# Particle Production in Highly Excited Matter

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# CASCADE MODELS AND PARTICLE PRODUCTION. A COMPARISON

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## 1. INTRODUCTION

### 1.1. GENERALITIES

Cascade models have been used successively for heavy ion collisions in a wide range of energy stretching to a few tens of MeV/u to a TeV/u. Depending upon the energy range, they carry different physics are elaborated to different degrees and used by physicists of different backgrounds. The present review aims to provide a synthesis of the cascade models and hopefully to help cascade users of one domain of energy to understand the problems encountered in another domain.

The common feature to all cascade models (and this will be used as a definition) is that they picture a multiscattering process as a succession of binary collisions (and possibly decays), well separated in space-time. A physical situation dominated by this kind of well separated collisions is denoted as a collision regime. It is rather easy to see where this collision regime will occur in heavy ion collisions by looking at Fig. 1, where the de Broglie wavelength of one of the incident nucleons and its classical mean free path (corrected for the Pauli principle) in nuclear matter are displayed as functions of its incident energy. At very low energy the de Broglie wavelength $\lambda\!\!\!^{-}_B$ is almost as large as the radius of the nucleus. The dynamics will be dominated by mean field effects. For an incident energy larger than say 20 MeV/u, $\lambda\!\!\!^{-}_B$ is less or of the order of the internucleon distance and the mean free path is substantially larger than this quantity : this is the onset of the collision regime. We will denote as *Nucleon Cascade* the corresponding process



as well as any model aimed to describe it. If one goes further in energy, one crosses particle production thresholds. For a certain range of energy the multiple scattering process will involve elastic and inelastic nucleon-nucleon collisions as well as collisions between the produced hadrons. We will call this process the *Hadronic Cascade*. Ultimately at higher energy $\lambda\!\!\!^{-}_B$ becomes smaller than even a fraction of the size of a nucleon. The interaction will therefore involve the substructures of the nucleon, namely the partons. We will refer to this process as the *Partonic Cascade*. We will see that in this cascade the elementary process should be generalized to include string formation.

A tentative comparison between the various cascades is performed in Table 1.

**Table 1.** Comparison between the various heavy ion dynamics

| | Mean Field | Nucleon Cascades | Hadronic Cascades | Partonic Cascades |
|---|---|---|---|---|
| $E_{lab}$ | $\leq$ 20 MeV/u | 0.02-2 GeV/u | 2-10 GeV/u | 10-200 GeV/u |
| Dynamics | mean field | NN collisions | NN collisions + hadron prod. | string dynamics |
| Typical length | a few fm | 1 fm | 0.5 fm | 1/10 fm |
| Typical energy | $h\,\omega_{osc}$ | 100 MeV | a few hundreds MeV | ? |
| Typical time | a few fm/c | ~ 1 fm/c | < 1 fm/c | ? |
| Structure | nucleon wave function | (point) nucleon | nucleon & resonances | partons & strings |
| Input | $V_{eff}$ | $\sigma_{eff}$ | $\sigma_{el}$ & $\sigma_{inel}$ | string formation and fragmentation probabilities |
| Energy loss | Fermi dynamics | collisions | particle production | string formation |
| Models | TDHF<br>FMD | BUU INC<br>VUU<br>LV<br>BL<br>KB<br>QMD<br>FMDC | INC<br>ARC<br>QMD | VENUS<br>DPM<br>FRITIOF |
| Equation of state | <--- Fermi liquid equation of state ---------------------------------------------> | <hadronic fluid> | | String fluid |

Several remarks are however in order. First the separation in energy (first line) should be taken as indicative only, as there is no sharp transition from one dynamics to the other but rather in some cases a continuous passage : e.g. the nucleon cascade is often mixed with mean field effects. Second, the typical lengths, energies, times are also given for illustrative purpose only. Third, the basic ingredients of the various cascade models are given. Fourth, we give the currently used models by their acronyms. We refer to the literature for a general description : TDHF[1], FMD[2], BUU[3], VUU[4], LV[5], BL[6], KB[7], QMD[8], FMDC[9], INC[10], ARC[11], VENUS[12], DPM[13], FRITIOF[14]. This list is not exhaustive, specially for the partonic cascade (for which we provide a much longer list in section 4). To be clear we reserve the name intranuclear cascade (INC) to the models which do not introduce mean field (except perhaps for initial static potentials), but which on the other hand introduce inelastic collisions involving the lightest hadrons[9,15].

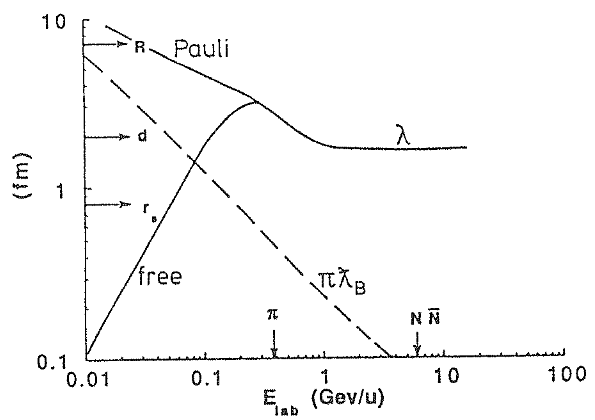


**Figure 1.** Comparison of the mean free path λ (classical and Pauli corrected) and the de Broglie wave length at given incident energy with the radius of the nucleus (R), the nucleon interdistance d , and the nucleon size $r_S$ .

In the next sections, we make a comparison between the various cascades, especially from the point of view of the physics they embody and not from the point of view of their capacity of reproducing the data. One can in fact say that all the cited cascade codes are quite successful, at least for the kind of observables they are built for (a code built for describing inclusive quantities cannot describe correlations of course). The paper is organized as follows. Section 2 is devoted to the nucleon cascades. The recent important developments concerning medium corrections, statistical fluctuations and quantum effects are discussed. This section is rather developed although the domain of energy is not the one of the main stream of the conference, but it is the one where the theory is the most advanced and we think that the status of this chapter and the presentation of the remaining difficulties may be instructive for the high energy domain. Section 3 contains a brief description of what should be a hadronic cascade and a discussion of the concept of the formation time. A model for the J/ψ will be presented as an illustrative example. Section 4 describes partonic cascades. The emphasis is put on



string dynamics. The VENUS model is chosen as the prototype of the codes using this feature as their basic premise. A short comparison between the existing codes is done. But before we proceed to section 2, we will say a few words on the underlying equation of state for each type of cascades.

## 1.2. (NUCLEON) NUCLEAR MATTER EQUATION OF STATE (T = 0)

The theoretical knowledge about the equation of state (e.o.s.) is summarized in Fig. 2. Using two-body forces only, both the variational methods[16] and the perturbative (Brueckner) method[17] yield an incorrect ground state saturation. Only when three-body forces are taken into account that the proper saturation is achieved, in both cases.

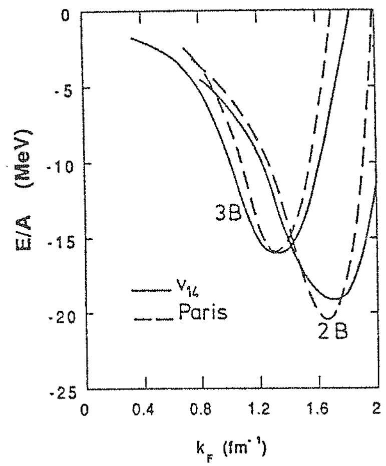


**Figure 2.** Nuclear matter (T = 0) binding energy per nucleon versus Fermi momentum. The full lines are the results of Ref. 16 for the $v_{14}$ potential and the dashed lines are those of Ref. 17 for the Paris potential. Curves (2B) are obtained with two-body potential only. Curves (3B) correspond to the addition of three-body forces.

We want to complement by several remarks : (1) saturation is obtained at the expanse of phenomenological attractive forces. They provide an extra attraction at equilibrium density of the order of 5-10 MeV. We want to stress this point which is not very well known and which shows (with the remarks below) the need for an experimental derivation of the e.o.s. ; (2) the variational result of Fig. 2 is well described up to $\rho = 3\,\rho_0$ , by a purely quadratic dependence upon the density, with the so-called compressibility modulus K = 220 MeV. However, there is no theoretical indication that the dependence should be simply parabolic on the indicated range of density ;

(3) saturation in the calculations of Fig. 2 is provided by the repulsive nature of the NN potential at small distance. However in relativistic theories, the saturation appears as a relativistic effect ; (4) one may wonder whether the hadronic degrees of freedom may influence the e.o.s., even at zero temperature. In fact the three-body forces introduced in the calculations of Fig. 2 are already arising from some resonance degrees of freedom. If one introduces the delta degrees of freedom, one obtains an extra repulsion of the order of 10 MeV around the saturation point[19,20].

The e.o.s. enters the cascades through the relation, assumed in general and verified at the level of some approximations in certain theories, between the average field U experienced by the nucleons and the binding energy per particle[21]. In equilibrium situations, the mean field depends upon the local density and the velocity of the particle with respect to the local environment. This property is usually assumed to remain valid in non equilibrium situations.

## 1.3. HADRONIC MATTER ($T \neq 0$)

We start first with the meson gas. If one considers this matter to be formed of any number of mesons of any mass, it is easy to show that the internal energy can be written as (we have left out the unnecessary complications due to BE statistics)

$$U = \frac{VT}{2\pi^2} \int_0^\infty dm\, \rho(m)\, m^3 K_2\left(\frac{m-\mu}{T}\right) , \qquad (1.1)$$

where $\rho(m)$ is the meson mass spectrum, $V$ is the volume, $T$ is the temperature and $K_1$ is the modified Bessel function. It is then clear that the specific heat, which is nothing but the temperature derivative of the internal energy, will be a function of the mass spectrum of the mesons. So it is presumed that one could be able to extract this meson mass spectrum from heavy ion collisions in the energy regime where the hadronic cascade is valid. This mass spectrum may show a limiting temperature, if, as advocated by Hagedorn, the heavier mesons are essentially made of lower mesons. If this is true one has basically $\rho(m) = \rho(m_1)\rho(m_2)$ if $m = m_1 + m_2$ . The approximate solution to this equation is

$$\rho(m) = \exp(\alpha m) \ . \qquad (1.2)$$

The true equation for $\rho(m)$ has been worked out by Hagedorn[22] and it is found that the solution tends asymptotically to (1.2) with $\alpha^{-1}$ of the order of the pion mass, the lowest mass of the mesons. From Eq. (1.2) it can be seen that the energy density becomes infinite when $T$ approaches $\alpha$ from below.

For the hadronic gas, there is no qualitative difference. Roughly speaking, Eq. (1.1) should be replaced by



$$U(V,T) = U(V,0) + U_{th}(V,T) \qquad (1.3)$$

where the first term refers to the compression of the cold matter (Fig. 2) and is discussed in section 1.2, and where the second term has the properties of the quantity given in (1.1) and just discussed above.

## 1.4. STRING MATTER ($T \neq 0$)

It is not clear yet what is called string matter and it is not obvious that his concept is relevant for the partonic cascades. Nevertheless, one may consider a matter made of the strings defined in section 4, assuming that they can decay and fuse. Therefore the developments are similar to those for the hadronic gas, except that the relevant degree of freedom is not the mass by itself, but the surfaces of the string (see section 4). One would then roughly obtain a string surface spectrum as

$$\rho(\sqrt{S}) = \exp(\alpha' \sqrt{S}) \qquad (1.5)$$

with this time $\alpha'$ related to the lowest allowed string surface. The e.o.s. is then linked to this spectrum and the string constant.

# 2. NUCLEON CASCADES

## 2.1. COLLISION DYNAMICS

We call by this name a situation where the motion of the nucleons (particles in general) is dominated by two-body collisions (and possibly two-body decays). The basic tool for handling this problem (common to all cascades) is numerical simulation. The basic rules are the following : (1) a collision occurs if the minimum distance between two nucleons is such that

$$\pi d_{min}^2 \leq \sigma_{tot}(s) \qquad (2.1)$$

where $s$ is the squared c.m. energy of the colliding pair ; (2) at each collision one has conservation of energy and momentum ; (3) the final state of the colliding pair is chosen at random according to the experimental angular distribution (see Ref. 23 for detail) ; (4) Pauli blocking is implemented (statistically) by checking the final phase space occupancies ; (5) physical quantities are calculated by ensemble averages.

It is generally stated that this simulation embodies the physics of the Boltzmann (Uehling-Uhlenbeck) equation, namely

$$\left(\frac{\partial}{\partial t}+\vec{p}.\vec{\nabla}\right)f(\vec{r},\vec{p},t)=$$

$$\int\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}\left[\sigma_{34\to 12}v_{34}f_3f_4\overline{f_2}\,\overline{f_2}-\sigma_{12\to 34}v_{12}f_1f_2\overline{f_3}\,\overline{f_4}\right] \quad (2.2)$$

where $\overline{f_i}=1-f_i$, $f_i=f(\vec{r}_i,\vec{p}_i,t)$ being the one-body distribution function. However, the simple simulation goes beyond the physics of the Boltzmann equation (as already underlined in Ref. 23) on the following points : (a) it propagates the one-body distribution function, but also any s-body distribution functions, with $s>1$ ; (b) it does not rely on the molecular chaos hypothesis ; on the contrary in several cascade codes, namely those using the prescription (2.1) and not the mean free path picture, the correlations due to collisions are propagated ; (c) it contains fluctuations.

We must add that these effects have not been studied explicitly, except for the considerations of section 2.4. In order to discuss these points, one needs a theoretical framework. The latter is sketched in the next section.

## 2.2. THEORETICAL FRAMEWORK

### 2.2.1. Classical Approach

The starting point is the so-called BBGKY hierarchy for the distribution functions[24]. For a system governed by the hamiltonian

$$H=\sum\frac{p_i^2}{2m}+\frac{1}{2}\sum_{i<j}V_{ij} \quad (2.3)$$

the first equations of the hierarchy can be written as

$$\left(\partial_t-L_1^0\right)f_1=\int dx_2\,L'_{12}f_2(x_1,x_2) \quad (2.4)$$

$$\left(\partial_t-L_1^0-L_2^0\right)f_2=L'_{12}f_2(x_1,x_2)+\int dx_3\left(L'_{13}+L'_{23}\right)f_3(x_1,x_2,x_3) \quad (2.5)$$

$$\left(\partial_t-L_1^0-L_2^0-L_3^0\right)f_3=\left(L'_{12}+L'_{13}+L'_{23}\right)f_3(x_1,x_2,x_3)+$$
$$\int dx_4\left(L'_{14}+L'_{24}+L'_{34}\right)f_4(x_1,x_2,x_3,x_4)\,. \quad (2.6)$$



In these equations, $x_i$ stands for $\vec{r}_i,\vec{p}_i$ . The s-body distribution function $f_s$ is normalized as

$$\int dx_1\,dx_2\ldots dx_s\,f_s(x_1,x_2,\ldots,x_s)=\frac{A!}{(s!(A-s)!)}\,. \quad (2.7)$$

Finally, the L-operators are given by

$$L_i^0=-\frac{\vec{p}_i}{m}.\vec{\nabla}_i\;,\;L'_{ij}=-\vec{\nabla}_iV_{ij}.\left(\vec{\nabla}_{\vec{p}_i}-\vec{\nabla}_{\vec{p}_j}\right). \quad (2.8)$$

The simplest approximation is to cut the hierarchy by assuming that the r.h.s. of the second equation of the hierarchy (2.5) vanishes and that $f_2(t\to\infty)=f_1(x_1,t\to\infty)f_2(x_2,t\to\infty)$ . In that case $f_2$ factorizes in a product of two $f_1$ distribution functions at any time. By putting this form into Eq. (2.4), the latter becomes

$$\left[\partial_t-L_1^0-\left(\vec{\nabla}_{\vec{r}}U\right).\vec{\nabla}_{\vec{p}}+\left(\vec{\nabla}_{\vec{p}}U\right).\vec{\nabla}_{\vec{r}}\right]f_1(\vec{r},\vec{p},t)=0 \quad (2.9)$$

where $U$ is the average potential

$$U(\vec{r},\vec{p},t)=\int dx_2\,V(x_1,x_2)f_1(x_2,t)\,. \quad (2.10)$$

If the potential depends upon positions only, $U$ depends upon $r$ only, and Eq. (2.9) is equivalent to a Vlassov equation, describing a set of particles moving freely in the mean field they generate. If $V$ depends upon momenta, the mean field will depend upon the velocity. So mean field with this property are not necessary of quantum nature.

The next approximation consists in retaining the first term on the r.h.s. of Eq. (2.5). Formally, its solution writes

$$f_2(t)=\exp\left\{\int_{t_0}^{t}\left(L_1^0+L_2^0+L'_{12}\right)dt\right\}f_2(t_0)\,. \quad (2.11)$$

So, even if $f_2(t_0)$ is uncorrelated, $f_2(t)$ will contain correlations. The correlations are built when the particles are interacting. If we assume that the particles are uncorrelated at time $t_0\to-\infty$ , we are facing a typical scattering problem. The particles will be correlated at any time, but when the interaction between them will be over, they will look again as uncorrelated except for energy and momentum conservation. Classically, there will be spatial correlations due to the fact that the particles follow their trajectories in a correlated manner. Actually, one obtains

$$\lim_{\substack{t \to \infty \\ t_0 \to -\infty}} \exp\left\{ \int_{t_0}^{t} \left( L_1^0 + L_2^0 + L_{12}' \right) dt \right\} f_1(x_1, t_0) f_1(x_2, t_0) = L_{12}' f_1(x_{1-}) f_1(x_{2-}) -$$

$$\int d^3p_3 \, d^3p_4 \left[ T_{12 \to 34} f_1(x_{1-}) f_1(x_{2-}) - T_{34 \to 12} f_1(x_{3-}) f_1(x_{4-}) \right] \quad (2.12)$$

where the minus indices indicate that the time should be taken at $-\infty$ and where one should understand the integrations as

$$\int d^3p_3 \, d^3p_4 \, T_{12 \to 34} = \int d^3p_3 \, d^3p_4 \int d^2\vec{b} \, \delta^3(\vec{p}) \, \delta(E) \, \delta\left( \theta(b) - \theta^*(\vec{p}_1, \vec{p}_3) \right) \quad (2.13)$$

where $b$ is the impact parameter, the delta functions stand for energy-momentum conservation, $\theta(b)$ is the deflection function and $\theta^*$ is the angle between $\vec{p}_1$ and $\vec{p}_3$ in the c.m. One usually relies at this point on a Markovian approximation, assuming that the collision time is much shorter that the equilibration time. So one may consider to look at time scale intermediate between these two times. Then the infinite limits appearing in the last relation may be reduced to the (instantaneous) collision time. One then obtains the structure of the classical Boltzmann equation

$$\left( \partial_t + \frac{\vec{p}}{m} \cdot \vec{\nabla} - (\vec{\nabla} U) \cdot \vec{\nabla}_{\vec{p}} \right) f_1(\vec{r}, \vec{p}, t) =$$
$$\int d^3p_2 \, d^3p_3 \, d^3p_4 \left[ W_{34 \to 12} f_1(x_3, t) f_1(x_4, t) - W_{12 \to 34} f_1(x_1, t) f_1(x_2, t) \right] \delta^3(\vec{p}) \, \delta(E) \, . \quad (2.14)$$

One may go even one step further by approximating $f_3$ in (2.5) by products of $f_1$ and $f_2$. One then gets particle scattering in the mean field, with single-particle energy $e(p) = \frac{p^2}{2m} + U(p)$ .

### 2.2.2. Quantum Approach

We will not give the derivation here, since it is rather lengthy. However, we want to say that using the Keldysh Green function formalism leads to a powerful and useful



formalism, since it yields a generalized Boltzmann type equation with many advantages : (1) it does not rely on the molecular chaos hypothesis (this is not necessary classically either, see Ref. (24) for detail) ; (2) it yields the connection with the (dynamical) strength function ; (3) it introduces naturally medium effects through a T-matrix approximation, describing the scattering through the medium. The following transport equation is obtained (see Refs. 25 and 26 for detail)

$$\left\{ \left[ \partial_t + \frac{\vec{p}}{m} \cdot \vec{\nabla} - \left( \vec{\nabla} U(\vec{r}, \vec{p}, t) \right) \cdot \vec{\nabla}_{\vec{p}} + \left( \vec{\nabla}_p U(\vec{r}, \vec{p}, t) \right) \cdot \vec{\nabla} + \frac{\partial U}{\partial t} \cdot \frac{\partial}{\partial \omega} \right] + \right.$$
$$\left. - \left[ \vec{\nabla} U_C(\vec{r}, \vec{p}, t) \cdot \vec{\nabla}_{\vec{p}} + \left( \vec{\nabla}_p U_C(\vec{r}, \vec{p}, t) \right) \cdot \vec{\nabla} + \frac{\partial U_C}{\partial t} \cdot \frac{\partial}{\partial \omega} \right] \right\} f(\vec{r}, \vec{p}, t, \omega)$$
$$= \int \frac{d^3p_2}{(2\pi)^3} \frac{d\omega_2}{2\pi} \int \frac{d^3p_3}{(2\pi)^3} \frac{d\omega_3}{2\pi} \int \frac{d^3p_4}{(2\pi)^3} \frac{d\omega_4}{2\pi}$$
$$\left[ W_{p\,p_2 \to p_3\,p_4} f(\omega) f_2(\omega_2) \bar{f}(\omega_3) \bar{f}(\omega_4) - \right.$$
$$\left. W_{p_3\,p_4 \to p\,p_2} f_3(\omega_3) f_4(\omega_4) \bar{f}(\omega) \bar{f}_2(\omega_2) \right] \delta^3(\vec{p}) \, \delta(\omega) \quad (2.15)$$

where the e.o.s. effects are carried by the mean field $U$ , medium effects are influencing $U_C$ and especially the transition probabilities $W$ but also the effective mass and where the purely quantum effects are linked with any $\omega$ variable. The one-body distribution function $f_1$ is related to $f(\vec{r}, \vec{p}, t, \omega)$ by the relation

$$f_1(\vec{r}, \vec{p}, t) = \int \frac{d\omega}{2\pi} f(\vec{r}, \vec{p}, t, \omega) \, . \quad (2.16)$$

In the classical case the $\omega$-dependence of $f$ reduces to

$$\delta(\omega - e(p)) = \delta\left( \omega - \frac{p^2}{2m} + U \right) . \quad (2.17)$$

The transport equation (2.15) illustrates the difficulty of the exercise of extracting the equation of state. Even if one could solve exactly Eq. (2.15) and compare the predictions with perfect data, the e.o.s. could be extracted (or constrained) only if the other input of the equation are perfectly well known. It is far from being the case as we are going to illustrate.

## 2.3. MEDIUM EFFECTS

The virtue of the approach given above is that it provides expression in the instantaneous T-approximation for quantities like $U_C$ and $W$. Their calculation in a collision is far outside of the scope of feasibility. However, they can be identified as well known quantities in equilibrium situations. Calculations in these circumstances may be used as a guide for non equilibrium situations. For instance, the quantity $U_C$ reduces to the so-called correlation potential in equilibrium nuclear matter. Its numerical value is about + 20 MeV at normal conditions and decreases slowly with temperature[27]. Its behaviour in other conditions has never been studied. Actually, it has never been introduced explicitly in numerical codes, in spite of the fact that it is not negligible compared to $U$.

The quantity $W$ can be identified as (up to some kinematical factor) the in medium cross-sections. In an equilibrium situation $W_{p p_2 \to p_3 p_4}$ can be related (through unitarity) to the imaginary part of the optical potential felt by a nucleon of momentum $p$. The in medium cross-sections have been compared with free space cross-sections for smaller temperatures and various densities in Ref. 28. The effect is far from being negligible. Following this work, an effort has been made to identify which observables are more sensitive to medium effects than to the e.o.s. For instance, it has been clearly shown that collective flow is much more sensitive to medium effects[29].

The medium corrections to particle production are even less known and presumably more important. In particular, it has been shown that pion production yield is very sensitive to many medium effects and can certainly not be used as a mean to determine the e.o.s. as it was once believed.

## 2.4. FLUCTUATIONS

Solving Eq. (2.15), even classically, cannot be considered as providing the $f_1$-solution of the classical BBGKY hierarchy. The reason is that the fluctuations in the initial state and the probabilistic character of the collisions introduce randomness in the evolution. Eq. (2.15) should be viewed as the equation for the average one-body distribution function only. This quantity fluctuates from one event to the other. According to Refs. 30 and 31, these fluctuations reflect in the correlated part $\delta f_2$ of the two-body distribution function

$$f_2(x_1, x_2, t) = f_1(x_1, t) f_1(x_2, t) + \delta f_2 \tag{2.18}$$

as it should since the product $f_1 f_1$ leads to the collision term of the Boltzmann equation (see section 2.2.). In fact, the formal solution of Eq. (2.4) with (2.18) is



$$f_2(t) = f_1 f_1 + \int_{t_0}^{t} dt' \, e^{iL_0(t-t')} L_{12} f_1(t') f_1(t') + e^{L_0(t-t_0)} \delta f_2(t_0) . \tag{2.19}$$

When inserted into (2.14), the first term gives the Vlassov term, the second term the usual collision term and the last one gives the effect of the propagation of correlations. This term is rapidly varying and it has been proposed to replace its effect by a fluctuating force giving so rise to the so-called Boltzmann-Langevin equation

$$\left( \partial_t + \frac{\vec{p}}{m} . \vec{\nabla} - (\vec{\nabla} U) . \vec{\nabla}_{\vec{p}} \right) f = I(f) + K(f) \tag{2.20}$$

where $I(f)$ is the usual collision integral and where $K(f)$ is a fluctuating force, whose average is zero and whose correlation function, taken as Markovian $C(f)\,\delta(t - t')$, is related to the correlation function of the third term in (2.19). Expressions using reasonable approximations are given in Ref. 31. Basically, the correlation of the fluctuating force is given by the *sum* of the gain and loss terms of the collisional integral. In practice, this force is assumed to change at random the moments $(\text{in } \vec{p})$ of the distribution function[31].

An interesting consequence of these fluctuations deals with the subthreshold particle production. In this case, the results are sensitive to the tail of the distribution functions, at the very end of the available phase space. The fluctuations of $f_1$ will thus increase the production yield. This is shown in Fig. 3 for the subthreshold $K^+$ production.

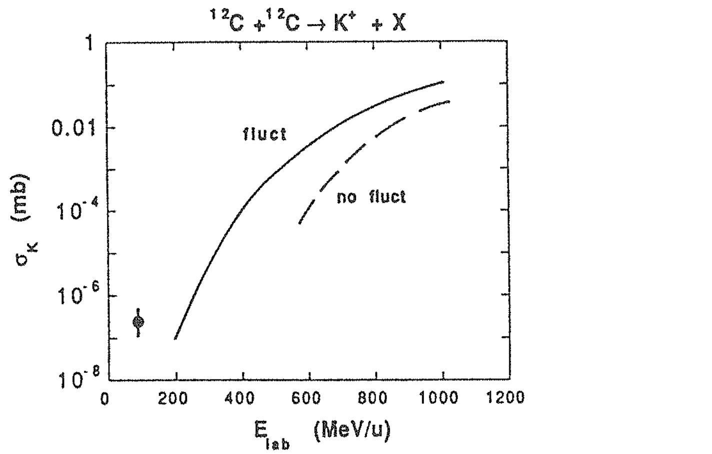


**Figure 3.** Subthreshold $K^+$ cross-section calculated with Eq. (2.2) (dashed line) and Eq. (2.20) (full line) and compared to the measurement of Ref. 55. Adapted from Ref. 56.

Several problems remain to be solved : (1) what is the dynamical content of the fluctuations ? (we speak here about statistical, not quantum fluctuations) ; (2) what is the importance of the "numerical noise" (use of quasi-particles) in each of the existing codes?; (3) what is the nature of the fluctuations which are measured ? The answer to the second question can be provided by checking with typical physical situations, where the fluctuations are known (equilibrium e.g.).

## 2.5. QUANTUM EFFECTS

As discussed in section 2.2.2, the main quantum motion effect implies that a particle with momentum $\vec{p}$ does not have a definite energy. As a consequence, one has in all generality

$$f(\vec{r}, \vec{p}, t, \omega) = a(\vec{r}, \vec{p}, t, \omega) F(\vec{r}, \vec{p}, t, \omega) \qquad (2.21)$$

where $a$ is the spectral function which reduces to (2.17) in the classical limit. The reason for this energy spread comes from the uncertainty principle : between two collisions the energy is undetermined, and the smaller the time between successive collisions is, the larger the uncertainty is. The typical form of $a$ is

$$a(\omega) = \frac{\mathrm{Im}\,\Sigma^+}{\left(\omega - \frac{p^2}{2m} - \mathrm{Re}\,\Sigma^+\right)^2 + \frac{\left(\mathrm{Im}\,\Sigma^+\right)^2}{4}} \qquad (2.22)$$

where $\Sigma^+$ is the particle self-energy. The effect of the spreading on the relaxation time has been studied in Ref. 26 by linearizing Eq. (2.15) for the case of a particle moving in infinite matter. For small times $\left(\lesssim \left(\mathrm{Im}\,\Sigma^+\right)^{-1}\right)$, the quantum system relaxes quicker than the classical one. For larger times, it is the contrary. The quantum correction may reach 50 % [26]. The quantum effect does not necessarily decrease for increasing energy, since the collision rate is expected to increase.

# 3. HADRONIC CASCADES

## 3.1. INTRODUCTION

The hadronic cascade implies nucleons, mesons and resonances. Some cascade codes included the lowest hadrons ($N$, $\pi$, $\Delta$) for a long time, but we are speaking here of extended versions with many hadrons. Theoretically, a hadronic cascade is assumed to



correspond to many coupled transport equations (one for each species) with collision and creation/destruction terms. From this point of view, it is just an extension of the nucleon cascade. Several physical problems arise however. The first one is to know whether we have evidence for resonance formation. Up to now, the only answer to this question is provided by Ref. 32 concerning the $\Delta$-resonance. In this reference, the authors carefully analysed the $\pi$-N correlation in invariant mass and substracted the combinatorial background : they clearly isolated the $\Delta$-peak with a shape somehow distorted when compared to the free space $\Delta$-mass distribution. It was not possible however to ascribe this distortion to a dynamical process. Another important question concerns the possible medium effects on particle or resonance production. All hadrons being strongly interacting, it is expected that the cross-sections are largely renormalized in dense matter. The pion case is illustrative in this respect. Furthermore, it has been shown that the particle yield are strongly sensitive to these possible medium corrections[33]. Another important question relates to the possible limit of validity of the hadronic cascade. Recently, Pang et al.[11] have shown that this model is surprisingly successful at 14 GeV/c. The question is to know at which energy this picture break down and should be replaced by the string formation.

## 3.2. FORMATION TIME

Another important and difficult problem is the question of the formation time. The basic idea behind the formation time means that a produced particle reaches its (free) asymptotic state after some time only. It is usually stated that prior to the formation time, the "nascent" particle is not or weakly interacting. It should be stressed however that the aspects are not necessary mixed. However, if they are, a formation time would lead to an increase of the transparency and a decrease of inelasticity. This is illustrated in Fig. 4, where we show our results of an INC calculation for pion production (left) and the number of participants (right) as a function of the formation time, for some typical examples. The formation time is often presented as the time necessary for a point-like wave packet of a produced meson ($q\bar{q}$ pair) (like in photo-production) to reach its natural size. This aspect of the wave dynamics may be investigated by a Schrödinger equation, as is done in the next section. But, one should keep in mind that the formation time may be related in an evolution of the system in Fock space and not in Hilbert one, since it contains many more components than the $q\bar{q}$ one.

## 3.3. THE J/Ψ AS AN EXAMPLE

In hadronic reactions, the $J/\psi$ is supposed to be formed through the gluon fusion mechanism. As discussed in Refs. 34 and 35, the $c$-$\bar{c}$ pair is thus formed in a narrow wave packet, of a typical size of 0.2 fm. The further evolution of this wave packet, either in free space or in another medium, may be described[35] by the following equation

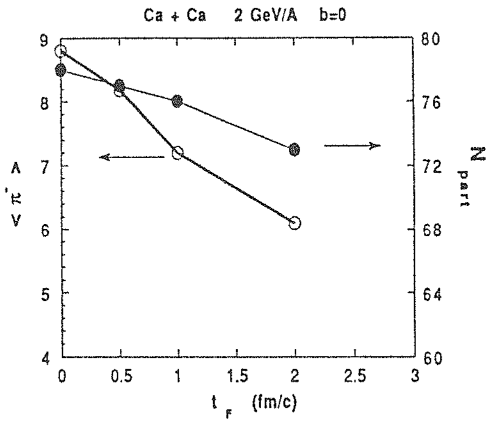


**Figure 4.** INC calculation of pion multiplicity (left scale) and of the number of participants (right scale) as functions of the formation time $t_F$, for central Ca-Ca collisions at 2 GeV/A.

$$i\hbar\partial_t\psi = \left\{-\frac{\hbar^2}{2m}\Delta + V(\vec{r},t) + iW(\vec{r})\right\}\psi(\vec{r},t) \qquad (3.1)$$

where $V(\vec{r},t)$ is the potential acting between the two quarks, possibly influenced by the medium and where $W(\vec{r})$ describes the loss of flux due to the coupling to the D-$\bar{D}$ channels. This model may be used to study the Schrödinger wave packet in various situations :

(1) *free space production.* Then $V(\vec{r},t)$ is the static charmonium potential. The size of the wave packet oscillates and the intensity of $J/\psi$ component is constant if $W = 0$. If not, the size of the wave packet will eventually remain intermediate between the one of the $J/\psi$ and that of the $\psi'$, and the intensity of the $J/\psi$ component remains constant (W couples the D-$\bar{D}$ channels to the static charmonium states above $\psi'$ only);

(2) *plasma phase.* The $V(\vec{r},t)$ equal to the Debye screened potential is practically vanishing. In that case, if one starts with $\psi(\vec{r},0) \propto e^{-r^2/2\sigma^2}$, it can be shown, that the $J/\psi$ intensity will behave like[35,36] :

$$\frac{P_{J/\psi}(t)}{P_{J/\psi}(0)} \approx 1 - \frac{t^2}{\tau^2}, \qquad (3.2)$$

where $\tau \approx m(r_{J/\psi} - \sigma)/p \approx \Delta r/v$, v being the initial average relative velocity ;



(3) *nuclear matter.* In that case, the c-$\bar{c}$ pair may be viewed as travelling inside a ensemble of random dipole color fields, corresponding to the quark-diquark configurations of the nucleons. The c-$\bar{c}$ pair being also a color dipole, the potential to be introduced in Eq. (3.1) can then be written as $V(\vec{r},t) = \sum_i \lambda_i \vec{r}.\vec{e}_i \theta_i(t) + V_{charmonium}$, where $\vec{e}_i$ and $\lambda_i$ are the direction and intensity of the color field inside nucleon i , respectively, and where $\theta_i$ is different from zero (and equal to unity) when the c-$\bar{c}$ pair crosses this nucleon. All these quantities are random. Eq. (3.1) can be solved for each realization of these configurations and ensemble averages can be performed[35]. A typical result for $\sigma \approx 0.2$ fm is shown in Fig. 5. The interesting result is that the $J/\psi$ intensity decreases exponentially, with a slope proportional to $\lambda_i$, as expected, except at early times where it barely decreases (actually parabolically) for a time of the order of $\tau$ (see above).

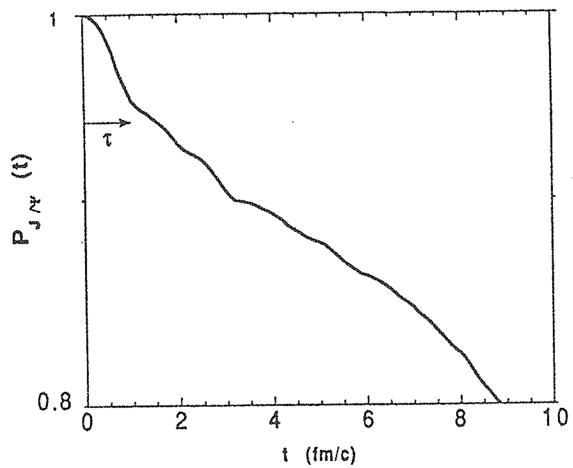


**Figure 5.** Time evolution of $P_{J/\psi}(t)$ in nuclear matter as function of the proper time (t) of the travelling c-$\bar{c}$ pair.

## 3.4. DISCUSSION

From the three cases discussed above, it is clear that the size of wave packet is not a proper means to define a formation time (example (1)). One may define a formation time as the time after which the newly produced particle behaves ordinarily in subsequent interactions. Obviously, the formation time will depend upon the kind of interaction and upon the nature of the particle. A c-$\bar{c}$ pair in a plasma will dissolve, as expected, after time $\tau$ (example (2)). A c-$\bar{c}$ pair travelling inside nuclear matter will be absorbed after time $\tau$ also (example (3)). The last example provides the simplest model of color transparency, since for a time $\tau$ , the interaction of one of the quarks due to the medium is more or less cancelled by the presence of the other quark. However, this property relies heavily on the dipole character of the interaction which makes the interaction probability proportional to the size of the object. In this simple example, color transparency appears as a "geometrical" quantum effect similar to the Perkins effects in quantum electrodynamics[37]. We remind that, even if $\tau$ is small, the effect at high energy may be important, as $\tau$ is a proper time and should be multiplied by the appropriate Lorentz factor in order to have the observable effect.

The fact that most of the hadron-hadron cross-sections are increasing with the size of each of the hadrons[38] may indicate that the dipole interaction is more or less justified and that the formation time with prior increased transparency should be implemented in hadronic cascade. However, a full justification and a derivation of its value remain to be done.

## 4. PARTONIC CASCADE

### 4.1. STRING DYNAMICS

Here the basic process involves the substructures of the hadrons. The most useful concept here is the classical relativistic string, that we review quickly. Afterward, we briefly describe how this feature is implemented in the VENUS model for heavy ion collisions and we draw a synopsis of the various codes handling partonic cascade.

A string can be viewed as a one-dimensional object with ends carried by massless particles[39]. The most popular example is the linear or yo-yo string, which produces an oscillating behaviour in space-time, as illustrated by Fig. 6. The string has an energy carried by the string itself and by the ends (quarks), a momentum carried by the ends only and a mass m which is equal to the area of the representative rectangle in Fig. 6.

A string can break into two or several strings. The most popular model is the Artru-Menessier model[40], which assumes iterative binary breaking of the string. For a single step (see Fig. 6), the probability of breaking at a given point of space-time is supposed to follow an exponential law

$$P = e^{-\alpha_0 A}, \qquad (4.1)$$

where $\alpha_0$ is some parameter and where A is the area of the absolute past after the so-called yo-yo point, i.e. the area of the rectangle under the breaking point (B in Fig. 6).

Energy and longitudinal momentum are conserved at each breaking point. Flavor is produced with some adjustable probability

In the VENUS model, the mass of the produced strings does not correspond to some known hadron. The breaking process is terminated if a string cannot break in two strings with masses larger than the pion mass.



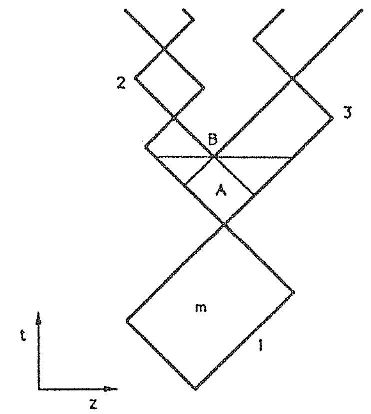


**Figure 6.** Breaking of string 1 into two strings (2 and 3). The heavy lines represent the trajectories of the ends of the strings.

### 4.2. THE VENUS MODEL[41,42]

In this model, the dynamical process is basically divided into three steps, namely the formation of strings, the independent fragmentation of strings and the string re-interaction, producing hadrons. The number m of strings is determined at random according the probability $\sigma_m / \sigma_{tot}$ , where $\sigma_m$ and $\sigma_{tot}$ are respectively the m-Pomeron exchange and the total cross-sections calculated from Gribov-Regge theory (this ensures unitarity). A Pomeron exchange can be viewed as a soft process permitting color exchange between two nucleons. The process is exemplified in Fig. 7. Strings are fragmented according to the Artru-Menessier model. Finally, strings which cross each other can fuse to make a cluster of definite mass. Finally, the clusters are supposed to produce hadrons according to a purely thermal model.

It should be noticed that the valence quarks but also the sea quarks generate strings. In the VENUS model, $p_\perp$ motion is produced at random at a string breaking point with an adjustable average amplitude. Also, minijets ($q\bar{q}$ pairs with $p_\perp^2 > 5\ \mathrm{GeV}^2$) are produced at high energy, assuming exchange of hard pomerons with appropriate cross-section. The VENUS model (as well as the other ones) can be applied to N-N and nucleus-nucleus (A-B) collisions.

### 4.3. COMPARISON WITH OTHER MODELS

We present synthetically this comparison in Table 2.

**Table 2.** Comparison between Partonic Cascade

| Model | First Interactions | Fragmentation | Soft/Hard | A-B Interaction | Rescattering |
|---|---|---|---|---|---|
| VENUS[41] | Color Exchange (Pomeron) | Area law | S/H | Gribov-Regge | Cluster model |
| DPM (Orsay)[43] | " | $j \rightarrow j + h$ | S/H | " | — |
| DPM (DTUJET)[44] | " | " | S/H | " | n-h resc. |
| QGSM[45] | " | Regge model | S | " | Gribov-Regge |
| FRITIOF[14] | Longit. Excitation | $s \rightarrow s + h$ (JETSET) | S | Geometry | — |
| ATTILA[47] | " | " | S | " | — |
| PYTHIA[48] | PQCD | " | H | " | — |
| HIJING[49] | PQCD/FRITIOF | " | S+H | " | — |
| RQMD[50,51] | (FRITIOF) | " | S | " | h-h resc. |
| SPACER[52] | (FRITIOF) | $s \rightarrow s + h$ | S | " | h-h resc.* |
| HERWIG[53] | PQCD | $g \rightarrow q\bar{q}$ | H | " | — |
| GEIGER[54] | PQCD | parton cascade | H | " | — |

The first interactions may be either a color exchange (due to Pomeron) as in VENUS or a "longitudinal excitation" as in FRITIOF, the latter requiring that all partons in a string originate from one nucleon, and implying large momentum transfer. The momentum distribution of the final products of the string breaking is not very sensitive to this difference. Going more to the high energy side, some models assume a perturbative QCD phase as the first interactions.

The fragmentation of the string may follow the area law (4.1), or the Feynman-field picture of two jets radiating hadrons ($j \rightarrow j + h$) independently, or the JETSET procedure, in which a string decays into a hadron and another string ($s \rightarrow s + h$).

The third column in Table 2 indicates whether soft and/or hard processes are included. The hard process refers here to mini-jets, whose importance increases beyond



100 GeV. The fourth column deals with the partonic cascades, developed for N-N collisions, are implemented in heavy ion collisions. There are essentially two choices : either the strings are generated according to the Gribov-Regge theory or they are simply following geometrical arguments (nucleons in a row).

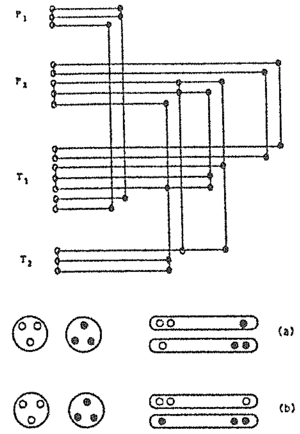

**Figure 7.** Upper part : formation of strings in the collision of projectile nucleons ($P_1$ , $P_2$) with target nucleons ($T_1$ , $T_2$), due to the exchange of four Pomerons. Note that sea quarks participate as well as valence quarks and that strings may carry single quarks or diquarks at their ends. Lower part : formation of strings due to color exchange (a) or to momentum exchange (b).

The last column in Table 2 refers to the (final state) rescattering, i.e. the possible rearrangement of the objects issued from the fragmentation. When it is included, the rearrangement may be based on cluster model like in VENUS or on hadron-hadron rescattering. Sometimes the latter are handled in r-space, assigning positions to the fragments of the original strings (SPACER, RQMD).

A special remark should be made for HERWIG which assumes that the PQCD phase is dominant on the (softer) string breaking phase, the fragmentation of the latter being replaced by gluon splitting. The approach by Geiger[55] is of the same spirit. Partons which are supposed to be deconfined (or rather decorrelated) from the beginning are followed in space-time. The equilibration is very fast, mainly because of the large density of partons. In both approaches, partons have large virtuality and it is then perhaps not allowed to consider local interactions. Nevertheless, these approaches are very promising in the perspective of constructing a transport theory in the deconfining regime.

## 5. CONCLUSION

We have viewed the cascade models from low energy to high energy, dividing them into nucleon cascades, hadronic cascades and partonic cascades from the low energy side to the high energy side.

The cascade models are very successful. Furthermore, they have the virtue (except perhaps at high energy) of providing a picture of the space-time evolution of the system. Potentially, the comparison of their predictions with experiment will eventually reveal some information on the dense phase realized in the collisions. However, open problems are still remaining.

Despite of their great success, the connection between these models and basic theory is still vaguely understood. In this perspective, the situation for the nucleon cascade is by far the most advanced : a consistent link between theory and the transport equation supposedly solved by the nucleon cascade model is established. However, three main problems remain : medium corrections have to be properly evaluated, fluctuations are still to be understood and quantum effects should be evaluated correctly. None of these effects seem to be very small. As we indicated, these uncertainties hamper seriously any attempt to extract the e.o.s. of cold nuclear matter.

For the hadronic cascade, the situation is worse since the connection with theory is loose. Furthermore, medium effects on hadron-hadron collisions and on hadron effective masses are expected to be large, but nothing is known about that. The other important problem is the question of the formation time and of the possible associated enhancement of transparency. This point would deserve serious investigation.

The partonic cascade is perhaps in even worse status, although here one may have some confidence from the success of the models for N-N scattering. The connection with theory (QCD) is rather vague, at least on the low energy side. The soft aspects of QCD are here described by means of string formation and fragmentation. The independence of strings and the way deconfinement and chiral symmetry are treated are here the most serious problems. Going to the high energy side, the situation may be better since PQCD should be more and more valid. However, one is then dealing with partons of large virtuality, the interaction of which is still not well described.

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