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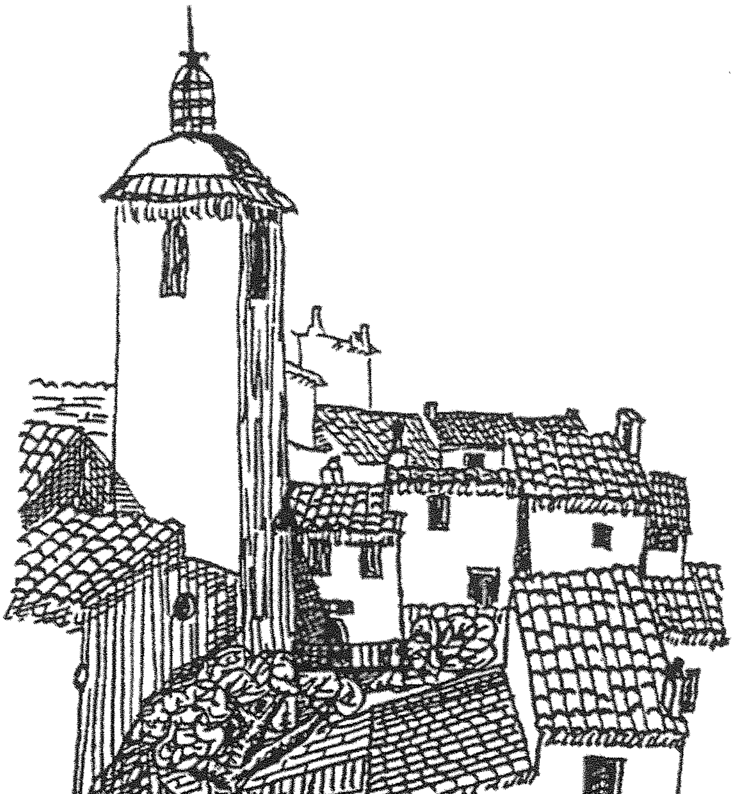

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New Perspectives in Intranuclear Cascades Calculations

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Abstract : The intranuclear cascade model is briefly reviewed. The underlying physical picture, the theoretical basis and the state of the art in the description of proton-induced spallation reactions are discussed. Currently investigated and other potential improvements of the model are presented. The need for taking account of quantum correlations is underlined.

1. Introduction

The intranuclear cascade (INC) model has recently received a new impetus, due to the renewed interest in proton induced spallation reactions, triggered itself by the perspective of building hybrid systems of nuclear energy production. Historically, this is the third stage of development of the INC model, after the first one which occurs in the fifties and the second one, which was driven by the high energy heavy ion collisions at the BEVALAC in the eighties. Here we plan to investigate the potential improvements of the model (Section 5), but previously, we will first describe shortly the model (Section 2), insisting on its versatility, present its theoretical foundations (Section 3) and the state of the art for proton-nucleus reactions in the GeV range (Section 4).

2. Physical picture and versatility

In short, the INC model pictures the nuclear collision process as a succession of binary baryon-baryon collisions. There are however basically two lines of approach, illustrated in figure 1. In the first one, all particles are propagating freely until two of them reach their minimum distance of approach, when they can scatter on each other if this distance is small enough $\left(d_{min} \leq \sqrt{\sigma_{tot}/\pi}\right)$. In the other line of approach (BERTINI), the target is seen as a continuous medium providing the particles with a mean free path $\lambda = (\rho\sigma)^{-1}$. After a path, determined stochastically according to an exponential law, the particle is supposed to scatter on a nucleon, which is promoted from the continuum and which is given a mean free path also. In the first type of approach, there is a time ordering of the collisions, but not in the second (more modern versions, like ISABEL propagates active particles by small steps).

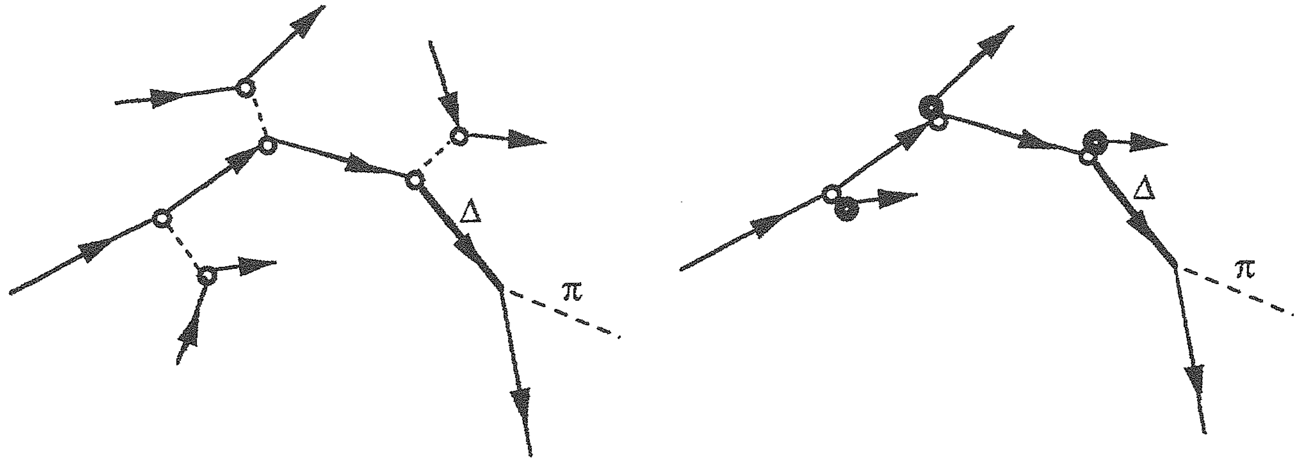

Fig. 1. Schematic representation of the INC models of the first type (left) and of second type (right). In the latter case, nucleons promoted from the continuum are indicated by heavy dots.

Detailed descriptions of the INC model can be found elsewhere (see Refs. [1,2] for the first type and Refs. [3,4] for the other ones). It is sufficient to say here that features like Fermi motion, Pauli blocking, inelastic collisions (through Δ-excitation), (constant) mean fields, are included. Collisions are described stochastically, with final states selected according to known data ($\sigma_{inel}/\sigma_{el}$ for the inelasticity, $d\sigma/d\Omega$ for the scattering angle). Therefore, observables are given by ensemble averages.

The conditions of validity of the INC picture are presumably the following :

$$\frac{\lambda\!\!\!^{-}_B}{v} << \tau_{coll} << \Delta t_{coll} \tag{2.1}$$

where $\lambda\!\!\!^{-}_B$ is the de Broglie wave length of the nucleons, v the average relative NN velocity, τ_{coll} the collision time and Δt_{coll} is the time interval between two successive collisions. One should notice that condition (2.1) is marginally verified in nuclear collisions.

The INC model has a great versatility, which covers three aspects :

(i) observables : inclusive and exclusive cross-sections, correlations (as the fate of all particles are followed), fluctuations and fragment production (if an ad hoc procedure is added) ; (ii) the systems : heavy ion collisions, p-nucleus, $\bar{p}$ annihilation on nuclei, π-induced reactions,... ; (iii) it also provides a physical picture for the reaction process. We will illustrate this point for ~ 1 GeV proton-induced reactions. The most important points are given below.

(1) The available incident energy is progressively shared, by the proton itself, the kinetic energy of the ejectiles, the pions and the target :

$$W_p^0 = W_p(t) + K_{ej}(t) + W_\pi(t) + E^*(t) \ , \tag{2.2}$$

where $E^*(t)$ is the excitation of the target. As shown in Ref. [5], the proton travels through the nucleus in 10 fm/c or so. It has transferred an energy $W_p(t \to \infty) - W_p^0$, basically to the nucleus which gets for a while a high excitation energy (see Figure 2). The latter is largely removed, and rather quickly (in 20 fm/c or so), by emission of a few fast nucleons and pions. The remaining excitation energy is further removed much more slowly (as indicated by the change of slope in Figure 2) and by emission of slow nucleons, very much akin to an evaporation process. Therefore the usual procedure is to stop the cascade at the time t_{sl} determined by the change of slope and to crank an evaporation model, knowing the properties of the residue at this time (basically $E^* = E^*(t_{sl})$).

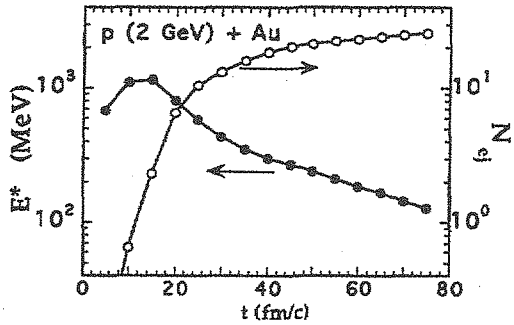

Fig. 2. Time evolution of the target excitation energy (left scale) and of the number of ejectiles (right scale) in b = 2 fm collisions for the indicated system.

(2) The nuclear density is not very much affected.

(3) The maximum relative energy transfer $\frac{W_p^0 - W_p(\infty)}{W_p^0}$ occurs between 1 and 2 GeV incident energy, whereas the E^* is slowly increasing with incident energy.

(4) On the average, the proton loses energy with a rate which is universal ; therefore, one can define a nuclear stopping power, which is given by Figure 3.

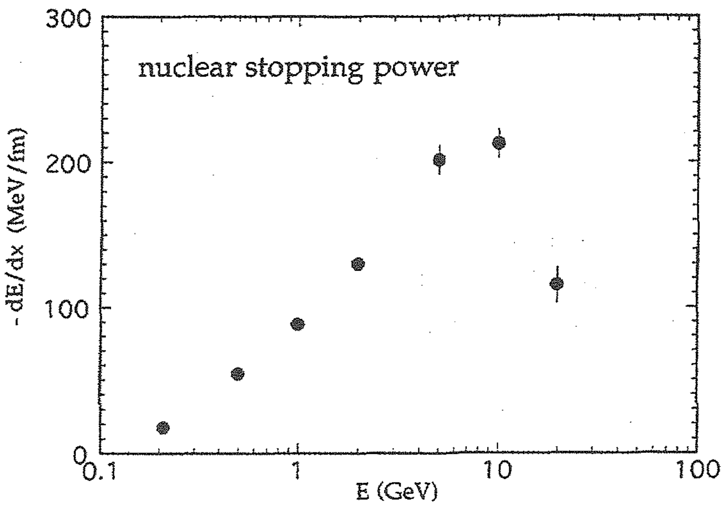

Fig. 3. Nuclear stopping power for nucleons. From Ref. [5].

(5) The number of ejected particle peaks also around 2 GeV.
(6) There are a lot of fluctuations in the observables indicated in Eq. (2.2).

3. Theoretical basis

Condition (2.1) being reminiscent of the condition of validity for the Boltzmann equation, there should be a relationship between INC and the latter. Actually, in the last ten years, there has been a considerable progress in deriving a Boltzmann-like equation in the nuclear case. Information can be found in Refs. [6,7]. Let us just state here the main results. Starting with the BBGKY hierarchy of the reduced density matrices, cutting this hierarchy at a suitable and reasonable place and using Wigner transforms and the so-called weak gradients approximation, one arrived at the following transport equation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}.\vec{\nabla} f - \vec{\nabla} U.\vec{\nabla}_p f + \vec{\nabla}_p U.\vec{\nabla} f =$$

$$\int \frac{d^3p_2}{(2\pi)^3} \int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} \left| G(12 \to 34) \right|^2 \{ff_2(1-f_3)(1-f_4) - f_3f_4(1-f_2)(1-f)\}\, \delta^3(\vec{p})\, \delta(e(p)) \tag{3.1}$$

where f stands for $f(\vec{r}, \vec{p}, t)$, basically the probability of finding a nucleon at place $\vec{r}$ with momentum $\vec{p}$ at time t, f_i for $f(\vec{r}, \vec{p}_i, t)$. The δ-functions stand symbolically for the momentum and energy conservation laws. In addition

$$e(p) = \frac{p^2}{2m} + U(p) \tag{3.2}$$

$$U(p) = \int \frac{d^3p'}{(2\pi)^3} \langle \vec{p}\,\vec{p}\,' | G(\rho(r)) | \overline{pp}\,' \rangle f(r, \vec{p}, t) \tag{3.3}$$

and G is the Brueckner matrix, solution of

$$G = V + V \frac{Q}{E - H_{12}} G, \tag{3.4}$$

where Q is the Pauli operator acting in the intermediate states. This G-matrix describes the scattering of two nucleons, but influenced by the other nucleons in the medium. The Boltzmann equation is the limit where $U = 0$, the factors $(1 - f)$ are missing and G is replaced by T.

It has been shown that the INC is solving the Boltzmann equation (with the

1 - f blocking factors) on the average. Mathematically, the stochastic occurrence of collisions corresponds to the Monte-Carlo evaluation of the collision integral in (3.1). Let us stress anyway that the INC model is doing more than just solving the Boltzmann equation as it propagates N-body correlations, whereas the latter deals with the one-body distribution only.

4. Comparison. State of the art

A detailed comparison between the currently used INC codes lies beyond the scope of this paper. Let us stress two points, which clearly contrast the two types of INC. First, the criterion for collisions is related to the one-body density in the second type (BERTINI-like) whereas it involves the two-body correlations in the first type. Second, the termination of the cascade is somewhat arbitrary (no particle having more than 1.2 times the Fermi kinetic energy) in the second type, whether it is dictated by the resulting flow of particles in the first type (see Fig. 2).

As far as the predictions are concerned, we give in Fig. 4 an idea of the kind of results one obtains for spallation reactions. The predictions of a BERTINI cascade calculation supplemented by a standard evaporation code [8] are compared to the paradigm of the data, namely those for Amien et al. [9]. Globally, the agreement is satisfactory, but locally, predictions may be off by a factor 2. It is interesting to note (see Fig. 5) that the spectrum may be split almost abruptly into a cascade part for E ≥ 20 MeV and an evaporation part which largely dominates the low energy part. Results with the Liège code are equally good [10]. They however underestimate the cross-section at smaller angles. On the other hand, this code reproduces [11] surprisingly well the 257 MeV data [12], as illustrated in Fig. 6, the conditions (2.1) being not really fulfilled in this case.

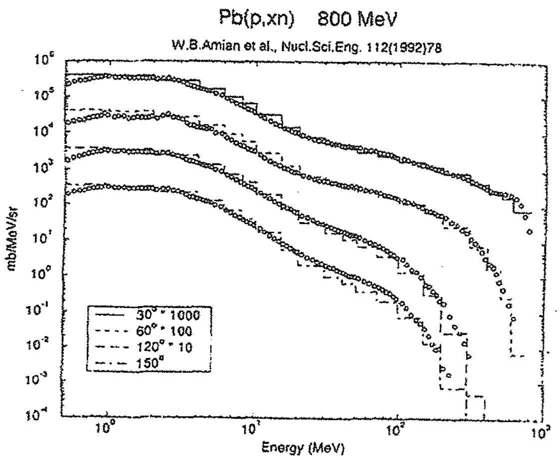

Fig. 4. Neutron double differential cross-section data (Ref. [9]) compared with the INC calculations of Ref. [8].

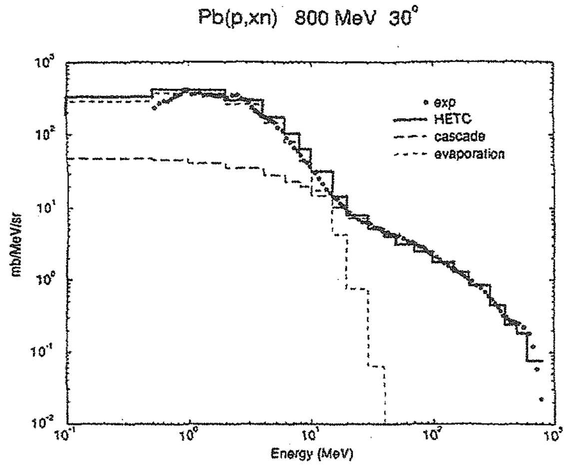

Fig. 5. Splitting of the neutron spectrum in cascade and evaporation contributions (from Ref. [8]).

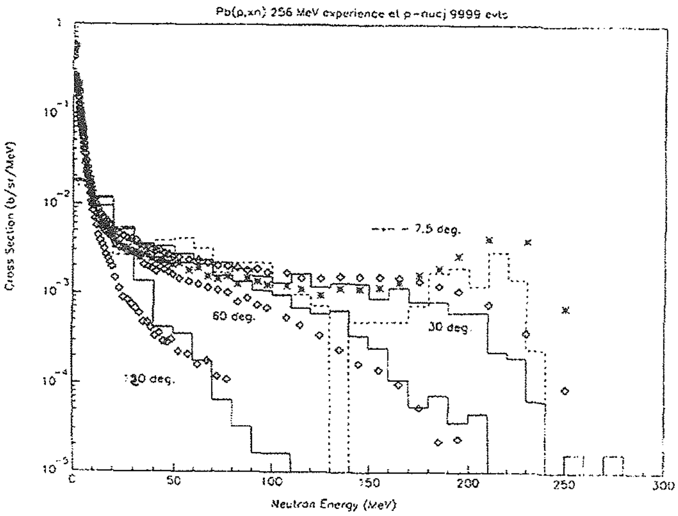

Fig. 6. Comparison of neutron double differential cross-section data (Ref. [12]) with the predictions of the Liège INC model (from Ref. [11]). Only the cascade component is shown.

5. Improvements. Open problems

5.1. Ingredients

Eq. (3.1) provides a clue for possible improvements. First one has to use in-medium cross-sections instead of free space ones. On the average their ratio, for a nucleon of momentum k travelling in an infinite Fermi sea, can be defined as

$$\alpha(k,\rho)=\overline{\frac{\sigma_{\rm med}}{\sigma_{\rm free}}}=\frac{\mathrm{Im}\,\overline{\left\langle \overrightarrow{kk'}\,|G|\,\overrightarrow{kk'}\right\rangle}^{k'}}{\mathrm{Im}\left\langle \overrightarrow{kk'}\,|T|\,\overrightarrow{kk'}\right\rangle}\ . \tag{5.1}$$

Two calculations of this quantity are compared in Fig. 7. They do not agree for $\rho=\rho_0$. The reduction of the cross-section could lie between 1 and 0.5. A recalculation of this effect is currently under way, with special attention to the full set of kinematic variables [13].

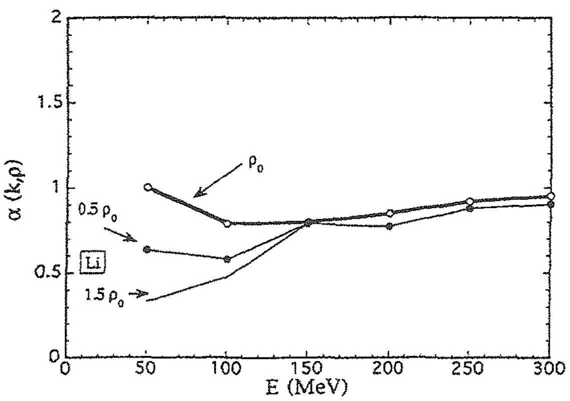

Fig. 7. Quantity $\alpha(k,\rho)$ as a function of the incident particle energy : predictions of Ref. [24] (heavy line) and those of Ref. [22].

The average nuclear mean field is momentum-dependent. This is a well established fact. If this dependence is quadratic, Eq. (3.2) shows that this feature is equivalent to having a constant effective mass

$$e(p) = \frac{p^2}{2m} - U_0 + ap^2 = \frac{p^2}{2m^*} - U_0 \ , \tag{5.2}$$

with $m^*/m \approx 0.7\text{-}0.8$ (at low momentum at least). This may have at least two consequences : (i) first, the energy conservation delta function appearing in (5.1) can be written as

$$\delta(e(p)) = \frac{m^*}{m}\,\delta\left(\frac{p^2}{2m} + \frac{p_2^2}{2m} - \frac{p_3^2}{2m} - \frac{p_4^2}{2m}\right) . \tag{5.3}$$

The INC simulations correspond to using the second delta function. Even requiring that the sum of $e(p)$ is exactly conserved would not be sufficient, as it does not include the phase space distortion factor ; (ii) the bottom of the average well U_0 may be different for nucleon and delta's (it is not well known for the latter). Therefore, a difference may play the role of shifting the effective inelastic threshold.

5.2 Treatment of quantum effects

Figures 8 and 9 show that nuclear matter does not behave as a free Fermi gas. Figure 8 gives the theoretical predictions [14] for the momentum states occupation. The Fermi sea is depleted by as much as 20 %. This is confirmed for nuclei [15]. Fig. 9 gives the probability of finding a particle with momentum k and energy ω [16]. Clearly, this shows that in an interacting Fermi liquid, the particles are off their energy shell, as their energy and momentum are not univocally related. It is only on the average that $\bar{\omega} = e(p)$, the latter being given by Eq. (3.2).

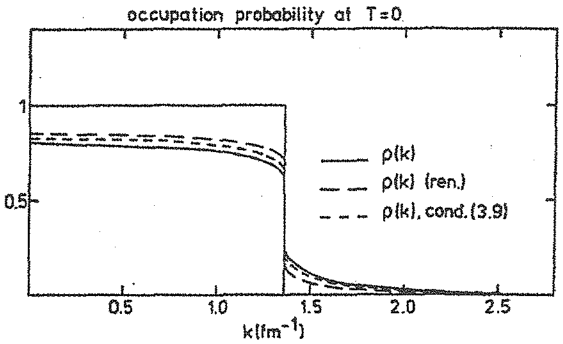

Fig. 8. Momentum distribution in normal nuclear matter, as calculated in Ref. [14]. The various lines correspond to different approximations.

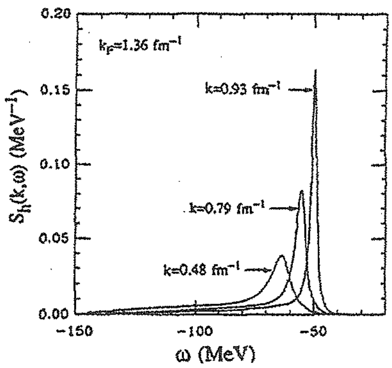

Fig. 9. Hole structure function, i.e. probability of having nucleon with momentum k and energy ω , for normal nuclear matter for three values of k. Adapted from [16].

This aspect, due to quantum correlations, has been neglected in the derivation of the nuclear Boltzmann equation in Sect. 3 (actually, all quantum motion aspects have been washed out by the small gradient approximation). It may be introduced by using the so-called causal Green's function

$$G^{<}(\vec{r}, t, \vec{r}', t') = \langle T\psi^{+}(\vec{r}, t)\psi(\vec{r}', t')\rangle \tag{5.4}$$

instead of the (one time) one-body density matrix

$$\rho(\vec{r}, \vec{r}', t) = \langle \psi^{+}(\vec{r}, t)\psi(\vec{r}', t)\rangle . \tag{5.5}$$

In eqs. (5.4) and (5.5), the brackets indicate quantum averages. Under a Wigner transform, $G^{<}$ becomes $f(\vec{r}, \vec{p}, \omega, t)$, whose interpretation is the probability of having a particle at $(\vec{r}, t)$ with energy-momentum $(\omega, \vec{p})$. Only in the non interacting case does the ω-dependence reduce to a simple delta function $\delta(\omega - e(p))$. It is however more practical to use the quantity $f(\vec{r}, \vec{p}, t, \tau)$ obtained by Wigner transform (5.4) in space and using the substitution $\frac{t+t'}{2} \to t$, $t - t' \to \tau$. In this case the transport equation can be written as [17,18]

$$\frac{df}{dt} = \int \frac{d^3p_2}{(2\pi)^3} \int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} |\langle p\vec{p}_2| G |\vec{p}_3 \vec{p}_4\rangle|^2$$
$$\int_{-\infty}^{0} [f f_2(1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]_{t,\tau} \cos\left[\left(\frac{p^2}{2m} + \frac{p_2^2}{2m} - \frac{p_3^2}{2m} - \frac{p_4^2}{2m}\right)\frac{\tau}{h}\right] \delta(\vec{p}) \tag{5.6}$$

where we did not write explicitly the drift term, and where the explicit dependence of the bracket upon t and τ is indicated. If the collisions are unfrequent the integrand does not depend very much upon the variable τ and one recovers the delta function for energy conservation. The physics is in fact rather transparent : because of the Heisenberg principle $\Delta E\,\Delta\tau \gtrsim \frac{\hbar}{2}$, particles remain off-shell for small $\Delta\tau$. Therefore, quantum correlation effects increase with the collision rate. Eq. (5.6) has been solved (by direct integration) only in very simple configurations, like collisions between two infinite uniform nuclear matters in relative motion [17,19]. The effect of correlations may change the relaxation times by as much as 30 %. Simulations of the collisions using ω and $\vec{p}$ as independent variables are however still lacking.

5.3 Transition from hard to soft processes

For the moment, the transition between hard (cascade) processes and soft (evaporation) processes is introduced in a somewhat heuristic manner. It is however unavoidable as one has to go from an independent particle continuous phase space to a A interacting particle phase space. The density of states for the latter is known to correspond to $\rho \propto e^{2\sqrt{aE^*}}$ with a $\simeq$ A/8 - A/10, whereas a =

A/16 corresponds to the former case (provided the same volume is considered). A possible improvement can be provided by the introduction of an intermediate regime, which could be the exciton model [20]. The latter basically differs from the cascade by the use of a (partly) discrete single-particle phase space, and may be indicated when the de Broglie wavelength of the particles becomes of the order of the size of the nucleus, i.e. when all the particles have a momentum below ~ 120 MeV/c. However, presently, this step cannot be introduced without a few ad hoc parameters, to be fixed phenomenologically.

6. Conclusion

We have shown that the INC model has a very high predictive power, a sound theoretical basis and a great versatility. For p-induced spallation reactions, the predictions of the INC model are globally satisfactory. Nevertheless, improvements are necessary for some applications. They can be provided by using in-medium cross-sections, a momentum-dependent field, by introducing off-shell dynamics and perhaps an intermediate exciton-like regime, as we have discussed briefly. However, this list is not exhaustive.

References

1. J. Cugnon, Nucl.Phys. **A389** (1982) 191c
2. J. Cugnon and M.-C. Lemaire, Nucl.Phys. **A489** (1988) 781
3. H.W. Bertini et al., Phys.Rev. **131** (1963) 1801
4. Y. Yariv and Z. Fraenkel, Phys.Rev. **C20** (1979) 2227
5. J. Cugnon, Nucl.Phys. **A462** (1987) 751
6. W. Botermans and R. Malfliet, Phys.Lett. **215B** (1988) 617
7. W. Botermans and R. Malfliet, Phys.Rep. **198** (1990) 115
8. O. Bersillon, unpublished
9. W.B. Amian et al., Nucl.Sci.Eng. **112** (1992) 78
10. C. Volant, private communication
11. C. Volant, unpublished
12. M.M. Meier et al., Nucl.Sci.Eng. **110** (1992) 289
13. R. Sartor, private communication
14. P. Grangé, J. Cugnon and A. Lejeune, Nucl.Phys. **A473** (1987) 365
15. Nikhef Annual Report **68**, Nikhef, Amsterdam, 1991
16. B.E. Vonderfecht, W.H. Dickoff, A. Polls and A. Ramos, Nucl.Phys. **A555** (1993) 1
17. P. Danielewicz, Ann.Phys. **152** (1984) 239
18. R. Malfliet, Int. Workshop *"Dynamical Fluctuations and Correlations in Nuclear Collisions"*, Aussois, March 1992
19. H.S. Kohler, Phys.Rev. **C51** (1995) 3232
20. M. Blann, Phys.Rev. **C28** (1983) 1648
21. J. Cugnon, A. Lejeune and P. Grangé, Phys.Rev. **C35** (1987) 861
22. G.Q. Li and R. Machleidt, Phys.Rev. **C49** (1994) 566