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MICROSCOPIC CALCULATION OF THE TEMPERATURE DEPENDENCE OF THE EFFECTIVE INTERACTION*

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Abstract: The effective interaction between two nucleons inside nuclear matter is calculated microscopically for several densities and several temperatures, assuming it is given by the Brueckner *g*-matrix, derived from the separable Graz version of the Paris potential. Particular attention is paid to the temperature dependence and to the way of putting the effective interaction into a Skyrme functional. It is found that the latter task can be achieved with a very good accuracy and that the Skyrme coefficients are weakly and quadratically temperature dependent. The *g*-matrix issued from the Paris potential turns out to be rather different from the standard Skyrme forces in singlet odd states. The charge dependence is investigated through the study of neutron matter. The connection with the temperature dependence of the surface energy is also discussed.

1. Introduction

The study of hot nuclei is of large current interest [1]), both experimentally and theoretically. The main issue is to know what is the limiting temperature, i.e., the temperature beyond which a nucleus ceases to keep its cohesion (at least for a time which makes its observation possible). From the theoretical side, this problem has been attacked with statistical [2,3]) as well as with dynamical calculations [4,8]). In both cases (and in the latter, this is not the only one), an uncertainty arises because of the poor knowledge of the temperature dependence of the nuclear effective forces. Recently, nuclear matter calculations [9,10]) have shown that, fortunately, the effective interaction on the whole does not depend very much on the temperature, when the latter is raised from 0 to ~10 MeV. To quantify this statement, it can be said that the change is at the most ~2–4%. However, a point still needs to be clarified. The effective interaction is a complicated quantity which depends upon spin, isospin and momentum of the nucleons. It may then happen that, even if the interaction does not globally depend very much upon the temperature, some of its pieces can display a stronger dependence. The purpose of this paper is to investigate this

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problem, studying in more detail nuclear matter calculation at zero and finite temperatures. As a commonly used form of the effective interaction is the Skyrme functional, we proceed as follows; we calculate the effective interaction by the extended Brueckner formalism at various temperatures and densities. The effective interaction is identified with the g-matrix calculated for the Paris potential. We investigate whether the latter can be cast into a Skyrme functional. This turns out to be the case, as it has also been demonstrated for simpler interactions [11]). We extract the parameters of the Skyrme functional and study their temperature and density dependence. The Brueckner approach for the two-body interactions alone, even truly realistic, does not give proper saturation properties. Therefore, our results for the temperature dependence are expected to be more reliable than those for the density dependence. We will focus primarily on the first ones.

The paper is organized as follows. Sect. 2 is a reminder of the basic theoretical background about effective interactions and the g-matrix. In sect. 3, we present and analyze our results. Finally, sect. 4 contains our discussion and our conclusion.

2. Theoretical background

The effective interaction, which we identify with the Brueckner g-matrix, depends upon many variables, even in the case of an infinite uniform system of density ρ and temperature T. In fact, we may write it as [12])

$$\langle \boldsymbol{k}|g|\boldsymbol{k}'\rangle = f(k, k', \cos\theta, W, \rho, T, K)\,, \tag{2.1}$$

where $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the initial and final relative momentum between the two interacting nucleons, θ is the angle between these two vectors, W is the so-called starting energy and K is the total momentum of the pair. The g-matrix depends upon the latter quantity through the energy denominators and through the occupation numbers (for $T \neq 0$) entering the Bethe-Goldstone equation (actually, there is an additional dependence upon the angle between $\boldsymbol{K}$ and $\boldsymbol{k}$, which is however negligible, in the limit of a constant effective mass). On the energy shell, expression (2.1) drastically simplifies, since $k = k'$, and W is directly linked to K. Therefore one may write

$$\langle \boldsymbol{k}|_{\mathrm{ON}}|\boldsymbol{k}'\rangle = f_{\mathrm{ON}}(k, \cos\theta, K)\,. \tag{2.2}$$

From now on, we will not write explicitly the overall dependence upon ρ and T.

According to the physical problem one is interested in, one is led to further simplification by averaging over the single-particle states. Here we will restrict ourselves to the effective interaction relevant to static or quasistatic properties of the nuclei. Then it is natural to consider the following quantity

$$\langle \boldsymbol{k}|G_{\mathrm{ON}}|\boldsymbol{k}'\rangle = \frac{\displaystyle\int \mathrm{d}^3k_1 \int \mathrm{d}^3k_2\, \delta(\boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k}) n(k_1) n(k_2) \langle \boldsymbol{k}|g_{\mathrm{ON}}|\boldsymbol{k}'\rangle}{\displaystyle\int \mathrm{d}^3k_1 \int \mathrm{d}^3k_2\, \delta(\boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k}) n(k_1) n(k_2)}\,, \tag{2.3}$$

where $n(k)$ is the occupation number probability, namely

$$n(k)=\left[1+\exp\left(\frac{e(k)-\mu}{T}\right)\right]^{-1}. \tag{2.4}$$

In the last expression, $e(k)$ is the single-particle energy of state k and μ is the chemical potential. Eq. (2.3) may be written as

$$\langle \boldsymbol{k}|G_{\mathrm{ON}}|\boldsymbol{k}'\rangle=\frac{\int \mathrm{d}K\,\omega(K) f_{\mathrm{ON}}(k,\cos\theta,K)}{\int \mathrm{d}K\,\omega(K)}, \tag{2.5}$$

where the function $\omega(K)$ is given in the appendix.

We would like to put this effective force in a Skyrme like functional. For a general Skyrme force, the (non-antisymmetrized) matrix element between a bra $\langle \boldsymbol{k}|$ and a ket $|\boldsymbol{k}'\rangle$, has the following simple form [13]) (on-shell)

$$\begin{aligned}\langle \boldsymbol{k}|V_{\mathrm{SK}}|\boldsymbol{k}'\rangle=&[t_0+\tfrac{1}{6}t_3\rho^{\alpha}+t_1k^2+t_2k^2\cos\theta]\\&+P_{\sigma}[t_0x_0+\tfrac{1}{6}t_3x_3\rho^{\alpha}+t_1x_1k^2+t_2x_2k^2\cos\theta],\end{aligned} \tag{2.6}$$

where P_σ is the spin state projection operator. For simplicity, we have neglected here the spin-orbit term, which is not very important in nuclear matter. It may be useful at this point to study expression (2.6) in each substate. One readily has, using antisymmetrized states

$$\text{(SO)}\qquad \langle \boldsymbol{k}|V_{\mathrm{SK}}|\tilde{\boldsymbol{k}}'\rangle=2t_2k^2(1-x_2)\cos\theta, \tag{2.7a}$$

$$\text{(TO)}\qquad \langle \boldsymbol{k}|V_{\mathrm{SK}}|\tilde{\boldsymbol{k}}'\rangle=2t_2k^2(1+x_2)\cos\theta, \tag{2.7b}$$

$$\text{(SE)}\qquad \langle \boldsymbol{k}|V_{\mathrm{SK}}|\tilde{\boldsymbol{k}}'\rangle=2t_0(1-x_0)+\tfrac{1}{3}t_3(1-x_3)\rho^{\alpha}+2t_1(1-x_1)k^2, \tag{2.7c}$$

$$\text{(TE)}\qquad \langle \boldsymbol{k}|V_{\mathrm{SK}}|\tilde{\boldsymbol{k}}'\rangle=2t_0(1+x_0)+\tfrac{1}{3}t_3(1+x_3)\rho^{\alpha}+2t_1(1+x_1)k^2. \tag{2.7d}$$

Of course, the quantity (2.5) has *a priori* a more complicated functional dependence upon k and $\cos\theta$ (and ρ as well) than the one expressed in (2.6). Therefore it is not possible to cast exactly (2.5) into (2.6) and one has to adopt some procedure. Here we look at (2.5) as a power series of k and $\cos\theta$. We can then identify the leading terms to expression (2.6) and study the importance of the deviation from the latter form. For matter of convenience, we will consider below specific linear combinations of the various substates. We first consider the following quantity

$$F_1=\sum_{\substack{LSTJ\\ L_{\mathrm{even}}}}\varepsilon(LST)(2J+1)(2T+1)\langle \boldsymbol{k}|G_{\mathrm{ON}}(LST)|\tilde{\boldsymbol{k}}'\rangle P_L(\cos\theta), \tag{2.8a}$$

where $\varepsilon(LST)=\frac{1}{2}[1+(-1)^{L+S+T}]$ and $G_{\mathrm{ON}}(LST)$ is the quantity G_{ON} projected on state LST. If we limit ourselves to $L\leqslant 4$, we have

$$F_1=F_1^{(0)}+F_1^{(2)}\cos^2\theta+F_1^{(4)}\cos^4\theta. \tag{2.8b}$$

The quantities $F_1^{(i)}$ are functions of k^2 and ρ (and T of course). We may always write

$$F_1^{(i)} = 12[\xi_1^{(i)}(\rho) + \eta_1^{(i)}(\rho)k^2] + O(k^4, \rho)\,, \tag{2.8c}$$

where the second term is at least of the order of k^4. The first term is written so because if the g-matrix G_{ON} was exactly a Skyrme interaction, one would have the following identifications

$$\xi_1^{(0)} \equiv t_0 + \tfrac{1}{6}t_3\rho^\alpha, \qquad \eta_1^{(0)} \equiv t_1, \qquad \xi_1^{(i\geq 2)} \equiv \eta_1^{(i\geq 2)} \equiv 0\,. \tag{2.8d}$$

Similarly, we can write

$$F_2 = \sum_{\substack{STJ \\ L_{odd}}} \varepsilon(LST)(2J+1)(2T+1)\langle k|G_{ON}(LST)|\tilde{k}'\rangle P_L(\cos\theta)\,, \tag{2.9a}$$

$$F_2 = F_2^{(1)}\cos\theta + F_2^{(3)}\cos^3\theta\,, \tag{2.9b}$$

$$F_2^{(i)} = 36[\xi_2^{(i)}(\rho)k^2] + O(k^4, \rho)\,. \tag{2.9c}$$

In the same conditions as for eq. (2.8d), one would have

$$\xi_2^{(1)}(\rho) \equiv t_2\,, \qquad \xi_2^{(3)}(\rho) \equiv 0\,. \tag{2.9d}$$

We will also consider

$$F_3 = \sum_{\substack{STJ \\ L_{even}}} \varepsilon(LST)(-1)^{S+1}(2J+1)(2T+1)\langle k|G_{ON}(LST)|\tilde{k}'\rangle P_L(\cos\theta)\,, \tag{2.10a}$$

$$F_3 = F_3^{(0)} + F_3^{(2)}\cos^2\theta + F_3^{(4)}\cos^4\theta\,, \tag{2.10b}$$

$$F_3^{(i)} = 12[\xi_3^{(i)}(\rho) + \eta_3^{(i)}(\rho)k^2] + O(k^4, \rho)\,. \tag{2.10c}$$

In the same conditions as for eq. (2.8d) , one would have

$$\xi_3^{(0)} \equiv t_0x_0 + \tfrac{1}{6}t_3x_3\rho^\alpha, \qquad \eta_3^{(0)} \equiv t_1x_1, \qquad \xi_3^{(i\geq 2)} \equiv \eta_3^{(i\geq 2)} \equiv 0\,. \tag{2.10d}$$

Finally, we have

$$F_4 = \sum_{\substack{STJ \\ L_{odd}}} \varepsilon(LST)(-1)^{S+1}(2J+1)(2T+1)\langle k|G_{ON}(LST)|\tilde{k}'\rangle P_L(\cos\theta)\,, \tag{2.11a}$$

$$F_4 = F_4^{(1)}\cos\theta + F_4^{(3)}\cos^3\theta\,, \tag{2.11b}$$

$$F_4^{(i)} = 36[\xi_4^{(i)}(\rho)k^2] + O(k^4, \rho)\,, \tag{2.11c}$$

and the limiting relations

$$\xi_4^{(1)}(\rho) \equiv t_2x_2\,, \qquad \xi_4^{(3)}(\rho) \equiv 0\,. \tag{2.11d}$$

Actually, the quantity F_1 is simply

$$F_1 = \sum_{SM_S} \sum_{TM_T}{}' \langle k\,SM_S\,TM_T|G_{ON}|\widetilde{k'\,SM_S\,TM_T}\rangle\,, \tag{2.12}$$

where the prime means that only the even partial waves are retained. It is easy to find similar definitions for F_2, F_3 and F_4. We give in table 1 the explicit coefficients entering expression (2.12) for the most important quantities $F_i^{(j)}$ when a projection on the total angular momentum J is performed.

TABLE 1

Coefficients entering the expansion of the quantities $F_i^{(j)}$ (eqs. (2.9)) in terms of the (non-antisymmetrized) partial-wave matrix elements of the g-matrix

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3D_1$	$^3D_2$	$^3D_3$	$^3F_2$
$F_1^{(0)}$	6	6					−15	−3	−5		
$F_1^{(2)}$							45	9	15		
$F_2^{(1)}$			54	6	18	30					−45
$F_2^{(3)}$											75
$F_3^{(0)}$	−6	6					15	−3	−5		
$F_3^{(2)}$							−45	9	15		
$F_4^{(1)}$			−54	6	18	30					−45
$F_4^{(3)}$											75

3. Numerical results

3.1. INTRODUCTION

We have calculated the various quantities $F_i^{(j)}$ defined in the previous sections for several densities ranging from $\sim\frac{1}{4}\rho_0$ to $\sim 2\rho_0$, where ρ_0 is the saturation density and for various temperatures extending from $T=0$ to $T=15$ MeV. These variations encompass the conditions encountered in medium-energy heavy-ion collisions and the so-called coexistence zone for nuclear matter. The interaction used is the Graz potential. It is a separable potential which has the same structure and yields the same phase shifts as the Paris potential [14]). It has also been shown to give roughly the same results for the Brueckner g-matrix and for the rearrangement term [15]) as the original Paris potential.

3.2. SKYRME-LIKE PARAMETERS

We first analyzed the quantities $F_i^{(0)}$ in the following way. We first verified that to a good accuracy, they closely fit (see later for a discussion) eqs. (2.8d), (2.9d), (2.10d) and (2.11d) except for the quantity $\xi_1^{(0)}$. The latter requires some discussion. In the extensive Skyrme phenomenology, the form (2.8d) is assumed and the exponent α is determined by fitting some properties of several nuclei. However, it should be realized that only the vicinity of ρ_0 is investigated. Here, we are interested in a very broad range of densities, especially in the $\rho<\rho_0$ side. If one tries a low density expansion, one has good reasons to assume that $\xi_1^{(0)}$ can be expanded as a rapidly converging series in k_F (this is the relevant parameter and not ρ),

$$\xi_1^{(0)} = \tilde{t}_0 + \tfrac{1}{6}\tilde{t}_3^{(1)}\rho^{1/3} + \tfrac{1}{6}\tilde{t}_3^{(2)}\rho^{2/3} + \cdots . \tag{3.1}$$

We therefore analyzed $F_1^{(0)}$ with expression (2.8d) and also with (3.1) limited to the first three terms. We first tried to fit the Skyrme parameters to the ξ functions for each temperature under study [8]). This procedure reveals that the exchange coefficients are practically temperature independent (see fig. 1). On the contrary,

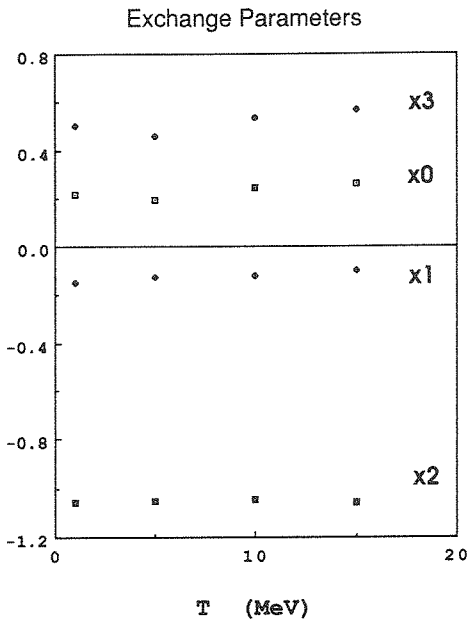

Fig. 1. Variation with temperature T of the exchange parameters for the microscopically calculated effective interaction. See text for detail.

the quantities t do show some dependence. However, due to the fitting procedure which introduces some distortion and to the error bars inherent to our calculation, the temperature dependence so extracted is, at least for some of the parameters, not purely quadratic in T, although the original ingredients of the calculation, the matrix elements in definite partial waves, turn out to be quadratic to a very good approximation. This is shown in fig. 2 for two of them. We therefore adopt the following procedure. We assume that the exchange coefficients are temperature-independent and that the other parameters are quadratic, as $(i=0, 1, \ldots)$

$$t_i = t_i(T=0)[1+\beta_i T^2] \tag{3.2}$$

in the domain of temperature under study, and fit the coefficients to the ξ functions for all temperatures at the same time. The results are shown in table 2. The coefficient which shows the largest temperature dependence is t_3' which changes by 10% when T changes from 0 to 5 MeV. Note that that on general grounds [16]) one expects an analytical dependence upon the temperature, with a development which starts like eq. (3.2).

3.3. COMPARISON WITH EXISTING SKYRME FORCES

In table 3, we compare the coefficients obtained from our calculation with those of existing Skyrme forces. There is a large spreading of the values of the parameters

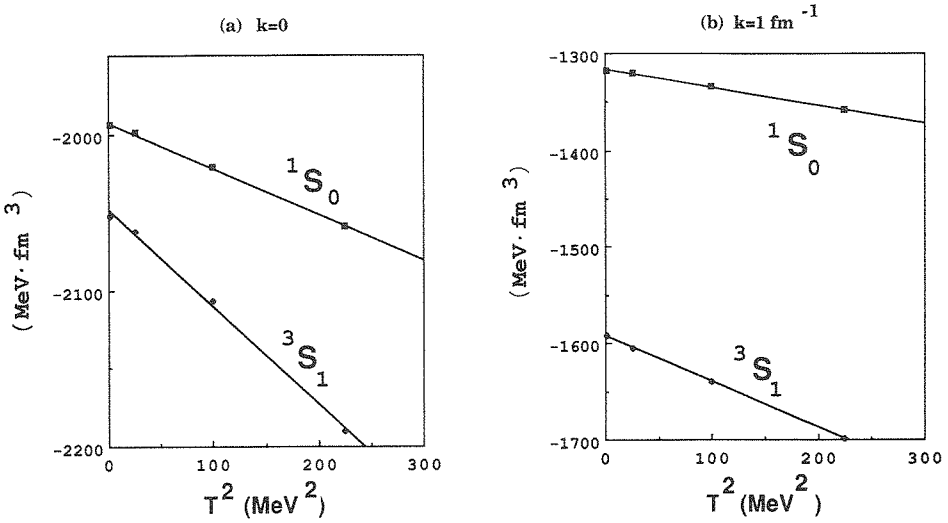

Fig. 2. Matrix elements of the g-matrix (averaged as in eq. (2.3)) in two partial waves for (a) $k=0$ and (b) $k=1\,\mathrm{fm}^{-1}$, calculated for $\rho=\rho_0$ and various temperatures. The linearity in T^2 is illustrated. See text for detail.

TABLE 2

Values of the parameters of the Skyrme functional (eqs. (2.6), (3.1) and (3.2)) fitted to the calculated g-matrix for temperature ranging from 0 to 15 MeV and for density between $\frac{1}{4}\rho_0$ to $2\rho_0$

	t_i $(T=0)$	β_i $(10^{-3}\,\mathrm{MeV}^{-2})$
t_0 (MeV·fm^3)	−1492	0.878
t_1 (MeV·fm^5)	192.5	0.423
t_2 (MeV·fm^5)	148.4	−0.010
t_3 (MeV·fm^4)	7011	0.808
t_3' (MeV·fm^5)	−1588	−4.445
x_0	0.218	
x_1	−0.130	
x_2	−1.054	
x_3	0.450	
x_3'	0	

among the various cases. This reflects the fact that the observables used to constraint the phenomenological Skyrme interactions do not uniquely define a force. As we already mentioned in the introduction, the parameters obtained in our analysis cannot be considered (at least for all of them) as a theoretical tool for removing definitely the undeterminancy. It is certainly useful to give a guideline for some parameters, but the lack of good saturation properties inherent to the Paris potential prevents us from attaching too much weight to those relevant parameters (t_0 and t_3).

TABLE 3

Comparison (at $T=0$) between the Skyrme-like parameters for our calculated effective interaction with those of some phenomenological effective interactions: SIII force [13]), SkM force [26]), the modified SIII force [19]) and the force of ref. [17]). Column (A) corresponds to eq. (3.1), whereas column (B) corresponds to the standard form (2.6)

	Our results		SIII	SkM	SIII*	Ref. [17])
	(A)	(B)				
t_0 (MeV·fm^3)	−1492	−1500	−1128.75	−2645	−1121	−1789
t_1 (MeV·fm^5)	192.5	192.5	395	385	400	301
t_2 (MeV·fm^5)	148.4	148.4	−95	−120	−533	502
t_3 (MeV·fm^6)		6200	14000	15595	14000	12764
$\tilde{t}_3^{(1)}$ (MeV·fm^4)	7011					
$\tilde{t}_3^{(2)}$ (MeV·fm^5)	−1588					
α		$\frac{1}{3}$	1	$\frac{1}{6}$		$\frac{1}{3}$
x_0	0.22	0.2	0.45	0.09	0.43	0.353
x_1	−0.13	−0.16	0		0.35	−2.5
x_2	−1.05	−1	0		−0.98	−1.7
x_3	0.45	0.47	0	0		0.475

The difficulty of the analysis lies in the fact that most physical properties are linked to several parameters at the same time. For some of them the relationship is not even known. Nevertheless we will make some comments.

A striking feature of table 3 is that t_2 (and x_2) are quite different (opposite sign) in our results and in the celebrated SIII and SkM forces. However, as pointed by Tondeur [17]) (who proposed for t_2 and x_2 qualitatively similar values as ours), the positive sign of t_2 and the negative sign of x_2 are dictated by the repulsive nature of the effective force in odd states, in particular in the singlet ones, a feature which is not tested separately in the abundant "skyrmiology".

It is interesting to consider more physically meaningful combinations of parameters. Some of them are given in table 4. The first quantity is linked to compression and the second one determines the effective mass, more precisely the k-mass [18]), which comes from the nonlocality of the interactions. The Paris potential yields the same mass as SIII, but not as SkM, which has a slightly larger mass. The interaction of ref. [17]) has the particular feature of having an effective mass = 1. The third quantity in table 4 is related to the gradient terms in the Skyrme energy functional (in symmetric systems) and therefore is connected to the surface energy. It will be discussed later on. The next three quantities enter the expression of the isospin symmetry energy. Similarly, the last three quantities contribute to the spin symmetry energy [19]). For the (isospin) symmetry energy, the most important is the first of these three coefficients. One can see that the Paris potential yields about the same value as SIII. Actually, detailed calculations [20,21]) reveal that the energy symmetry is the same in both cases (~30 MeV). Concerning the spin symmetry energy, the Paris potential yields about the same value as for the isospin symmetry energy [22]).

TABLE 4

Same as table 3, but parameters are grouped in expressions corresponding to the physical quantities of the first column. See text for detail

		(A)	SIII	SkM	SIII*	Ref. [17])
B/A	$t_0+\frac{1}{6}t_3\rho_0^{1/3}$	−937	164	−1144	171	−610
m^*	$3t_1+5t_2+4t_2x_2$	702	710	555	708	−0.6
$(\nabla\rho)^2$	$9t_1-5t_2-3t_2x_2$	1370	4030	4065	4697	2759
E_T	$t_0(2x_0+1)$	−2100	−2145	−3121	−2085	−3052
	$4t_2+5t_2x_2-3t_1x_1$	−58.7	−380	−480	340	−1.5
	$t_3(2x_3+1)$	11834	14000	15595	14000	24870
E_S	$t_0(-2x_0+1)$	−900	−129	−2168	−161	−525
	$4t_2+5t_2x_2-3t_1x_1$	−236	−380	−480	620	−4516
	$t_3(-2x_3+1)$	366	14000	15595	14000	739

This in fact results from subtle compensations between x_0, x_1 and x_3 contributions. The value of x_0 in SIII combined with vanishing x_1 and x_3 gives a smaller spin symmetry energy, a well known deficiency of the standard Skyrme interactions, partially corrected in SkM.

3.4. DENSITY DEPENDENCE

We are here primarily concerned with the quantity $\xi_1^{(0)}(\rho)$ defined in eq. (2.8c). We study a large enough range of density to describe it accurately with expression (3.1). The quantity $\xi_1^{(0)}$ is given in fig. 3. The latter clearly shows the quadratic dependence upon $\rho^{1/3}$. When focusing on $\rho\approx\rho_0$, as it is usually done, a linear dependence is largely sufficient. That is why static properties are generally reproduced by Skyrme forces with $\alpha=1$. Only the description of vibrations requires a different exponent.

3.5. DEVIATIONS FROM SKYRME FUNCTIONAL

We observe in general that the quantities $F_i^{(j>2)}$ are generally much smaller (at least one order of magnitude) than the leading terms. As an illustration, we give in fig. 4 the comparison at $T=0$ between the quantities $F_1^{(2)}$ and $F_1^{(0)}$. This more or less justifies the use of the Skyrme functionals in the study of static and quasistatic properties, since only the values of $k\leqslant k_F$ are relevant (see the appendix).

3.6. SURFACE ENERGY

It has been established [23]) that for fermion systems described by a energy functional of the Skyrme-type, i.e., by an energy density of the form

$$\mathscr{H}=\frac{\hbar^2}{2m}\tau+F(\rho)+B|\nabla\rho|^2+G\rho\tau, \tag{3.3}$$

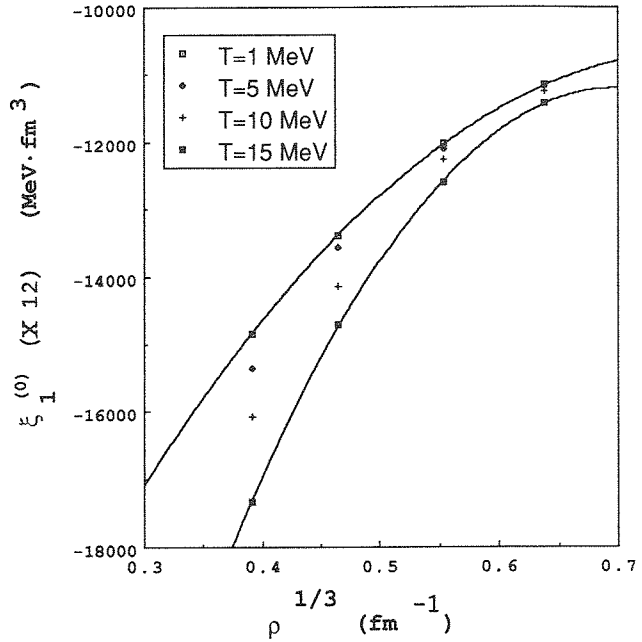

Fig. 3. Quantity $\xi_1^{(0)}$ (eqs. (2.8a–c) as calculated microscopically for various temperatures (symbols) and the fits of two of them (full lines) by quadratic forms in $\rho^{1/3}$ (see eq. (3.1)).

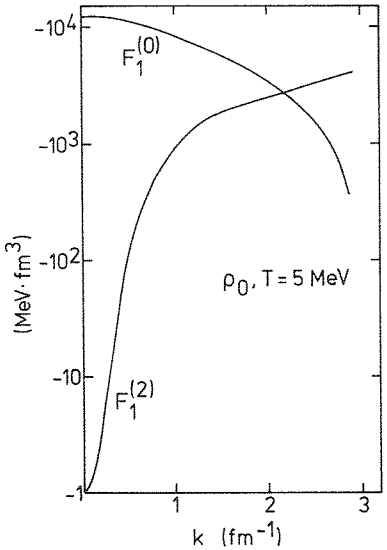

Fig. 4. Variation with the relative momentum k of the quantities $F_1^{(0)}$ and $F_1^{(2)}$ (see text), for normal density and $T = 5$ MeV.

where $F(\rho)$ is some function of the density and τ is the kinetic energy density, there exists a simple relation between the surface tension and the coefficients of the functional (3.3), provided semiclassical approximations are made for relating τ and ρ. It writes at $T=0$

$$\sigma = a\rho_0 b\,, \tag{3.4}$$

with

$$a = \frac{\hbar^2}{2m}\beta + \frac{2}{5}\rho_0[G(\beta-\gamma)+B]\,, \tag{3.5}$$

where β and γ are the coefficients in the so-called Weiszäcker term ($\beta=\frac{1}{6}$, $\gamma=\frac{1}{3}$). One sees that a is directly related to the gradient terms. On the contrary b is related to bulk properties only: density, binding energy, compressibility modulus. Its explicit form depends upon the detail of the function $F(\rho)$ and can be found in ref. [23]) for some typical examples of $F(\rho)$. The temperature dependence of σ (defined as the work necessary to create a unit surface at constant T) has been studied in ref. [24]). It can be cast into

$$\sigma \approx \sigma_0(1-\beta' T^2)\,, \tag{3.6}$$

for temperature-independent coefficients in (3.3). If B and G depend upon temperature, eq. (3.6) should be replaced by

$$\sigma \approx \sigma_0[1-(\beta'+\beta'')T^2]\,, \tag{3.7}$$

where β'' is the T^2 coefficient of the expansion of a (eq. (3.5)) due to the temperature dependence of G and B. It is related to the coefficients β_i of eq. (3.2). Using table 2, we find $\beta'' \approx 0.2\times 10^{-3}$ MeV^{-2}, which should be compared to the value of $\beta' \approx 10^{-2}$ for standard Skyrme force [24]). We can thus conclude that the temperature dependence of the effective interactions has negligible effects on the surface energy.

3.7. NEUTRON MATTER

We also performed the same analysis for neutron matter. It turns out that the extracted values of the Skyrme parameters are the same as those of nuclear matter (to less than ~4%) at the same density.

4. Conclusion

We have calculated microscopically the effective interaction between two nucleons inside nuclear matter. We assumed that the latter can be identified with the g-matrix, calculated with the Graz version of the Paris potential, when averaging over the total momentum of the nucleons is performed. We looked at its expansion as function of both the relative momentum and the cosine of the angle between the incident

and final relative momentum and found that it largely reduces to the Skyrme functional for practical purposes. We compared the calculated effective interaction with conventional Skyrme forces. We found that they are very similar, except for two features:

(1) the coefficient x_2 is totally different, which indicates that the Skyrme forces are not microscopically founded for the singlet odd states;

(2) the calculated coefficients t_0 and t_3 are different from the ones of the best Skyrme forces. This is of course due to deficient saturating property of the Paris potential at this approximation of the Brueckner-Bethe expansion. Moreover, we found a more complicated density dependence, even at low density.

We paid particular attention to the temperature dependence of the effective interaction. We think our method is very well suited for this study, since this dependence arises from the "heating" of the Pauli operator in a Brueckner approach. We found that the exchange parameters are temperature-independent and that the other ones are weakly dependent in the 0-15 MeV range, including those which are connected to the surface energy.

The method is very promising and should be extended to the case of scattering nucleons in the intermediate energy domain.

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Appendix A

EXPRESSION FOR THE NUMBER OF PAIRS WITH RELATIVE MOMENTUM k

The number of pairs with relative momentum $\boldsymbol{k}$ per unit volume is given by

$$\frac{\mathrm{d}n}{\mathrm{d}^3k}=\frac{1}{\rho}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_2}{(2\pi)^3}\,n(k_1)n(k_2)\delta(\boldsymbol{k}_1-\boldsymbol{k}_2-\boldsymbol{k})\,, \tag{A.1}$$

where $n(k)$ is the occupation number probability (eq. (2.4)). Using c.m. coordinates $\boldsymbol{K}=\frac{1}{2}(\boldsymbol{k}_1+\boldsymbol{k}_2)$, $\boldsymbol{q}=\boldsymbol{k}_1-\boldsymbol{k}_2$, one can put (A.1) into

$$\frac{\mathrm{d}n}{\mathrm{d}^3k}=\frac{1}{(2\pi)^6\rho}\int\mathrm{d}^3K\,n(|\mathbf{K}+\tfrac{1}{2}\mathbf{k}|)\mathrm{n}(|\mathbf{K}-\tfrac{1}{2}\mathbf{k}|)\,. \tag{A.2}$$

Of course, $\mathrm{d}n/\mathrm{d}^3k$ does not depend upon the direction of $\boldsymbol{k}$ for an homogeneous system. If we call ξ the angle between $\boldsymbol{K}$ and $\boldsymbol{k}$, one obtains

$$\frac{\mathrm{d}n}{\mathrm{d}^3k}=\frac{1}{(2\pi)^5\rho}\int_0^\infty\mathrm{d}K\,K^2\int_{-1}^{+1}\mathrm{d}(\cos\xi)n(|\boldsymbol{K}+\tfrac{1}{2}\boldsymbol{k}|)n(|\boldsymbol{K}-\tfrac{1}{2}\boldsymbol{k}|) \tag{A.3}$$

or

$$\frac{\mathrm{d}n}{\mathrm{d}^3k}=\frac{1}{\rho}\int_0^\infty \mathrm{d}K\,\omega(K) \tag{A.4}$$

in the notation of sect. 2. The second integral in (A.3) can be performed analytically [25]) for $T\neq 0$ when a quadratic approximation is used for the single-particle energy $e(k)=a+bk^2$. We just reproduce the formula here for $\omega(K)$

$$\omega(K)=\frac{1}{(2\pi)^5}\frac{2K^2}{1-\exp(2A)}\left\{\frac{1}{B}\ln\frac{1+\exp(A-B)}{1+\exp(A+B)}+1\right\} \tag{A.5}$$

with

$$A=\frac{a+b(K^2+k^2/4)-\mu}{T},\qquad B=\frac{bkK}{T}. \tag{A.6}$$

The limit of relation (A.5) for $T\to 0$ is not easy to work out, but explicit formulae are also given in ref. [25]). The formula for $\omega(K)$ is then

$$\omega(K)=\frac{2K^2}{(2\pi)^5}\times\begin{cases}1 & \text{for } 0\leqslant\tfrac{1}{2}k\leqslant k_\mathrm{F}-K,\\ (k_\mathrm{F}^2-k^2/4-K^2)/kK & \text{for } k_\mathrm{F}-K\leqslant\tfrac{1}{2}k\leqslant\sqrt{k_\mathrm{F}^2-K^2},\\ 0 & \text{for } \sqrt{k_\mathrm{F}^2-K^2}\leqslant\tfrac{1}{2}k\leqslant k_\mathrm{F}, \text{ or for } K>k_\mathrm{F}.\end{cases} \tag{A.7}$$

In this case, $\mathrm{d}n/\mathrm{d}k^3$ can be calculated explicitly. One readily obtains

$$\frac{\mathrm{d}n}{\mathrm{d}k^3}=\frac{1}{(2\pi)^5\rho}\times\tfrac{1}{3}(k_\mathrm{F}-\tfrac{1}{2}k)^2(k_\mathrm{F}+\tfrac{1}{4}k)\,. \tag{A.8}$$

For $T\neq 0$, the integration has been performed numerically.

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