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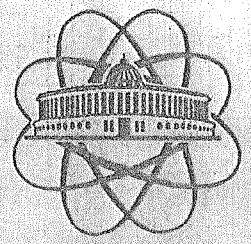

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PROPAGATION OF A PIONIUM INSIDE MATTER

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ABSTRACT

The interaction of a pionium atom colliding with an ordinary atom is described by looking at the evolution of the internal wave function driven by the time-dependent Coulomb field. This evolution is studied by solving the associated time-dependent Schrödinger equation. It is shown that the Born approximation is equivalent to the one photon exchange picture when the Coulomb form factor is taken into account. Multiphoton exchange contribution is evaluated for an illustrative case.

1. Introduction

The propagation of a pionium and of similar "atoms" inside matter have been studied for several years. In his seminal work, Mrówczyński [1] studied the elementary interaction with an ordinary atom in the one photon exchange approximation. Using this formalism, several ionisation and excitation cross-sections have been calculated by Denisenko and Mrówczyński himself [2], and by Afanasyev and Tarasov [3, 4]. The same approximation has been studied in a semi-classical picture by D. Trauttman [5]. Here, we present a model, based also on a semi-classical picture, but which allows to study the effect of multiphoton exchanges to all orders. The paper is dividorganized as follows. In section 2, we present the model. In section 3 we establish the connection with the one photon exchange models. We present typical results in section 4 and we discuss the future developments in section 5.

2. The model

Fig. 1 describes schematically the interaction of a pionium with an ordinary atom, as seen in the c.m. frame of the pionium. We assume the trajectory of the atom nucleus along the x-axis with a constant velocity v and with an impact parameter b. To simplify the presentation, we disregardin a first step the effects of the electrons and the magnetic forces. We will return to these points later. The internal wavefunction ψ of the pionium is then governed by the time-dependent Schrödinger equation

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}(H_0 + V(t)), \tag{1}$$

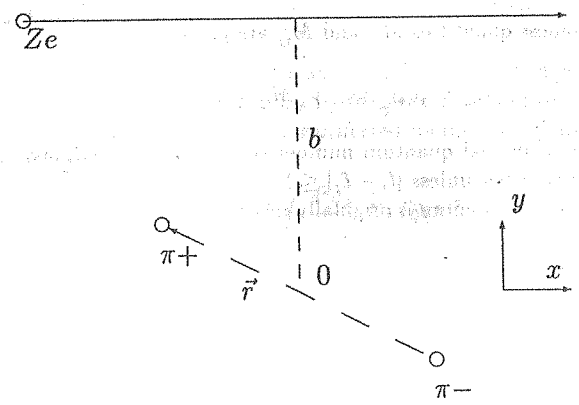

Fig.1. Schematic representation of a collision of a pionium with an atom, in the rest frame of the pionium.

where H_0 is the intrinsic pionium Hamiltonian and where $V(t)$ describes the Coulomb force between the pionium and the external charge. We limit ourselves for the moment to the dipole approximation

$$V(t) = \frac{\gamma Z e^2}{(b^2 + \gamma^2 t^2)^{3/2}} (b\vec{e}_y + \beta t \vec{e}_x) . \vec{r}, \tag{2}$$

where $\vec{r}$ is the internal coordinate of the pionium, Z is the charge of the nucleus and γ its Lorentz factor. Using the expansion of ψ in terms of the eigenstates ψ_i of H_0, i.e.

$$\psi = \sum_i c_i e^{-\frac{i}{\hbar}\epsilon_i t} \psi_i, \tag{3}$$

one obtains a set of coupled differential equations for the coefficients c_i. It is useful to introduce at this point dimensionless quantities

$$\tau = \gamma\beta t \ , \ \tilde{b} = b/a \ , \ \eta = \frac{Ze^2}{\hbar v}, \tag{4}$$

where a is the pionium Bohr radius. The equations for the c_i's write

$$\frac{dc_i}{d\tau} = \frac{-i\eta}{\tilde{b}(1+\tau^2)^{3/2}} \sum_j exp\left[-i\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right)\frac{\eta\tilde{b}\tau}{\gamma Z}\right]\left(M_{ij}^T + \frac{\tau}{\gamma} M_{ij}^L\right) c_j(\tau), \tag{5}$$

where the dimensionless quantities M_{ij}^T and M_{ij}^L are given by

$$M_{ij}^T = \langle\psi_i|\frac{y}{a}|\psi_j\rangle \quad , \quad M_{ij}^L = \langle\psi_i|\frac{x}{a}|\psi_j\rangle \tag{6}$$

and where n_i is the principal quantum number of state i. The following selection rule holds: the coupling vanishes unless $|\ell_i - \ell_j| \leq 1 \leq \ell_i + \ell_j$ and $(-)^{\ell_i - m_i} = (-)^{\ell_j - m_j}$.

Assuming that the pionium is originally in state ψ_i, the Born approximation yields

$$c_f(t \to \infty) = -i\frac{\eta}{\tilde{b}}\int_{-\infty}^{+\infty} \frac{e^{iA_{if}\tau}}{(1+\tau^2)^{3/2}}\left(M_{if}^T + \frac{\tau}{\gamma}M_{if}^L\right)d\tau, \tag{7}$$

which gives

$$|c_f(t \to \infty)|^2 = \frac{4\eta^2}{\tilde{b}^2}\left\{|M_{if}^T|^2 A_{if}^2 K_1^2(A_{if}) + \frac{|M_{if}^L|^2}{\gamma^2}A_{if}^2 K_0^2(A_{if})\right\}, \tag{8}$$

where $A_{if} = \frac{1}{2}\frac{\eta}{Z}\frac{\tilde{b}}{\gamma}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ and where K_ν is the modified Bessel function of order ν.

It is interesting to look at the limiting behaviour of Eq. (8). If $\tilde{b} << 2\tilde{\alpha}^{-1}$, $\tilde{\alpha} = \alpha|n_f^{-2} - n_i^{-2}|$, α being the fine structure constant, Eq. (8) becomes

$$|c_f(t \to \infty)|^2 \approx \frac{4\eta^2}{\tilde{b}^2}|M_{if}^T|^2. \tag{9}$$

If $\tilde{b} >> 2\tilde{\alpha}^{-1}$, then

$$|c_f(t \to \infty)|^2 \approx \frac{\pi\eta^2}{\tilde{b}}\tilde{\alpha}e^{-2\tilde{\alpha}}|M_{if}^T|^2. \tag{10}$$

These equations are valid in the ultrarelativistic case, for which the longitudinal contribution is always much smaller than the transverse one.

The transition cross-section is given by

$$\sigma_{i\to f} = 2\pi a^2\int_0^\infty \tilde{b}d\tilde{b}|c_f(t \to \infty)|^2. \tag{11}$$

At small impact parameters, this integral is divergent when the Born approximation (8) is used. This divergence disappears when the exact solution is used. However, at very small b's, the coupling in Eq. (5) becomes extremely large and solving these equations constitutes a serious numerical problem. At large impact parameters, integral (11) is convergent, because of the behaviour shown in Eq. (10), which translates the fact that the perturbation is not large enough to induce transitions in the pionium. There is another cut-off factor induced by the electron screening. Taking the latter into account by a Molière form factor [6], introduces another cut-off at large $\tilde{b}$, i.e. for $b > a_{sc} \approx 121\ \hbar c/m_e Z^{1/3}$ the probability $|c_f|^2$ is vanishingly small. We do not discuss this point here any further.

The divergence at small b of the Born approximation is due to the neglect of the quantum motion of the external charge. How to cure this situation is described in the next section.

3. The one-photon exchange

The interaction of a pionium with an atom has been studied field-theoretically in the one-photon exchange approximation by Mrówczyński and others [1, 3, 4]. We just quote the result in the high energy limit (transferred momentum $\vec{q}$ mainly tranverse and negligible total energy transfer):

$$\sigma_{i\to f} = 4\eta^2\int\frac{d^2q_\perp}{q_\perp^4}|F_{if}(\vec{q}_\perp)|^2, \tag{12}$$

with

$$F_{if}(\vec{q}_\perp) = 2i\langle\phi_f|\sin(\frac{1}{2}\vec{q}_\perp.\vec{r})|\phi_i\rangle. \tag{13}$$

Using the transformation $q_\perp = \frac{1}{b}$, Eq. (12) transforms into

$$\sigma_{i\to f} = 2\pi a^2\int_0^\infty \tilde{b}d\tilde{b}\ 4\eta^2|\tilde{F}_{if}(b)|^2, \tag{14}$$

with

$$\tilde{F}_{if}(b) = 2i\langle\phi_f\ |\sin\left(\frac{1}{2}\frac{y}{b}\right)|\phi_i\rangle. \tag{15}$$

This expression is to be compared with Eqs. (11) and (6). In the one-photon exchange picture, there is no cut-off at large impact parameter, presumably because of the neglect of the energy transfer. This is, of course, harmless as the Molière form factor will ensure convergence at large $\tilde{b}$. On the low impact parameter side, the proper handling of the quantum motion of the atom amounts to replacing M_{if}^T (Eq. (6)) by $\tilde{b}\tilde{F}_{if}(b)$. It is evident that these quantities are closer and closer to each other when $\tilde{b}$ is getting larger and larger.

4. Numerical results

We present here a few illustrative preliminary results. We solved the set of Eq. (5) for the case of an incoming pionium in its ground state interacting with Cu atoms. The typical evolution of some of the coefficients $c_j(\tau)$ is displayed in fig. 2 for $\tilde{b} = 1.5$ and $\gamma = 20$. The full line corresponds to j being $n = 2$, $\ell = 1$, $m = 1$, and the symbols (squares) give the result in the Born approximation. The importance of higher orders is illustrated by the dottedlower curve, which gives the coefficient c_j for j being $n = 2$, $\ell = 0$, $m = 0$. In the Born approximation, this transition is suppressed, because of the selection rules indicated below Eq. (6). The transition is possible in at least two steps: $100 \to 211 \to 200$.

We also investigated the impact parameter dependence and calculated integrated cross-sections. In order to avoid the divergence at small b and according to the discussion of the previous section, we substitute M_{ij}^T by $\tilde{b}\tilde{F}_{ij}(b)$, that we approximate by $\tilde{b}\ sin\left(\frac{1}{b}\langle\phi_j|y|\phi_i\rangle\right)$. We show in fig. 3 the value of $|c_{ij}(t \to \infty)|^2$ for i being the ground state and j being the $n = 2$, $\ell = 1$, $m = 1$ state. The dots give the approximate value given by Eq. (9). The heavy (upper) line gives the Born approximation (Eq. (8)), using the Molière-Thomas-Fermi density for the atom (see [6]). This in fact introduces an extra factor of the form

$$f_M = \sum_{i=1}^{3}\alpha_i\left(1 + \frac{b_i}{\tilde{a}_{sc}}\tilde{b}\left(1+\tau^2\right)^{1/2}\right)\exp\left(-\frac{b_i\tilde{b}}{\tilde{a}_{sc}}\left(1+\tau^2\right)^{1/2}\right) \tag{16}$$

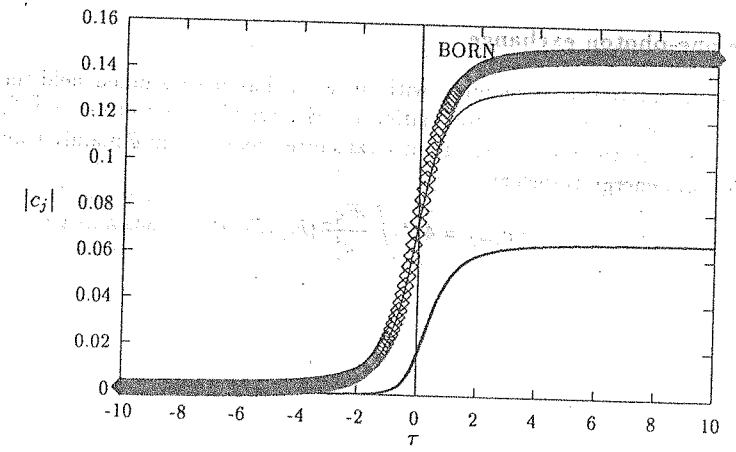

Fig. 2. Variation of $|c_j(\tau)|$ with the reduced time τ (Eqs. (3)-(5)) during the collision of a pionium originally in its ground state with a Cu atom. The upper full line corresponds to the exact 211 level amplitude. The squares represent the same quantity in the Born approximation. The lower full line corresponds to the 200 level amplitude. The parameters are $\tilde{b} = 1.15$ and $\gamma = 20$.

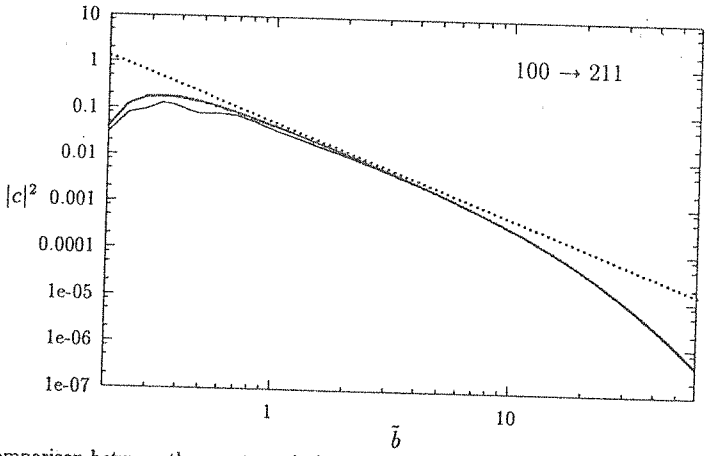

Fig. 3. Comparison between the exact result (thin full line), the Born approximation (thick full line) and expression (9) for the $100 \rightarrow 211$ excitation probability of a pionium after a collision with a Cu atom.

in the r.h.s. of Eq. (5). The numerical constants α_i, b_i are taken from Ref. 2 and a_{sc} is the reduced Thomas-Fermi radius of the atom

$$\tilde{a}_{sc} = \frac{a_{sc}}{a} = \frac{1}{a}\left(121 \frac{\hbar c}{m_e c^2} \frac{1}{Z^{1/3}}\right). \tag{17}$$

It can be seen from fig. 3 that expression (5) gives a good description of the Born approximation (and even of the "exact" solution) in a wide range of impact parameter.

Departure from the Born expression arise at $b \lesssim a$, due to the introduction of the form factor $\tilde{F}_{ij}$, and at large impact parameters $b \gtrsim a_{sc}$ ($\tilde{a}_{sc} = 39$ in the present case), due to electron screening. The cut-off imposed by the quantum spectrum of the pionium and illustrated by Eq. (10) is obscured by the electron screening in this case. The latter would occur for $\tilde{b} \gtrsim 50$ (see Eq. (10)). The dominance of the atom electron screening would even increase for larger values of Z.

The exact solution of Eq. (5) is given by the thin (lower) line. The separation of the two lines illustrates the difference with the Born approximation and hence, owing to our discussion of section 3, the importance of the multiphoton exchanges. As can be seen, the latter are important for $b \lesssim a$. The global importance of the multiphoton exchanges is shown in the Table below, giving the integrated cross-section for the $1s \rightarrow 2p$ transition and the comparison with various approximations. The difference between the exact result and the Born approximation amounts to a reduction of ~ 10 %.

$100 \rightarrow 211$ Cross-section (cm^2)		
$\tilde{F}$ form factor	BORN	3.223×10^{-21}
	exact	2.863×10^{-21}
sharp cut-off $b \geq 1$	BORN	2.389×10^{-21}
	exact	2.241×10^{-21}
one photon exchange (ref. [2])		3.0×10^{-21}
Eq. (18)		3.07×10^{-21}

This sizeable effect is due to the fact that the $b < a$ range accounts for $\sim 1/4$ of the total cross-section. For the same reason, a sharp cut-off, disregarding this range (instead of using the $\tilde{F}_{if}$ form factor), gives very inaccurate results. Our numerical results in Born approximation do not reproduce exactly the one-photon exchange result of Ref. 2. This is, of course, due to the fact that we approximate the effect of the Coulomb form factor $\tilde{F}$ at small b. In fact, we are able to reproduce the result of Ref. 2, if we used $\frac{\tilde{b}}{0.9}\tilde{F}_{if}(0.9\ b)$. We are presently working on a better formulation, but the importance of multiphoton exchange remains the same. Finally, it is worthwhile to note that using approximation (9), between $1 \leq \tilde{b} \leq \tilde{a}_{sc}$, that gives

$$\sigma_{i \rightarrow f} = 8\pi a^2 \eta^2 |M_{if}^T|^2 \ell n\ \tilde{a}_{sc}, \tag{18}$$

when inserted in Eq. (11), yields a very good numerical evaluation of the one-photon exchange result.

5. Conclusion

We have presented a simple formalism which enables us to evaluate the multiphoton exchange contribution to the excitation of a pionium colliding with atoms. Although our approach has to be improved at small impact parameters, due to the approximate treatment of the singularities in the Coulomb potential, the evaluation of the size of the multiphoton exchange is quite reliable. Furthermore, our approach is potentially able to account for quantum interference in multiple collisions with atoms. We will investigate this point in the near future.

On the numerical side, further refinements should be added to reach a one percent accuracy, necessary for instance to analyse the results of the DIRAC experiment [7]. We

have first to go beyond the dipole approximation (Eq. (2)). Also we have to introduce magnetic effects. In lowest order, the latter are negligible as there is no Zeeman effect on self-conjugate systems (opposite charges, equal mass) in a homogeneous magnetic field. Only the gradient of the magnetic field can induce some effects. The size of the latter has been evaluated [2] and found of the order of 1-2 %.

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