Nuclear Physics A489 (1988) 781–802
North-Holland, Amsterdam

MEDIUM EFFECTS IN PION PRODUCTION

J. CUGNON[1] and M.-C. LEMAIRE

Service de Physique Nucleaire à Moyenne Energie; CEN Saclay 91191 Gif-sur Yvette cedex, France

Received 19 May 1988

Abstract: Pion production is studied in the framework of the intranuclear cascade model, for three different systems: proton–nucleus, heavy-ion and pion-nucleus collisions. The aim of this work is to investigate the sensitivity of the pion yield to modifications of the input data describing the properties of the delta and pion formation, destruction and propagation due to the surrounding nuclear medium. For these systems both the target mass dependence at one incident energy and the energy dependence for few selected targets have been considered. It is found that the three systems behave quite differently. The sensitivity of the heavy-ion case is rather large and medium corrections can remove the original discrepancy between standard cascade results and experiments. The possible lack of absorption in the standard model for pion production is also discussed.

1. Introduction

Pion production in heavy-ion collisions is a topic of large interest in connection with the nuclear matter equation of state, and more generally with hadronic matter properties and nuclear collision dynamics [1-11]). Up to recently, the prominent point of view was the discrepancy between the experimental pion yield and the prediction of the intranuclear cascade (INC) model is related to the compression energy necessary to attain the high densities realised in the course of the reaction. This argument was reinforced by the observation that the pion yield in proton–nucleus reactions (where compression is presumably unimportant) is more or less well reproduced by the cascade (see below however). The idea that compression energy is the key explanation was pushed forward by the so-called VUU[6]) and similar approaches, which explicitly include this effect. They first concluded to a high compressibility of nuclear matter, but it seems that this conclusion was revised when it was realized that the compression energy may depend sizeably on the relative velocity of the particles. The situation is far from being settled. For instance, recent measurements done with the DIOGENE detector for α projectile [12]) show that a microscopic theory disregarding compression energy, namely the intranuclear cascade, over-estimates in general the pion yield, in as much as the target mass increases.

Most of the studies so far were undertaken on the basis of a production mechanism which is essentially the same as in free space. The possibility that the above-mentioned discrepancy can be due to medium effects on the mesonic degrees of

---

[1] Université de Liège, Institut de Physique au Sart Tilman, B-4000 Liège 1, Belgique.

freedom has been disregarded. We have in mind a possible modification of the pion and delta propagation, lifetime, creation and destruction probability due to the presence of a (possibly dense and hot) medium. This is a rather surprising situation, since this possibility was pointed out quite early ^{2}) and since some modifications do arise in ordinary nuclear matter, as many theoretical works have discussed $^{13-19}$). The purpose of this paper is to answer the following important question: is the size of the discrepancy between cascade and experiment such that it can be removed by assuming a different behaviour of pions and deltas in nuclear matter, as compared to free space? The medium corrections being not known precisely, especially for hot dense matter, we will rely on the following pragmatic attitude. We will look at several modifications, of reasonable size (in view of the fragmentary information available), and study their influence on the pion yield. We will also try to see whether the INC model can describe the pion yield for p-induced and heavy-ion reactions at the same time, using the same modification of π and Δ properties.

The INC model misses some physical effects (compression energy is the most obvious one), but our study of the in-medium modifications of the mesonic degrees of freedom can nevertheless be valuable for other approaches. Indeed, we concentrate on the collision dynamics and the effects on it are not expected to depend very much on the mean field and on other (rather) soft processes.

For a better understanding of the π production mechanism, we will also pay attention to pion absorption in nuclei, although the latter case deals with a completely different kinematic regime.

The paper is organized as follows. In sect. 2, the main features of the Liege INC code are recalled and the parametrization of the elementary cross sections is discussed. The results obtained for inclusive cross sections of pion production by protons are compared to the available experimental data in sect. 3, for several targets between carbon and lead and incident energies ranging between 500 MeV and 2.1 GeV. The influence of the basic input like the magnitude of the inelastic cross sections, the delta mass or lifetime on the calculated cross sections are examined. The case of the pion multiplicities calculated for central collisions between heavy ions is discussed in sect. 4. In sect. 5, we investigate the pion-nucleus interaction. Finally, sect. 6 is devoted to the discussion and the conclusion.

2. The intranuclear cascade model

The INC model that we have used here, has already been described in refs. ^{2,20,21}). The main points are briefly recalled in order to specify the modifications which have been implemented to get the standard version used for the present analysis.

The model is a Monte Carlo calculation describing the collision process as a succession of binary free collisions well separated in space and time. In the original version the nucleon positions are distributed randomly inside a sphere of radii .12 $A^{1/3}$. To take into account the effect of the diffuse edge of the nucleus a trapezoidal shape has been considered for the nuclear density profile.

$$\rho(r)=\begin{cases} r_0 & \text{if } 0<r<c-z \\ r_0(c+z-r)/2z & \text{if } c-z<r<c+z \\ 0 & \text{if } r>c+z \end{cases} \tag{1}$$

with $c=r_cA^{1/3}$ and $z=2.2a$.

Parameters c and a describe the charge distributions as determined from electron scattering measurements [22]). The corresponding values are listed in table 1 together with the reduced radius $RA^{-1/3}$ of the equivalent sphere distribution (giving the same root mean-squared radius).

$$R^2=c^2+\tfrac{7}{3}\pi^2a^2\,. \tag{2}$$

The nucleon momenta are distributed according to a sharp Fermi sphere with a radius of $p_F=270$ MeV/c. The projectile is boosted towards the target with the beam velocity. The particles move along straight lines until a pair of them achieve its minimum relative distance. The final momenta of the particles are determined at random in agreement with conservation laws and experimental cross sections. Inelasticity is taken into account by introducing the following reactions:

$$\mathrm{N}+\mathrm{N} \leftrightarrow \mathrm{N}+\Delta, \qquad \Delta \leftrightarrow \pi+\mathrm{N}\,. \tag{3}$$

The final channels are chosen at random in proportion of the total cross section. Various modifications of the code refs. [2,20,21,23]) have been used for the present analysis. They are listed below:

(a) *The isospin dependence of the elementary cross sections* has been taken into account. The pp and np total elastic and inelastic cross sections are parametrized so as to reproduce the experimental data. For the reactions involving a Δ, the probabilities of the different charge states are given by the appropriate Clebsch-Gordan coefficients.

TABLE 1

Reduced radii r_c and diffuseness used to describe the nuclei with the trapezoidal shape given by eq. (1); reduced radii r_R of the equivalent sphere distribution

Target	$r_c=cA^{-1/3}$	a	$r_R=RA^{-1/3}$
^{12}C	1.013	0.42	1.343
^{27}Al	1.024	0.54	1.274
^{48}Ti	1.055	0.54	1.274
^{63}Cu	1.066	0.54	1.249
^{108}Ag	1.082	0.54	1.211
^{207}Pb	1.095	0.54	1.179

All these quantities are defined in the text and are given in fm.

(b) *The spectator nucleons are frozen* as described in ref. [23]). Indeed, in the original version the spectators are given initially a velocity due to the combination of the nucleus motion and their Fermi motion, they are subsequently left free to move with this velocity. This introduces a spurious expansion of the nucleus even in the absence of perturbations. An improvement to the model has been done by freezing the spectators until they interact. Specifically, the Fermi motion velocity of the nucleons is recorded and given to them before they interact. Once they are involved in a collision, they are given back their original momentum.

(c) *The Pauli blocking factor* $(1-f)$ is calculated using a prescription similar to what is used in the VUU or BUU codes [3,6,24]). For each event, the occupation factor f is determined by examining the neighbourhood of the final-state phase space whenever a collision would otherwise occur. The number of particles is counted in a sphere centered at the final phase space coordinates of the colliding pair. The sphere has a radius of 2 fm in coordinate space and 200 MeV/c in momentum space. A similar procedure is applied to the nucleons resulting from delta decays.

(d) *A binding potential* has been taken into account so that the nucleons are no longer considered as free particles; the interaction has been implemented in a very simple way, as the nucleons are considered as initially moving in potential wells providing a binding field to the individual nuclei. Therefore one can add a scalar field with the following energy-momentum relation [2]):

$$E^2 = p^2 + (m+V)^2 . \tag{4}$$

Once the particle has made a collision the average field is destroyed. Except when it will be specified, all the calculations have been carried out with a binding potential of −40 MeV.

(e) *The* π^+p *cross section* is taken with the prescription of ref. [25]):

$$\sigma = \frac{\sigma_0}{1+4[(\sqrt{s}-E_0)/\Gamma]^2}\,\frac{q^3}{q^3+\mu^3} , \tag{5}$$

where s is the c.m. energy and q is the c.m. momentum. The numerical values are: $\sigma_0 = 326.5$ mb, $E_0 = 1215$ MeV, $\Gamma_0 = 110$ MeV and $\mu = 180$ MeV/c,

(f) In the original version of the code, *the delta lifetime* was calculated as:

$$\tau = h/\Gamma_0 \tag{6}$$

with $\Gamma_0 = 115$ MeV. The possibility of multiplying this lifetime by a constant value (without changing the delta mass width) has been included. However, except when it will be specified, we have used an option where the lifetime is related to the mass of the delta m_Δ through:

$$\tau = h/\Gamma(m_\Delta) , \tag{7}$$

$$\Gamma = \Gamma_0 \left[\frac{t}{t_0}\right]^3 \left[\frac{1+(R_1 t_0)^2}{1+(R_1 t)^2}\right] , \tag{8}$$

with the following numerical values: $\Gamma_0 = 109$ MeV, $R_1 = 1.09$ fm, $t_0 = 1.15\ \mathrm{fm}^{-1}$. The quantity t is the equivalent π-nucleon c.m. momentum:

$$t^2 = (m_\Delta^2 - (m_\pi + m_N)^2)(m_\Delta^2 - (m_\pi - m_N)^2)/(4\,m_\Delta^2)\,. \tag{9}$$

The implementation of this description was stimulated by the work of ref. [9]) where it was claimed that such prescription largely decreases the pion yields in comparison with the case of a constant value $\Gamma = 115$ MeV.

(g) *The nucleon–nucleon total inelastic cross sections* has been parametrized in two different ways. Recently, the SACLAY nucleon–nucleon group has reconstructed the total pp and np total inelastic cross sections [26]). These data were fitted with series of generalized Laguerre polynomials insuring a correct threshold behaviour. Therefore, it is interesting to compare the parametrization used in the standard version of the code to this recent description and test how the differences affect the calculated pion cross sections.

Let us first recall the parametrization included in the standard version of the code. For pp and nn collisions, at a momentum P given in GeV/c, the total nucleon–nucleon inelastic collision is written as:

$$\sigma_{\mathrm{in}} = 23.5 + \frac{24.6}{1+\mathrm{e}^{(-10P+12)}} - \frac{1250}{P+50} + 4(P-1.3)^2 \qquad \text{for } P < 1.5\ \mathrm{GeV}/c$$

$$\sigma_{\mathrm{in}} = 41 + 60(P-0.9)\,\mathrm{e}^{-1.2P} - \frac{1250}{P+50} + 4(P-1.3)^2 \qquad \text{for } 1.5 < P < 2\ \mathrm{GeV}/c$$

$$\sigma_{\mathrm{in}} = 41 + 60(P-0.9)\,\mathrm{e}^{-1.2P} - \frac{77}{P+1.5} \qquad \text{for } P > 2\ \mathrm{GeV}/c$$

For np collisions, the parametrization was:

$$\sigma_{\mathrm{in}} = 33 + 196\sqrt{|P-0.95|^5} - \frac{31.1}{\sqrt{P}} \qquad \text{for } P < 1\ \mathrm{GeV}/c\,,$$

$$\sigma_{\mathrm{in}} = 24.2 + 8.9\,P - \frac{31.1}{\sqrt{P}} \qquad \text{for } 1 < P < 2\ \mathrm{GeV}/c\,,$$

$$\sigma_{\mathrm{in}} = 42 - \frac{77}{P+1.5} \qquad \text{for } P > 2\ \mathrm{GeV}/c$$

In the parametrization recently derived in ref. [26]) the cross sections are expressed in terms of an effective amplitude $F(x)$ as:

$$\sigma_k(x) = \begin{cases} 0 & \text{for } T < T_k \\ (F^k(x))^2 & \text{for } T > T_k\,, \end{cases}$$

where T_k is the threshold energy for the considered reaction k and x depends on the kinetic energy as:

$$x = \ln (T/T_k)$$

The amplitude $F(x)$ is expanded into a series of orthonormal functions $L_n^2(x)$ as:

$$L_n^2(x) = x\,\mathrm{e}^{-x/2}\mathscr{L}_n^2(x)$$

where $\mathscr{L}_n^2(x)$ are generalized Laguerre polynomials of second order.

The ratios of the total inelastic cross sections calculated with the two methods are given in table 2. It clearly establishes that the threshold behaviour is not correctly described in the standard version. Its influence on the calculated pion cross sections in p-nucleus collision will be investigated in sect. 3. Except when it is specified the calculations are done with the original parametrization of the cross sections.

TABLE 2

Ratios R of the total inelastic cross sections calculated with the standard parametrization included in the code to the values derived from the parametrization of ref. [26])

PP		NP	
T_0 (GeV)	R	T_0 (GeV)	R
0.30	4.2	<0.35	<0
0.34	1.7	0.35	1
0.36	1.3	0.40	1.6
0.40	1.0	0.45	1.45
0.44	0.9	0.50	1.3
0.50	0.9	0.55	1.2
0.60	1.1	0.60	1.1
0.80	1.1	0.80	0.9
1.0	1	1.0	0.9
1.4	0.95	1.4	0.9
1.8	1	1.8	0.9
2.2	1	2.2	0.9

3. The proton–nucleus interaction

The validity of different hypothesis included in the cascade code has first been investigated by the calculation of pion-production cross sections in proton–nucleus collisions. Two aspects have been studied: how do the cascade calculations describe the target mass and incident-energy dependence of the pion-production cross sections?

3.1. TARGET MASS DEPENDENCE

On fig. 1, the calculated results (circles) are compared to the data (triangle) of ref. [27]). The corresponding calculations have been done assuming that the nuclear density distribution is simulated by a step function distribution with a nuclear radius parameter $r_0 = 1.12$ fm; the binding energy prescription introduced in ref. [2]), following which the nucleons suddenly lose 40 MeV at their first collision, has been used together with the assumption that the Δ lifetime depends on the center-of-mass momentum t as indicated in eqs. (7), (8), (9) and using the original parametrization of the inelastic cross sections. The agreement between experimental and calculated values is good up to Ti, then it deteriorates as the target mass A increases: the ratios (casc./exp.) of the π^+ cross sections are, 0.9, 0.96, 1.1, 1.3, 1.2 and 1.55 for C, Al, Ti, Cu, Ag and Pb targets, respectively. These values are very similar to those reported in ref. [12]) for α projectile at 800 MeV per nucleon incident energy, where the ratios (casc./exp.) of the total pion mean multiplicities were found respectively equal to 0.94, 1.1 and 1.4 for the carbon, copper and lead targets. Therefore, one can conclude that even without compression the cascade overestimates the pion yield as much as 55% for Pb target. The experimental ratio π^+/π^- is fairly well described by the cascade code. In particular for Pb target the same discrepancy is observed for π^- as for π^+, despite the fact that the production mechanism is less direct as the contribution from charge exchange is known to be important [28]).

The importance of the nuclear surface description on the pion-production cross sections has already been emphasized [28]). In the present code, it has been implemented in a simple way assuming a trapezoidal shape as described by eq. (1) with the parameter values given in table 1. The net effect is, as expected, an increase of the calculated cross sections by 20–40% leading to ratios (casc./exp.) of the π^+ cross sections of, respectively: 1.06, 1.24, 1.4, 1.6, 1.65 and 1.95 for C, Al, Ti, Cu, Ag

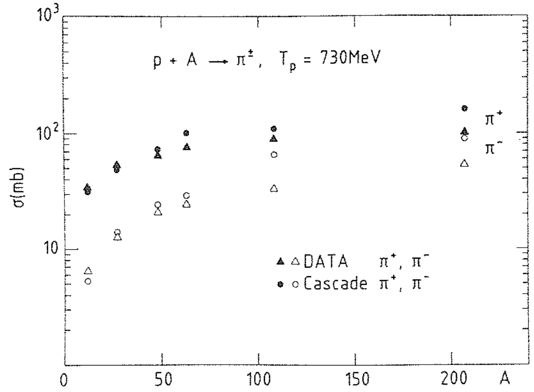

Fig. 1. Pion cross sections as predicted by the model, assuming sphere density profile, compared to the measurements of ref. [27]).

and Pb targets. As can be seen on fig. 2, the same behaviour is observed for π^- production cross-sections providing again a fairly good description of the π^+/π^- ratio for all targets.

The reason for the increase of the calculated pion cross sections, when the diffuse edge of the nucleus is taken into account, is clearly seen on fig. 3 where is displayed, for each impact parameter, the partial contribution to the cross section. The importance of the geometry on the absolute value of the cross section was already shown by the authors of ref. [28]). We have checked that the cross sections for pion production from a nucleus with a diffuse edge are roughly equal to those obtained from the equivalent uniform distribution of radius R given by eq. (2). In respect to this feature, the multiplicity is a better observable as, from its measurement, one gets rid of the geometrical aspect. Despite the fact that the absolute cross sections are increased by about 30% for the calculation with a diffuse edge as compared to a

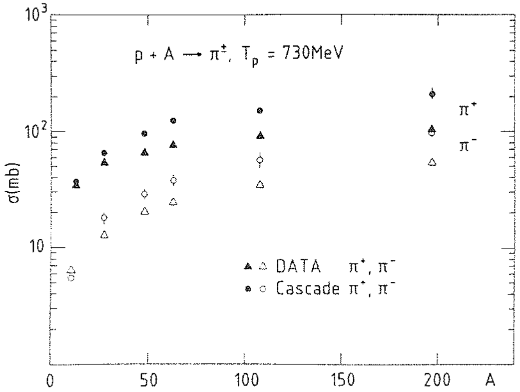

Fig. 2. Pion cross sections as predicted by the cascade model, assuming trapezoidal density distributions, compared to the measurements of ref. [27]).

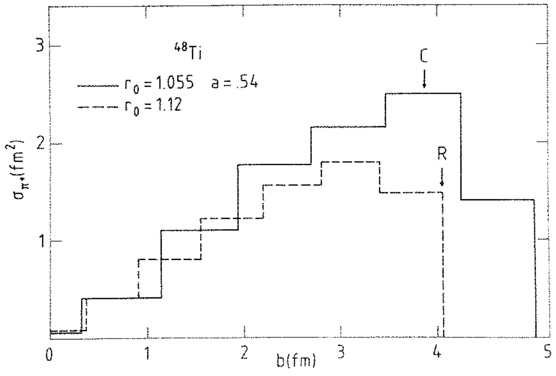

Fig. 3. Impact parameter dependence of the pion cross sections calculated in the cascade model assuming trapezoidal density distribution (solid line) or spherical density distribution with $r_0 = 1.12$ fm (dashed line).

calculation with a sharp edge (with $r_0 = 1.12$ fm), the target mass dependence of the ratios (casc./exp.) remains the same. Consequently, all the following calculations of proton–nucleus collisions have been done with diffuse nuclear surface.

Several test calculations have been done in order to study how the modifications of the delta properties, mass and width, or modifications of the inelastic cross-sections affect the π production cross sections and its mass dependence.

First, *a shift of the delta mass* by 50 MeV, towards higher mass values, has been tried. Its effect on the π-production cross sections is displayed on fig. 4 for both π^+ and π^-. It leads to a reduction of the cross sections by 30% for π^+ independently of target mass; for π^-, the reduction factor varies between 25% for the two extreme targets C and Pb to a maximum of 45% for the intermediate target mass Ag. Consequently, such a shift of the delta mass does not improve the target mass dependence description of the cascade calculations, as what is needed is no effect for the ^{12}C target and a reduction of a factor 2 for the Pb target.

Concerning *the delta lifetime* it is clear from table 3 that the use of eq. (7) instead of a constant value $\Gamma = 115$ MeV influences very weakly the calculated pion cross sections. Using a delta lifetime constant three times larger than the free value improve

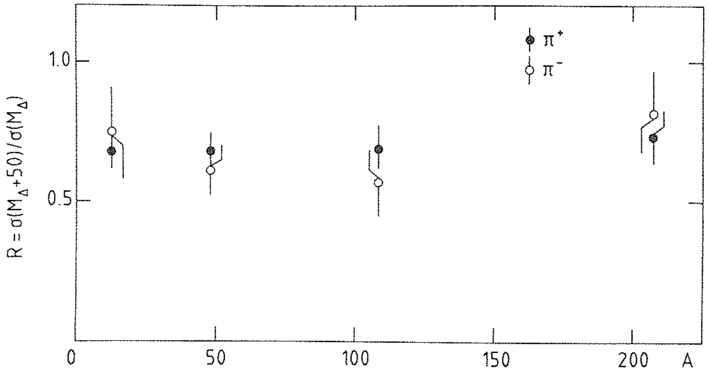

Fig. 4. Influence of a shift of the delta mass by 50 MeV towards higher mass values on the calculated pion cross sections.

TABLE 3

Ratios between pion cross sections calculated in the framework of the cascade model to experimental values for 3 choices of the delta lifetime (eqs. (6) and (7), with $\Gamma = \Gamma(m_\Delta)$, (eq. (8)), Γ_0 and $\frac{1}{3}\Gamma_0$ respectively (Γ_0 being the free delta width).

Target	π^+			π^-		
	$\Gamma(m_\Delta)$	Γ_0	$\frac{1}{3}\Gamma_0$	$\Gamma(m_\Delta)$	Γ_0	$\frac{1}{3}\Gamma_0$
^{12}C	1.08 ± 0.08	1.06 ± 0.08	0.99 ± 0.08	1.25 ± 0.08	0.85 ± 0.13	0.93 ± 0.13
^{48}Ti	1.47 ± 0.11	1.41 ± 0.11	1.23 ± 0.10	1.40 ± 0.11	1.36 ± 0.17	1.32 ± 0.16
^{108}Ag	1.78 ± 0.17	1.65 ± 0.16	1.47 ± 0.15	1.77 ± 0.17	1.62 ± 0.24	1.50 ± 0.19

the target mass dependence of the calculated cross sections; however, for ^{108}Ag the π^+ calculated cross section is still 40% too large.

The role of the inelastic cross sections has also been investigated with three different calculations. In the first one both NN ↔ NΔ and πN ↔ Δ are decreased by a factor 2. The results are displayed on fig. 5. It directly leads to a decrease by a factor 2 of the π^+ cross section from the carbon target. For heavier target masses the reduction factor weakens to reach only 30% for Ag. By examining separately the effect of each process, one can see that the main variation rises from the NN ↔ NΔ cross sections, the influence of the reduction of the πN ↔ Δ process is less than 10%. We believe, for the last two points, that the different behaviour in π^+ and π^- productions can be understood in terms of the importance of multiple scattering. For instance, single scattering favours π^+ production in proton-induced reactions. If multiple scattering does not contribute, a reduction of the inelastic cross sections generates a similar reduction in the production yield. This indeed happens for π^+ production from ^{12}C. The π^- production, which requires multiple scattering is less reduced, since the relative importance of multiple scattering is less affected by the reduction of the inelastic cross sections. For a heavy target, where multiple scattering is presumably important for both π^+ and π^-, the reduction is expected to be about the same for both charge states.

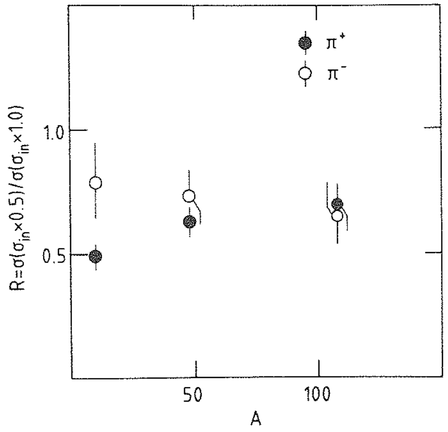

Fig. 5. Influence of a reduction of the inelastic cross sections by a factor 2 on the calculated pion cross sections.

The influence of the binding potential is displayed on fig. 6. Most calculations have been performed with a binding potential 40 MeV deep. From fig. 6 one can see that such a prescription reduces by 25% the calculated π^+ cross sections independently of target mass. The effect on the π^- is larger and exhibits a stronger dependence on target mass, reducing the cross sections by 55% for C target and only 45% for Pb target.

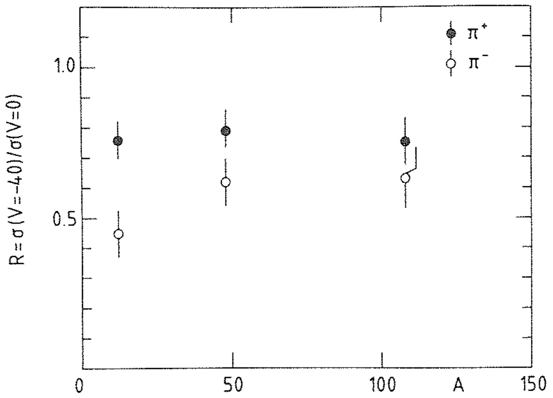

Fig. 6. Influence of a binding potential 40 MeV deep on the calculated pion cross sections.

The sensitivity to *the parametrization of the total inelastic cross sections* has also been investigated by modifying the code so that below 1 GeV kinetic energy the parametrization proposed in ref. 26) is taken into account. The ratios of the so-calculated cross sections to the experimental ones are listed as the R3 values of table 4. The calculations were performed with a binding potential of 40 MeV and taking into account the diffuse edge of the target nuclei. Consequently, they are directly comparable to the R2 values. It clearly shows that the improvement of the description of the elementary total inelastic cross sections does not affect significantly the calculated pion-production cross sections in p-nucleus collisions at 730 MeV.

A satisfactory description of the mass dependence has been obtained by simultaneously increasing the delta lifetime by a factor 3 and increasing the absorption cross sections $N\Delta \rightarrow NN$ by a factor 3 without changing the cross sections of the inverse reaction. The corresponding results are listed in table 5.

TABLE 4

Ratios between positve- and negative-pion cross sections calculated in the framework of the cascade model to experimental values assuming spherical density distributions for R1, trapezoidal density distribution for R2, both trapezoidal density distributions and the parametrization of ref. 26) for the total inelastic cross sections for R3

Target	π^+			π^-		
	R1	R2	R3	R1	R2	R3
^{12}C	0.90±0.06	1.06±0.08	0.92±0.07	0.80±0.06	0.83±0.13	1.2±0.16
^{27}Al	0.96±0.07	1.24±0.10	1.2±0.10	1.06±0.10	1.4±0.20	1.28±0.16
^{48}Ti	1.1±0.08	1.4±0.10	1.4±0.10	1.14±0.13	1.4±0.20	1.2±0.15
^{63}Cu	1.3±0.1	1.6±0.12	1.45±0.10	1.16±0.14	1.5±0.20	1.4±0.16
^{108}Ag	1.2±0.1	1.65±0.16	1.65±0.16	1.8±0.25	1.6±0.25	1.8±0.16
^{207}Pb	1.55±0.15	1.95±0.20	2.25±0.26	1.72±0.25	1.8±0.25	1.9±0.30

TABLE 5

Ratios between pion cross sections calculated in the framework of the cascade model to experimental values. The calculations have been carried out assuming the delta lifetime is increased by a factor 3 compared to its free value and the absorption cross section NΔ → NN is multiplied by a factor 3

Target	π^+	π^-
^{12}C	0.85 ± 0.09	0.70 ± 0.15
^{48}Ti	0.76 ± 0.09	0.59 ± 0.13
^{108}Ag	0.90 ± 0.11	0.72 ± 0.15
^{207}Pb	1.04 ± 0.13	1.21 ± 0.21

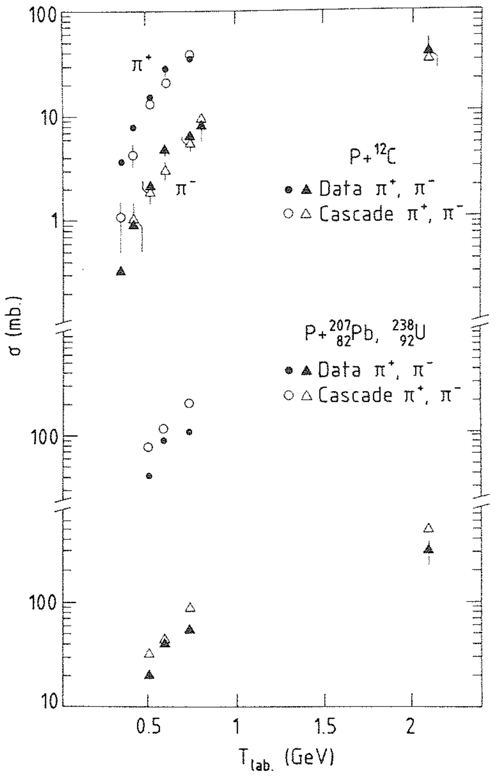

Fig. 7. Comparison between pion cross sections calculated with cascade model to the existing data on a wide range of incident energies.

3.2. ENERGY DEPENDENCE

The dependence of the pion cross sections on the incident energy has been studied between 330 MeV and 2.1 GeV for the two targets C and Pb on which such observable is available [27,29-31]). The comparison between the cascade results and the experimental data is displayed on fig. 7. From these results it is clear that the four sets of data on π^+, π^- on C and Pb targets the data measured at 585 MeV differ significantly from the behaviour of the data taken at the two neighbouring energies 500 and 730 MeV, the two latter being consistent with each other. It looks like that the cross sections measured at 585 MeV are too high by 30% and 50% for C and Pb target, respectively. For the low incident energies, below 500 MeV, the cascade calculations have been done only for C target. The calculated cross sections strongly underestimate the experimental values in contrast to the work of ref. [29]). Indeed, at 500, 400 and 330 MeV, the ratios between the calculated and experimental cross sections are, respectively, 0.9, 0.56, 0.3 for π^+ and 0.9, 1.07, 0. for π^-. With the parametrization of ref. [26]) for the total inelastic cross sections these numbers become 0.92, 0.47, 0.36 for π^+ and 0.9, 0.45, 0.18 for π^-. Above 500 MeV, both π^+ and π^- cross sections measured on C are well reproduced by the calculations. The discrepancy observed on Pb, the cascade cross sections are about 60% too high, has about the same magnitude at all incident energies.

4. The Ar-KCl central collisions

This heavy-ion system has been largely studied in connection with the equation of state [1-12,20,21]). Therefore, it is interesting to investigate the sensitivity of the negative pion multiplicity $\langle n_\pi \rangle$ to the different ingredients used in the Liege cascade code. Most of the cascade calculations have been done for $b=0$ impact parameter. The resulting pion multiplicities have been renormalized in order to compare to the experimental values which correspond to impact parameters ranging from $b=0$ to $b=2.4$ fm.

From fig. 8 and table 6, the great sensitivity of the calculated multiplicities to the *potential* used to bind the nucleons is confirmed [2]). The influenece of the binding potential increases with decreasing incident energy, ranging from 20% at 1800 MeV per nucleon to a factor 3 at 400 MeV per nucleon.

In contrast to the work of Kitazoe *et al.* [9]) the pion multiplicities calculated with *a delta lifetime depending on the delta mass* are very similar to those calculated with a width of the delta fixed at 115 MeV. The origin of such a difference is not understood. To investigate further the role of the delta lifetime, a drastic increase by a factor 3 has been done. It reduces the calculated pion multiplicity by 20% at 400 MeV per nucleon and 10% at 800 and 1800 MeV per nucleon.

From fig. 8 it is clear that the experimental pion multiplicities can be reproduced if the inelastic cross sections are arbitrarily divided by a factor 2 or if the delta mass is increased by 30 MeV. Modifications of these quantities, delta mass and elementary

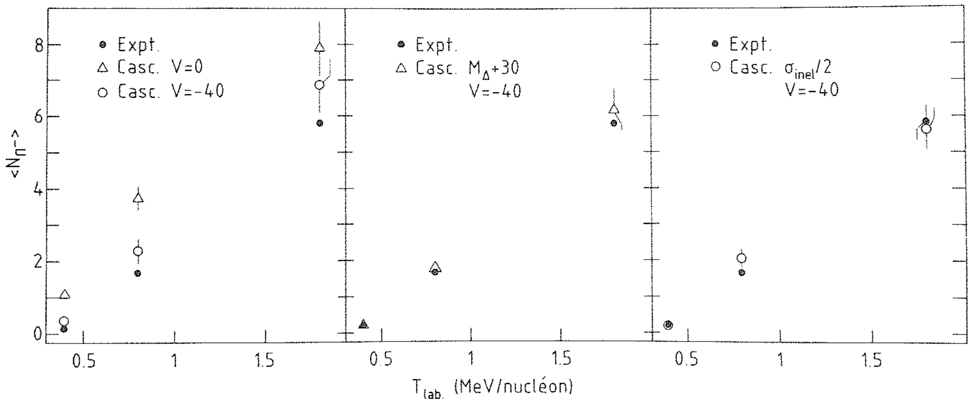

Fig. 8. Negative pion yield as predicted by the model compared to the streamer chamber measurements of ref. [1]) for central trigger and Ar + KCl collisions.

TABLE 6

Ratios between the π^- mean multiplicities calculated with the cascade model and the experimental values for Ar + KCl central collisions for the different assumptions given on the left of the table

T_{lab}(MeV/n):		400	800	1800
$V=0$	$\Gamma(M_\Delta)$	5.5	2.3	1.4
$V=-40$	$\Gamma(M_\Delta)$	1.6	1.4	1.2
$V=-40$	$\Gamma=115$ MeV	1.7	1.4	1.2
$V=-40$	$\Gamma=\frac{1}{3}\Gamma_0$	1.4	1.3	1.1
$V=-40$	$\sigma_{\text{inel}}/2$	1.0	1.2	1.0
$V=-40$	$M_\Delta=1262$ MeV	1.0	1.1	1.1

cross sections are expected in dense nuclear matter from relativistic Brueckner calculations [32]). For instance, the authors of ref. [32]) predict a 20 MeV shift of the delta mass when the density varies from normal nuclear density ρ_0 to $2\rho_0$. More theoretical work is required to specify how the delta properties and the elementary cross sections are modified in dense and hot matter, which are the conditions encountered in heavy-ion collisions at intermediate energy.

The prescription which provides the best target mass dependence of the pion cross sections in proton-nucleus collisions consists in increasing both the delta lifetime by a factor 3 and the absorption cross sections $N\Delta \to NN$ by a factor 3. Applied to the Ar + KCl system this prescription leads to calculated pion multiplicities 30% lower than the experimental values. However, from the work of ref. [32]), it is clear that these modifications of the free values has to be density dependent. So, it is normal that the modifications has to be different between proton and nucleus-nucleus collisions. For a comparison to the Ar + KCl case, we also performed a calculation for the La + La system at 730 MeV per nucleon [33]). It shows that for central collisions the ratio between the calculated and experimental pion multiplicity is 1.4, which is the same as for Ar+KCl.

5. Pion–nucleus interaction

Keeping in the same spirit, we also investigate the pion-nucleus interaction. We stress that the kinematical conditions are different from the previous cases. The pions are never very energetic, the density is close or smaller than normal and the nucleon distribution is barely disturbed. The most extensive measurements are those of Ashery *et al.* [34]). We will first concentrate on the A-dependence of the cross sections. As a matter of fact, the total non-elastic cross sections are about one-half of the total cross sections. This is a striking example of a quantum strongly absorbing ("black") target. Since a cascade model can describe the incoherent processes only, we will compare calculations with non-elastic cross sections only.

The pion-nucleus system at low energy requires a careful treatment of the average field. In the two cases investigated above, a struck nucleon is generally put far apart in the momentum space from its neighbour in position space. From the experience gained in the study of proton-nucleus scattering, one knows that the average field felt by such a nucleon is practically vanishing, for a relative energy larger than ≈100 MeV [refs. [35,45])]. This supports the procedure adopted in sects. 3, 4. On the contrary, in low-energy pion-induced reactions, a struck nucleon has always a momentum close to the Fermi momentum. Therefore, it still experiences roughly the same field as it did in the target ground state before it was struck. We thus here adopt the following procedure: a struck nucleon remains off-shell (in the sense of eq. (4)). The binding energy is removed when it leaves the nucleus, if it does.

The result of our standard INC calculation is given in fig. 9a, along with the experimental data. The prominent feature is that the INC underestimates the true pion absorption (by a factor 2.5-3) and overestimates the inelastic cross section. Many of the reasonable modifications mentioned in sects. 3 and 4 improve the situation considerably, but none of them gives a perfect reproduction of *all* the cross sections. The best modification (at the simplest level) is provided by multiplication of the delta lifetime by a factor 3 (fig. 9b) although the multiplication of the $NN \leftrightarrow N\Delta$ cross section by a factor ≈3 yields roughly to the same results. Other results are contained in table 7. Let us print out the content of table 7e. If one increases the inelastic cross section $\sigma(N\Delta \rightarrow NN)$ by a factor 3 and increases the lifetime of the delta by a factor 3 at the same time, the situation is further improved. The absorption cross section is very well reproduced. The single charge exchange then comes out rather good, as well as the inelastic cross sections. It is noticeable that the A-dependence of the cross sections is correctly reproduced by all our calculations, even when the absolute magnitude is missed. This indicates that the A-dependence is largely governed by geometrical features. The exponent in a power law turns out to be about 0.58, 0.71 and 0.31 for single charge exchange, absorption

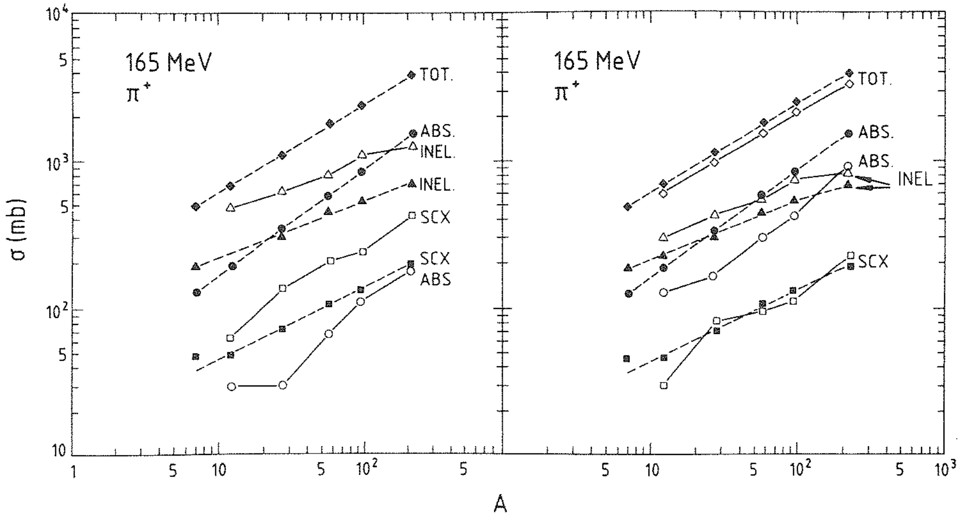

Fig. 9. Comparison between experimental π^+ absorption (ABS), single charge exchange (SCX), inelastic (INEL) and total (TOT) cross sections (full symbols) and the predictions on INC calculations (open symbols) for various nuclei. Part (a) refers to the standard version of the cascade, Part (b) refers to the case where the delta lifetime is multiplied by a factor 3. See text for the detail.

TABLE 7a

Theoretical predictions for π^+ absorption (σ_{abs}), single charge exchange (σ_{ex}), double charge exchange (σ_{dex}) and inelastic (σ_{in}) cross section on various targets using the standard version of the cascade

	σ_{abs}	σ_{ex}	σ_{dex}	σ_{in}
^{12}C	72	79	10	421
^{27}Al	137	134	19	454
^{56}Fe	234	192	28.5	637
^{93}Nb	337	232	40	866
^{207}Bi	541	448	91	922

TABLE 7b

Same as table 7a using an inelastic cross section $\sigma(\text{N}\Delta \leftrightarrow \text{NN})$ multiplied by a factor 3

	σ_{abs}	σ_{ex}	σ_{in}
^{12}C	118	59	389
^{27}Al	251	87	400
^{56}Fe	442	120	521
^{93}Nb	610	153	657
^{207}Bi	913	332	696

TABLE 7c

Same as table 7a, when the mass of the delta resonance is lowered by 30 MeV

	σ_{abs}	σ_{ex}	σ_{in}
^{12}C	64	82	406
^{27}Al	116	122	470
^{56}Fe	264	175	609
^{93}Nb	320	231	824
^{207}Bi	576	425	908

TABLE 7d

Same as table 7a, after multiplication of the delta lifetime by a factor 10

	σ_{abs}	σ_{ex}	σ_{in}
^{12}C	147	52	376
^{27}Al	273	65	398
^{56}Fe	478	98	510
^{93}Nb	636	133	690
^{207}Bi	1079	200	704

TABLE 7e

Same as table 7a, when the $\sigma(N\Delta \leftrightarrow NN)$ is multiplied by a factor 3 and the delta lifetime is enhanced by a factor 3 (upper figures), or when the $\sigma(\pi N \to \Delta)$ is multiplied by a factor 0.75 (lower figures)

	σ_{abs}	σ_{ex}	σ_{dex}	σ_{in}
^{12}C	183	49		331
	51	68	6	366
^{27}Al	341	55		336
	113	111	11	447
^{56}Fe	622	76		384
	231	170	19	615
^{93}Nb	864	91		502
	327	251	46	767
^{207}Bi	1212	192		582
	548	370	24	995

and inelastic processes, respectively. This is in good agreement with the observed values. Incidentally, we also computed the two step, incoherent double charge exchange (π^+, π^-) process for this particular case. The numbers are given in table 7.

The modification giving the best results (table 7e) appears somewhat unreasonable. See, however, the next section, to where we have postponed our discussion.

Let us finally notice that the energy dependence of the cross sections is grossly reproduced. For the ^{56}Fe target, the calculated absorption cross section peaks at $E_\pi \approx 225$ MeV instead of 180 MeV, experimentally, but in both cases, the maximum is rather broad. At very low energy ($E_\pi < 100$ MeV) the calculated cross section drops too quickly in comparison to experiment.

6. Discussion and conclusion

6.1. SUMMARY

We have looked at the sensitivity of the INC output to some modifications of the properties of pions and deltas which are handled by the model. We do not pretend by any means to have been exhaustive. For instance, we did not pay attention to the modification of the pion mass inside the medium. Our philosophy was the following: in view of the absence of precise evaluation of the medium effects on pion and delta formation as a function of temperature and density (see, however, ref. [32]), we have just studied the modification of the calculated pion yield under reasonable changes of the input data. We can summarize our results in the following way:

(a) the pion yield in central heavy-ion collisions is very sensitive to any modification of the input data. This sensitivity is the largest for the lowest incident energy. Good agreement can be achieved by either multiplying the inelastic cross sections by a factor $\frac{1}{2}$ or by raising the delta mass by about 30 MeV.

(b) the pion yield in proton-induced reaction is described within a factor 2, at 730 MeV incident energy, with the standard INC calculation. The largest discrepancies are observed for the heavy targets. None of the reasonable modifications applied individually can succeed to achieve a good agreement; the target mass dependence of the pion yield is not described satisfactorily.

(c) for the pion–nucleus interactions, the standard cascade model underpredicts the true absorption cross sections by a factor 2.5–3 (in spite of the fact that the latter is not measured independently from the single-charge-exchange cross sections). Here, owing to a rather large sensitivity, the discrepancy can be substantially reduced by reasonable modification of the input data, compatible with the expected size of the medium effects. However, a *total* removal of this discrepancy for all the non-elastic cross sections (table 7e) seems to require unreasonable modification of the input data. By this, we mean (somewhat arbitrarily) a cross section or a lifetime changed by more than a factor 3, or a Δ-mass changed by more than 50 MeV. This choice is based on the fact that in the literature, modifications of about these sizes are often quoted.

6.2. DISCUSSION

We have considered two types of modification. In the first type (e.g. the modifications contained in sect. 3.1, except for the last one), we conserve all the possible mechanisms as in the standard version, and we merely change the cross sections, or sometimes some kinematical quantity (threshold energy). These modifications change both direct and inverse cross sections consistently. The second kind of modification has the property of changing a cross section without touching the inverse. They may appear somewhat arbitrary since they seem to violate microreversibility. However, they should not be rejected at once and should rather be considered as mocking up the presence of new mechanisms occurring in the medium only (see below).

We want to stress that most of the modifications discussed above (except the one of tables 5 and 7e) have sizes which are currently given in the literature [32,35-37]). It should be noticed, however, that in general, only some of these modifications are studied at the same time and that there is no real agreement between all the estimates of these modifications. This is presumably due to the fact that these effects are not studied consistently and globally (except perhaps for refs. [32,38])). This should be done because all these modifications are reacting on each other.

Despite this situation, one may ask whether there is a modification which will remove the discrepancies for all the three cases we studied. We actually found one. If one multiplies the cross section $\sigma(N\Delta \rightarrow NN)$ (in this way only) by a factor 3 and the delta lifetime by a factor 3, the results on the pion-induced reactions agree fairly well with the data. Such a modification also gives good results in proton-induced reactions. In heavy-ion reactions, it also reduced the pion yield compared to the standard version. Actually, the reduction is even too large (by 30%), in agreement to the strong sensitivity of this case.

This modification seems to lie beyond what one may call reasonable. Furthermore, it seems to destroy the consistency of the INC model, since it requires some arbitrariness, changing a cross section without changing the reverse cross section. It should be noticed, however, that such a strong modification is required for having a very good agreement for the pion-induced and the proton-induced reactions. The heavy-ion case can be accommodated with less stringent modifications.

The mechanism of modification within our model can be understood in looking at the chemical reaction rates or fluxes. The pion production results in a transformation of nucleons into deltas and of deltas into pions and vice versa as it is sketched on fig. 10. In heavy-ion reactions, there is a large incoming flux (i.e. reactions going to the right) and a large outgoing flux (i.e. reactions going to the left); the resulting

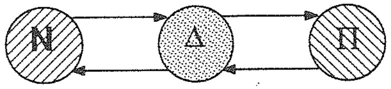

Fig. 10. Scheme for the pion production and absorption as described in the INC model.

pion yield is a small balance between the two fluxes. This has been known for a long time [21]). To be clearer, for Ar+KCl at 800 MeV/n, $b=0$ fm, $t=30$ fm/c the average number of delta produced and absorbed are 18 and 10, respectively. In proton induced reactions, the situation is roughly similar, but the fluxes are less important compared to the final yield: for Ti target, at 730 MeV, $b=2.7$ fm, $t=20$ fm/c the average number of delta produced and absorbed are 0.47 and 0.12, respectively. For pion-induced reactions, there is no incoming flux of course, and the outgoing flux is important, since typically the probability of absorption, at 165 MeV, is roughly 0.5 on the average. In all the three cases, the cascade overpredicts the pion yield.

In order to reduce the pion yield, one has to change the balance between the incoming and the outgoing fluxes. That is why the heavy-ion case is so flexible. A simultaneous change on both fluxes done by a decrease of the inelastic cross sections or an increase of the outgoing flux can provide the right pion yield. In the pion-induced reactions there is no choice - only the outgoing flux can be modified.

One may address the following question: should the two fluxes be changed (like modifying NN ↔ NΔ) or only the outgoing fluxes for the heavy-ion and proton-nucleus cases? We think that the second alternative should not be discarded for the following reasons. For α reactions studied at Diogene, there is an increasing discrepancy between INC and experiment with increasing target mass, a feature which also cannot be corrected by reasonable modification of the input data [39]). In proton induced reactions, the difficulty of reproducing correctly the A-dependence is also pointing out in favour of a stronger absorption. Of course, the unifying modification that we discuss in this section is not very satisfactory for three reasons (1) it implies rather strong modifications of the input data, (2) it introduces them in an inconsistent way, modifying a cross section without modifying the reverse one (note that in the pion-nucleus case, this does not matter since the NN → NΔ reaction is ineffective) (3) for the heavy-ion case, the nature of the modification is perhaps not physically acceptable. Indeed, although nothing is known with certainty it seems that the NΔ → NN is reduced and not enhanced in dense matter [32]). So we have to interpret our results and observations described above as pointing towards an enhancement of the absorption, but which should be implemented in a way which is different from the one used in our model. One immediately thinks at new channels of absorption, like the absorption on several nucleons. But, one has to distinguish between genuine two, three or four nucleon absorption and simple iteration of the processes $\pi+\mathrm{N}\rightarrow\Delta$, $\mathrm{N}\Delta\rightarrow\mathrm{NN}$, which are included in the model. This feature is often overlooked [10,40-42]). It seems, however, in pion-nucleus interaction, the genuine pion absorption on three nucleons can be as important as the contribution of the $\pi + N \to \Delta$, $N\Delta \to NN$ process [46]). Another possibility of enhanced absorption can be provided by the fact that all the particles are off-shell during the collision process which, in principle, opens their phase spaces. This possibility has not really been looked for, at least in the heavy-ion case. All these considerations strongly call for a consistent approach of the mesonic degrees of freedom in the heavy-ion case.

We make a final remark. It is not completely sure that the same modification of the pion production process should be made in the three cases under investigation since one is dealing with different regimes of pion energy and medium density. However, this argument does not destroy the presence of indications in favour of an enhancement of the pion absorption.

6.3. CONCLUSION

We have looked at the pion production mechanism in three different systems, using the INC model. We especially studied the sensitivity of the pion yield to modifications of the input data describing pion and delta properties. The modifications are coming from physical expected medium effects.

For the heavy-ion case, we found a strong sensitivity. A similar conclusion has been reached by other authors which have recently looked to modified delta creation cross section [43,44]). The original discrepancy between cascade and experiment can be easily removed by possible medium effects. This should be contrasted with the explanation in terms of compression effects. The sensitivity of the pion yield to the equation of state is far from being established [3,6,8,43]) and is certainly not as strong as the one we observed here. The size of all possible medium effects is not known very well. A consistent approach of all of them is certainly needed. In proton-induced reactions, the pion yield is less sensitive and it is hard to find out a good agreement with the data. The situation is even worse for pion-induced reactions. We discussed at length the meaning of our observations. In our opinion, there are strong indications that the pion absorption is underestimated. A coherent description of new absorption channels is required. This, however, goes beyond the scope of the present INC model.

We thank D. Lhôte for having stimulated this work and for valuable advice. We also are grateful to P. Siemens and R. Malfliet for interesting discussions and to M. Soyeur for helping us to correct the description of the pion-induced reactions from the first version of this manuscript.

References

1) R. Stock *et al.*, Phys. Rev. Lett. 49 (1982) 1236;
J.W. Harris *et al.*, Phys. Lett. **B153** (1985) 377;
J.W. Harris, "2nd Conf. on the intersections between particle and nuclear physics, Lake Louise (1986)
2) M. Cahay, J. Cugnon and J. Vandermeulen, Nucl. Phys. **A411** (1983) 524

3) G.F. Bertsch, H. Kruse and S. Das Gupta, Phys. Rev. **C29** (1984) 673
4) R. Malfliet, Phys. Rev. Lett. **54** (1985) 289
5) R. Malfliet and B. Schürmann, Phys. Rev. **C28** (1983) 113
6) H. Kruse, B.V. Jacak and H. Stocker, Phys. Rev. Lett. 54 (1985) 289
7) M. Sano *et al.*, Phys. Lett. **B156** (1985) 27
8) R.B. Clare, D. Strottman and J. Kapusta, Phys. Rev. **C33** (1986) 1288
9) Y. Kitazoe *et al.*, Phys. Lett. **B166** (1986) 35
10) G.E. Brown and P. Siemens, Stony Brook preprint (1987)
11) R. Stock, Phys. Reports **135** (1986) 259
12) D. L'Hôte *et al.*, Phys. Lett. **B198** (1987) 139
13) E. Oset and W. Weise, Nucl. Phys. **A319** (1979) 477
14) M. Hirata, J.H. Koch and F. Lenz, Ann. of Phys. **120** (1979) 205
15) Y. Orikawa, M. Thies and F. Lenz, Nucl. Phys. **A345** (1980) 386
16) M. Hirata *et al.*, Phys. Lett. **B128** (1983) 15
17) M. Thies, Nucl. Phys. **A382** (1982) 434
18) J. Delorme, M. Ericson, A. Figureau and C. Thevenet, Ann. of Phys. **A382** (1976) 273
19) G.E. Brown and W. Weise, Phys. Reports **22** (1975) 280
20) J. Cugnon, Phys. Rev. **C23** (1981) 2094
21) J. Cugnon, D. Kinet and J. Vandermeulen, Nucl. Phys. **A379** (1982) 553
22) R. Hofstadter, Rev. of Mod. Phys. **28** (1956) 214
23) J. Cugnon and D. L'Hôte, Nucl. Phys. **A452** (1986) 738
24) J. Aichelin, Phys. Rev. **C33** (1986) 537
25) A. Rittenberg *et al.*, Part. Data Group, Rev. of Mod. Phys. **43** (1971) 5114
26) J. Bystricky *et al.*, preprint DPhPE 87-03
27) D. Cochran *et al.*, Phys. Rev. **D6** (1972) 3085
28) M.M. Sternheim and R.R. Silbar, Phys. Rev. **D6** (1972) 3117
29) N.J. DiGiacomo *et al.*, Phys. Rev. **C31** (1985) 292
30) S. Nagamiya *et al.*, Phys. Rev. **C24** (1981) 971
31) J. Crawford *et al.*, Phys. Rev. **C22** (1980) 1184
32) B. Ter Haar and R. Malfliet, Phys. Rev. **C36** (1987) 1611
33) J.W. Harris *et al.*, Phys. Rev. Lett. **58** (1987) 463
34) D. Ashery *et al.*, Phys. Rev. **C36** (1987) 460
35) S. Homma, Nucl. Phys. **A446** (1987) 2412
36) K. Dreissigacker *et al.*, Nucl. Phys. **A375** (1952) 334
37) R. Cenni and G. Dillon, Nucl. Phys. **A422** (1984) 527
38) P. Siemens and M. Soyeur, private communication
39) D. L'Hôte, thesis, Université de Paris Sud (January 1987), unpublished
40) D. Ashery and J.P. Shiffer, Ann. Rev. of Nucl. Sci. **36** (1986) 207
41) J.R.D. McKeown *et al.*, Phys. Rev. Lett. **44** (1980) 1033
42) K.E. Hernandez and E. Oset, Nucl. Phys. **A455** (1986) 584, and to be published
43) L.C. Gale, Phys. Rev. **C36** (1987) 2152
44) M.B. Waldhauser *et al.*, UFTP-194 (1987)
45) J.P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Reports **C25** (1976) 83
46) L. Salcedo, E. Oset, M.J. Vicente-Vacos and C. Garcia-Recio, to be published