons hit the boundary and to keep transmission as simply given by the probability law (1). This procedure gives rise to other modifications. The incoming nucleon is placed, at t = 0, further away. The excitation energy is calculated through Eq. (6) on a larger volume $(r < R_{max})$ and the stopping time is larger.

2.3. *The Liège incld model*

A model for deuteron-nucleus interaction has been built, on a scheme similar to the incl2 one. The deuteron is generated by sampling the relative distance and the relative momentum on gaussian laws inspired by known properties of the deuteron. The deuteron is positioned, for a given impact parameter, such as touching the target volume.

3. Predictive power and performances

Before discussing these matters, let us draw the attention on a physics result. For a large domain of energy, the (average) energy loss per unit of (longitudinal) length of the incident nucleon just depends upon its actual energy but is independent of the target, of the impact parameter and even of the actual position inside the target. Whether the particle is just entering the nucleus or whether it has already travelled some distance does not matter provided its energy is the same. This does allow to define a universal stopping power of nuclear matter. The results of the calculation are given in Fig. 2.

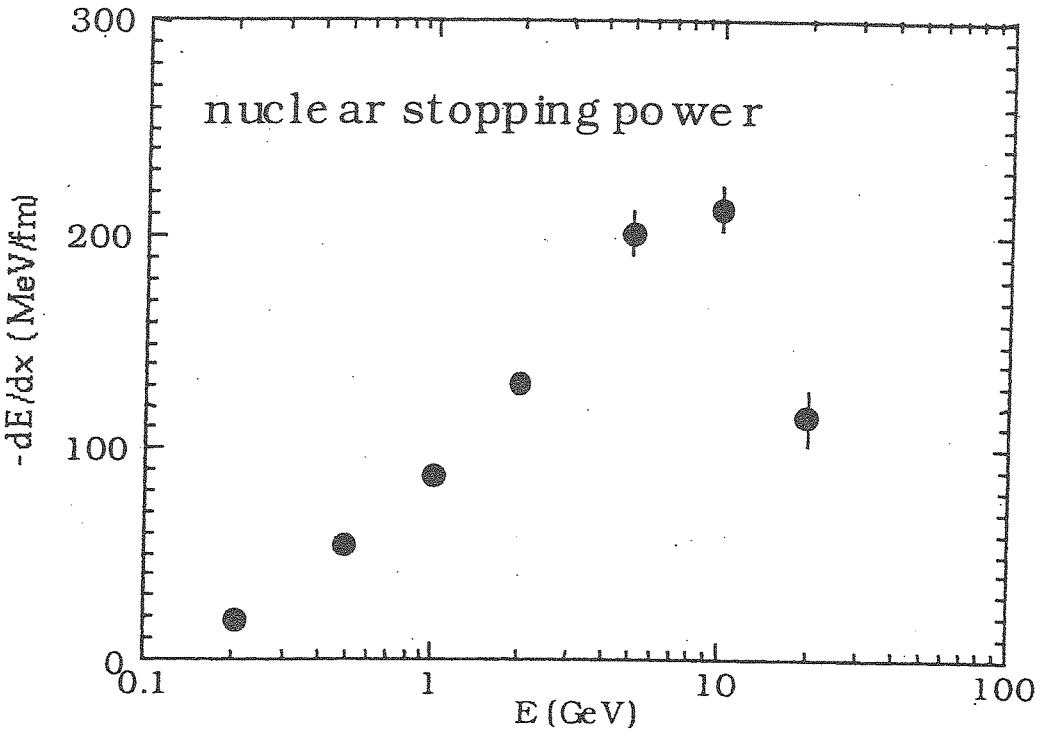

Fig. 2. *Nuclear stopping power of nuclear matter with respect to propagating nucleons, as deduced from INC calculations of Ref. [3]*

The output of the cascade model is the number of particles emitted, their type, their energy, their direction, the mass and charge of the remnant, its momentum and directions. When supplemented with an evaporation model, the Liège model can predict any cross-section. In the incl2 model, it is obvious that the total reaction cross-section is always smaller or equal to the « geometrical cross-section » πR^2, which is often inconsistent with experimental data. An option is to use these data as input and the model can then predict any other cross-sections. On the other hand, incl3 does predict, with good success, the total reaction cross-section in the GeV range. The success of the Liège INC model is testified by recent calculations [9,10,15]. We just give here the comparison of incl2 predictions with the recent measurements of neutron multiplicities in p+Pb collisions (see Table 1). Discussion of the comparison for differential cross-sections and isotopic production can be found elsewhere [9-10].

Let us mention that there are only two real parameters in the Liège model, left as so in the numerical codes. These are the depth of the mean field V_0 and the stopping time t_{stop}. In fact, for the letter, a single overall factor is considered multiplying the parametrized form alluded above. This is introduced in view of the not sharply defined change of slope in Fig.1. Of course, reasonable change of t_{stop} should not exceed 5 fm/c or so. Up to now, no other free parameter has been introduced. The sensitivity of the model to other ingredients (in particular, cross-sections, modified to account for some in-medium modifications) has been investigated, but no particular change induces drastic improvements of the agreement with the data. See however an interesting discussion on this topic in Ref. [10].

Table 1 Comparison of the incl prediction with the experimental data of Ref. [9] for neutron multiplicity and energy flux in neutron emission, in p+Pb collisions at three incident energies.

Energy	M_n^{exp}	M_n^{Th}	$E \times M_n^{exp}$	$E \times M_n^{Th}$
E = 800 MeV σ_R = 1738 mb				
0-2 MeV		4.9		4.8
2-20 MeV	6.4 ± 1.0	6.9	38. ± 4.	42.
20 - E_{max}	1.9 ± 0.2	2.2	199. ± 20.	211.
TOTAL		14.0		258.
E = 1200 MeV σ_R = 1740 mb				
0-2 MeV		5.8		5.8
2-20 MeV	8.2 ± 1.0	8.9	52. ± 6.	54.
20 - E_{max}	2.6 ± 0.3	2.8	314. ± 31.	309.
TOTAL		17.5		369.
E = 1600 MeV σ_R = 1747 mb				
0-2 MeV		6.0		6.0
2-20 MeV	9.9 ± 1.4	10.0	64. ± 8.	61.
20 - E_{max}	3.3 ± 0.4	3.1	403. ± 42.	422.
TOTAL		19.1		489.

4. Numerical details

The incl2, incl3 and incld models are translated in numerical codes, written as subroutines. The entrees do not need any particular comment, as they just define the initial state. The output includes a tag indicating whether the event is empty or not, and in the second case, the number of ejectiles and for each of them, its type, its energy and its direction of motion. The code gives also the mass, charge, excitation energy and momentum of the remnant. The recoil energy is calculated from the last quantity. In order to comply with energy conservation (as recoil is not accounted for explicitly in the cascade), this quantity is introduced in the energy balance expressed by Eq. (6) and the other quantities in the rhs of this equation are reduced proportionally. The justification of this procedure is provided by the small recoil energy compared to the available energy (one percent at the most in the GeV range).

Two versions of the incl2 routine and one of incld can be obtained from the authors on simple request. The incl3 routine is still under development. A programme including incl2 and the Dresner evaporation model [16] and preparing plenty of histograms and figures, to be generated readily by the package PAW++ from CERN, can also be obtained from the authors.

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