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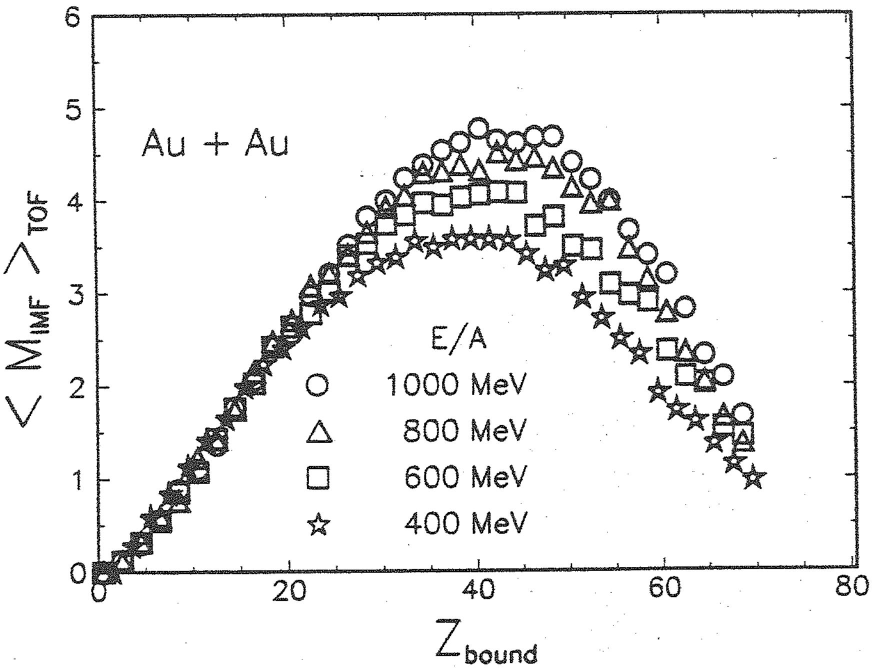
MULTIFRAGMENTATION



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REVIEW OF MULTIFRAGMENTATION THEORIES

J. CUGNON,

University of Liège, Institute of Physics B5, Sart Tilman B-4000 LIEGE 1, Belgium

ABSTRACT

The current theories are tentatively classified according to the nature of the degrees of freedom they incorporate and to the statistical or non statistical way these degrees of freedom are treated. The possible relationship of multifragmentation with bulk or surface instabilities is analyzed as well as the possible implications of the success of percolation models. Finally, the shortcomings of transport theories are underlined.

1. Introduction

Nuclear multifragmentation is the phenomenon by which an excited nuclear system breaks into several pieces of intermediate size. It is believed to make a bridge between the evaporation-fission regime and the vaporisation or explosion (fireball) regime. The phenomenon has been observed, but the conditions under which it occurs are not well delineated and the theoretical description of the phenomena is far from being satisfactory. Much attention has been paid to the decay of an idealized thermalized system, with many different approaches depending upon the degrees of freedom which are explicitly considered. Microscopic dynamimcal calculations try to follow the whole mechanism, but they are not very transparent and furthermore do not treat smooth dynamics satisfactorily. We briefly review the main approaches below.

2. Partition

The basic quantity in nuclear fragmentation is the probability of having n fragments of size $A_1, ..., A_n$ for given initial conditions (target, projectile, energy and impact parameter

$$P(n, A_1, ..., A_n). \qquad (1)$$

This is the quantity which would be measured by an ideal 4π detector (with a perfect impact parameter selection) and the quantity which should be predicted by an ideal theory. In fact, only measurements related to integrated forms of (1) has been performed and analyzed. For instance, the mass yield is given by

$$Y(A) = \sum_n n \sum_{A_2, ..., A_n} P(n, A_1, ..., A_n) \quad . \qquad (2)$$

Similarly, the distribution of the multiplicities n_i in an interval I_i of mass, considered in intermittency analysis, is given by

$$\Pi (n_1 ,..., n_r) = \sum_n \sum_{A_1 , ... , A_n} \prod_{i=1}^{r} \delta \left(\sum_i \theta (A_i , I_i) - n_i \right) P (n , A_1 , ..., A_n) , \tag{3}$$

where $\theta(Ai, I_i) = 1$ if A_i lies within the interval I_i and 0, if not. These "reduced" distributions have been compared to specific models of phase transition : for $Y(A)$, a power law was predicted [1, 2], but it is not believed that this power law is a unique property of a given phase transition ; for the distribution (3), signals for intermittency was looked for [3, 4], but it seems that the limited range of A and the mixing of different impact parameters preclude any conclusion. Not very much attention has been paid to the properties of quantities like (2) and (3) assuming "minimal" properties of (1). This would however be useful. It is often stated that a power law in (2) would arise from almost any multi-step fragmentation model. For instance, in ref. [5], it is shown that sequential binary decays (with some simple law for a binary splitting) yield a power law. However, a power law is not universal, as the same model with n binary splittings with a uniform law for each step yields [6]

$$Y(A) \propto \frac{(1 - A/A_0)^n}{n!} , \tag{4}$$

where A_0 is the initial mass, which is closer to an exponential than to a power law.

3. Classification of theories

A tentative, admittedly simplified, classification of the existing theories is provided by fig. 1.

	DEGREES OF FREEDOM		
	Collective	Collective/microscopic	Microscopic
STATISTICAL	← fission	liquid-vapor eq. (*) "chemical" eq. (*) fission-evaporation	→ fireball percolation
NON STATISTICAL	sequential fission multibarrier fission (*) bulk instabilities surface instabilities		transport theories (†)

Fig. 1 Schematic classification of existing multifragmentation theories. Fission and fireball models are given as extreme cases at low and high energies.

It is based on the degrees of freedom explicitly introduced in the theories and on the statistical or non statistical way of treating these degrees of freedom. Additional features would consist of the dynamical (i.e. time-dependent) or static (referred to in fig. 1 by a star) formulation of the theory and on the possibility to describe the formation of the excited system before it breaks up (indicated by a cross). We will successively describe the currently used theories.

4. Multibarrier fission models

These models have been inspired by the success of the liquid drop model for ordinary fission, expecting that ternary, quarternary,... fission will replace (abruptly or progressively) the binary fission. Like in the latter process, it is first important to know the barrier for splitting the system into several pieces. In ordinary fission, it is already clear that at least two collective degrees of freedom are necessary to describe the barrier correctly. For partition in several pieces, the computation of a multidimensional barrier appears as a formidable task. A remarkable work in this direction has been done recently by Haddad [7]. He showed that multifragmentation is energetically possible for $x = Z^2/A \geq 20 - 25$, but the barrier for the partition in n equal size fragments is depicted in fig. 2.

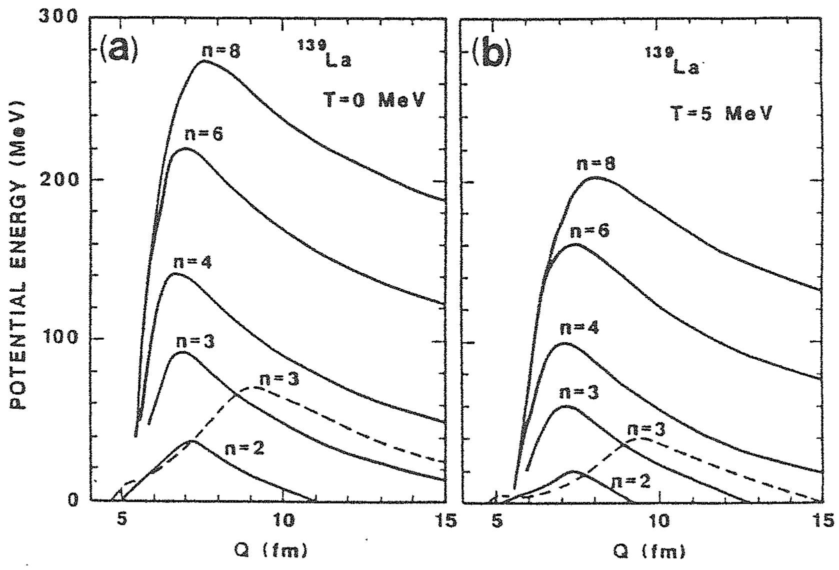

Fig. 2 Fragmentation barrier for n fragments in ^{139}La, given as a function of the r.m.s. radius Q of the n-sphere configuration and of the temperature.

In these calculations, the n spherical fragments are assumed to be closely packed and recess from each other (which corresponds to the r.m.s. radius Q of the configuration). These results strongly suggest that multifragmentation should appear progressively with more and more intermediate mass fragments. He also showed that, like in binary fission, non symmetrical configurations are more favourable. Theoretically, the multifragmentation would be described by a representative point moving in a multidimensional space. However, one is far from a complete description in this frame : the inertia of the system, the damping forces due to the coupling to the intrinsic (not collective) degrees of freedom and especially the role of the fluctuations which will smoothen the picture of successive thresholds are still to be worked out. One has however to underline that the progressive onset of multifragmentation seems to be well established. The data of ref. [8] points also to a gradual increase of the $3IMF$, $4IMF$,... frequency compared to the $2IMF$ frequency and are qualitatively explained by Haddad's calculations.
The same prediction of the excitation function of the $nIMF$ frequencies can also be obtained by a succession of binary splittings (with barrier effects) as embodied by the code $GIMINI$ [9, 10]. This does not contradict the ordering of the barriers in each (n) channels, but simply means that each channel ($n > 2$) can be reached by different paths.
It is an important issue however to determine whether multifragmentation occurs sequentially or simultaneously. This can be studied by looking at the possible Coulomb deflections between the fragments, which lead to different lower relative energy in the two modes. This has been done by the LPC Caen Group, who was able to determine that the emission between two IMF separated by $\sim$ 300 fm/c at low energy may become simultaneous as the energy increases [11].

5. Species in equilibrim

The next generalization would be to assume a partition with any number of fragments, including nucleons, mixing so collective variables (surface of clusters) and microscopic ones. This has be studied in the statistical models assuming a thermochemical equilibrium between various nuclear species. In the (grand canonical) statistical model of Gross [12], quantity (2.1) can be calculated readily

$$P\ (n\ ,\ A_1\ ,\ \dots\ ,\ A_n\ )\ =\ V^n \prod_{i=1}^{n} e^{-\beta \left[\sum_i E(A_i) - A_i \mu \right]}\ g_i\ \frac{(\ 2\pi\ \mathrm{MkT}\ )^{3/2}}{h}\ , \qquad (5)$$

with g_i being the internal partition function and β and μ being determined by assuming (average) energy and nucleon number in a volume V. In such a model, one has to supplement a recipe giving the values V, β, μ from initial conditions. The mass yield is given by

$$Y(A)\ \propto\ e^{-\beta A \left(\frac{E(A)}{A} - \mu \right)}\ g_A\ . \qquad (6)$$

In the liquid-vapor equilibrium model of Fisher [1], the premises are more or less the same (except that one deals with an infinite system from the beginning) and one obtains

$$Y(A) \propto e^{-\beta A\left(\frac{E(A)}{A} - \mu\right)} \tilde{g}_A \ . \tag{7}$$

Fisher studied this form assuming a bulk + surface form for the energy $E(A) = -AE_0 + WA^{\sigma}$ and $\tilde{g}_A$ representing all the ways to form a cluster of mass A :

$$\tilde{g}_A = e^{AS_0 + \omega A^{\sigma}} e^{-\tau \ln A} \ . \tag{8}$$

The first factor is straightforward, but the second is added from considerations on analytic properties of the grand partition functions and on the way to construct surface in lattice gas models. Eqs. (5.3) and (5.4) leads to the well-known formula

$$Y(A) \propto x^{A^{\sigma}} y^{A} A^{-\tau} \ . \tag{9}$$

If $T \rightarrow T_c, x, y \rightarrow 1$ and the power law is obtained, with τ related to the (thermodynamical) critical exponents.
There is a strong resemblance between Gross and Fisher models. In both cases, the same $E(A)$ form is used (in average in ref. [1]). Gross uses g_A values, calculated from Fermi gas model, which are presumably very close to (5.4) (it would be worthwhile to check this point). A big difference is the fact that Gross introduces a (linearized) Coulomb interaction. A somewhat clear consequence of this is the well marked transition from binary fission to ternary,... fission, observed in the results. This property should be brought closer to the results of ref. [7]. Apparently, in the statistical model, the system probes the partition thresholds (with thermal fluctuations) calculated at some effective values of the separation distance between the clusters, since the latter are confined in a volume V.
In summary, the success of the statistical models for heavy systems seems to underline the necessity of an explicit description of collective variables generalizing those used in fission.

6. Surface and bulk instabilities

The analysis of transport code outputs has let believe [13] that some kind of toroidal excited system is formed, which decays into pieces in a way suggesting Rayleigh (surface) instabilities. This has revived the interest for the study of nuclei of various forms and for a possible explanation of multifragmentation by these instabilities.
Geometrical configurations of incompressible fluids are metastable against fragmentation [14]. As an example a sheet of liquid of thickness D is metastable under the fragmentation into pieces of size λ, forming ultimately cylinders. The condition for metastability in various configurations are given below :

(1) sheet of thickness D → cylinders : $\lambda > 2\pi D$
(2) sheet of thickness D → spheres : $\lambda > \frac{3}{2}\sqrt{2\pi} D$
(3) Torus of radius R → spheres : $\lambda > \frac{9}{2} R$
(4) Idem + $\lambda = 2\pi\rho/n, \rho$ = main radius : $\rho > \frac{9n}{4\pi} R$.

These conditions are, in practice, rather drastic. Indeed it is hard to believe that the relevant values of D or R should be smaller than ~ 2 fm. This requires λ to be larger than 12 fm, 7.5 fm or 9 fm respectively for the above configurations, i.e. of the order of the size of the system. Therefore, this possibility should be ruled out. Of course, the metastability may be increased by the Coulomb forces. On the other hand, metastability does not mean instability, as the energy of the system may sometimes be increased, as the deformation is starting. Instability is guaranteed by metastability only for very small values of D or R, apparently [5].

The bulk (or spinodal) instabilities, as first suggested in refs. [15, 16] and often quoted since as a possible source for multifragmentation, have not been studied very much so far. A possible way is to study the expansion of an infinite piece of matter and the evolution of the fluctuations of the density around the time-dependent average value

$$\rho = \bar{\rho} + \sum_k \sigma_k \, e^{i\vec{k}.\vec{r}} \; . \tag{10}$$

The corresponding (time-dependent) Landau-Ginzburg hamiltonian, using σ_k as the coordinates is given by [17]

$$H = \sum_k \frac{b_k}{2} \dot{\sigma}_k^2 + \sum_k \left(a_2 + c\, k^2 \right) \sigma_k^2 + \sum_k a_k \, \sigma_k^4 \; , \tag{11}$$

where

$$a_2 = \left(\frac{\partial p}{\partial \rho} \right)_S \tag{12}$$

is given by the variation of pressure along the expansion. If the system reaches the spinodal, a_2 is negative. The quantity σ_k will grow exponentially

$$\sigma_k \propto e^{\Gamma_k t} \; , \; \Gamma_k = \frac{\left(\mid a_2 \mid - c\, k^2 \right)^{1/2}}{\sqrt{b_k}} \; , \tag{13}$$

for sufficiently small values of k. The quantities c and b_k can be obtained from the (i.e. Skyrme) energy density functional. In particular $b_k = b/k$, and

$$\Gamma_k = b^{-1} k \left(\mid a_2 \mid - c\, k^2 \right)^{1/2} \; . \tag{14}$$

The instability of density fluctuations is thus limited for large wave length (not relevant anyway for finite systems) and for $k > k_{min} = \sqrt{\mid a_2 \mid / c}$. Just to give an idea, in the middle of the spinodal region, for Skyrme III, $k_{min} = 2.3 fm^{-1}$, and $\Gamma_{k=0.5fm^{-1}} = 25\ (fm/c)^{-1}$. This is to be related to the time spent by the system in the spinodal region is of the order of $\sim$ 60 fm/c. The instability has plenty of time to grow. Of course, this depens upon the original fluctuations. Therefore, a transport equation for the fluctuations of σ_k is needed. This exists and is embodied in the so-called Cahn-Hilliard equation [18, 19]. Unfortunately, this equation is valid for small values of σ_k. When the departures from $\bar{\rho}$ are large, i.e. when the bulk is spotted with regions of low density , the evolution of the system may be described by surface dynamics [20]. There is up to now no theory dealing on the same footing any departure for uniform density, except perhaps some numerical simulation [21].

7. Percolation

Percolation model [22] appears as a scientific monster in multifragmentation theory : it seems to work better than any model [23, 24], although devoid of any (obvious at least) dynamics. In my opinion, the reason for this is that percolation model is able to pick up the fluctuations characteristic of some critical behaviours. The fact that it works pretty well with an extremely limited number of parameters is to be related to the universality in critical phenomena, which means that some aspects are almost independent of the dynamics. Several questions are however raised by this remark : (a) is the variation of the necessary fitted parameters with the kinematical conditions reasonable ? (b) what are the critical phenomena underlying multifragmentation ? Several candidates are: static liquid-vapor transition, dynamic spinodal decomposition, critical dispersion in phase space,... For instance, in the second case, the quantities Γ_k (eq. (14)) should show some critical behaviour, possibly characterized by critical exponents and they should fluctuate strongly in the course of the spinodal decomposition.

8. Transport theories

These theories are potentially the most promising ones, since they include all the degrees of freedom and since they follow the whole collision process in contradistinction with previously mentioned theories. Furthermore, transport theories like *BUU* or *QMD* (and also like *INC*) follow the *A*-body distribution function, which in principle is necessary to make definite predictions on multifragmentation. Formally, one has (classically)

$$P\ (n\ ,\ A_1\ ,\ \ldots\ ,\ A_n\ )\ =\ \int dx_1 \ldots dx_N\ f_A\ (\ x_1\ ,\ \ldots\ ,\ x_N\ ) \\ f_{A_1}\ (\ x_1 \ldots x_{A_1}\ )\ f_{A_2}\ (\ x_2 \ldots x_{A_2}\ )\ \ldots \qquad (15)$$

where f_A is the asymptotic *A*-distribution function of the whole system and where f_{A_i} is the full distribution function for the actual fragment A_i, x_j being the coordinates on nucleon j. In any scheme based on the *BBGKY* hierarchy, any $s \geq 2$ distribution function follows an equation of the form [25]

$$\frac{\partial f_s}{\partial t} + \sum_{j=1}^{s} \overline{L}_0\,(j)\; f_s = \sum\ (\ f_{s+1}\ )\ , \qquad (16)$$

where Σ is some kind of collision (source) term and where $\overline{L}_0$ propagates the f_s-function with the mean field dynamics. When the source term is negligible (at the end of the collision), it is mandatory to have a good description of the mean field. This seems not to be the case for the moment, especially for the fluctuations [26-29]. That is why most of the approaches rely on some percolation procedure to replace this delicate stage, even theories incorporating quantum motion [30, 31]. Furthermore, they have to assume that transport process should be stopped somewhere and replaced by evaporation, because the treatment of the individual degrees of freedom is then so complex that a statistical treatment is sufficient. It seems that for the time being transport theories cannot really provide a clear picture of multifragmentation. However, they are very useful in order to study the initial stages of the collision and to provide conditions prevailing when randomization of some collective or of all degrees of freedom is more or less realized.

As an example, transport theories plus percolation are better than ad hoc percolation models on lattices because the former can predict the excitation functions [32].

9. Conclusion

We have drawn attention on recent progress indicating that multifragmentation may appear as a barrier penetration in a complicated multiparameter space with increasing number of open channels as energy increases and that statistical models may pick up in some way the barriers at some effective mutual separation distance between the fragments. This scenario may be related to the instabilities, although surface instabilities should probably be ruled out. It is not clear for the moment that it can generate fluctuations typical of the critical behaviour underlying the successfull percolation model. This would require much attention in the future as well as the study of the relationship with microscopic transport theories.

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