Z. Phys. A – Atomic Nuclei 327, 187–192 (1987)

Zeitschrift für Physik A
Atomic Nuclei

Negative Binomial Multiplicity Distributions and Intranuclear Cascade

J. Cugnon
Institut de Physique B 5, Université de Liège, Belgium

Received November 21, 1986; revised version January 29, 1987

The multiplicity distribution of the ejected particles, as predicted by the intranuclear cascade model, are analyzed in terms of negative binomial shapes, which have recently been shown to be relevant for high-energy collisions. We discuss some physical features which are related to this distribution shape.

PACS: 24.90.d

1. Introduction

Recently, it has been discovered by the UA5 Collaboration [1–2] that the charged particle multiplicity distributions in the high energy $\bar{p}p$ collisions closely follow a negative binomial (NB) form. Further analyses at different energies [2, 3] and on different reactions [4] seem to establish the general validity of the NB, both for total multiplicities and in finite pseudorapidity intervals [5]. This universality replaces the previous less firmly established KNO scaling [6].

The NB distribution is characterized by two parameters. In Ref. 5 and in a recent publication [7], Van Hove and Giovannini examined the plausible origin of the NB and the physical implications of the variation of the two parameters with the energy and with the system under consideration. As for the first point, they give convincing arguments showing that negative binomials arise from an underlying cascade dynamics, provided the latter shows some particular properties that are explained below.

Many nuclear processes, including heavy ion collisions and hadron-nucleus interaction, at moderate energy (well below the UA5 experiment) are known to be described reasonably well by the intranuclear cascade (INC) model. Therefore, it is interesting to know *(i)* whether NB also applies to the experimental data of these kinds of processes and *(ii)* whether the INC model shares the properties described in [5]. This paper is devoted to the second point. In particular, we investigate the properties of the multiplicities within the framework of the INC model for several processes. We also try to discuss which circumstances are favourable to the manifestation of the NB forms and which induces a breakdown of its validity.

2. Negative Binomial and Cascade Dynamics

The NB is the following probability law (for multiplicity $n \geq 0$)

$$P(n)=\frac{k(k+1)\ldots(k+n-1)}{n!}\left(\frac{\bar{n}}{\bar{n}+k}\right)^{n}\left(\frac{k}{\bar{n}+k}\right)^{k}. \qquad (2.1)$$

There are two parameters: $\bar{n}$, which is the average multiplicity and $k(>0)$, which is related to the variance σ by

$$\sigma^2=\bar{n}+\frac{\bar{n}^2}{k}. \qquad (2.2)$$

As this equation suggests, the NB law reduces to a Poisson (P) law when $k \to \infty$.

The basic result of [5] consists in showing that NB arises from some special cascade events. To characterize these events, the authors of [5] introduce the concept of clans*. As Fig. 1a suggests, in a cascade event, any produced particle originates from a prima-

---

* This concept was already introduced in [13] within the INC description of heavy ion collisions. Therein, the clans are named interacting clusters. See later for detail

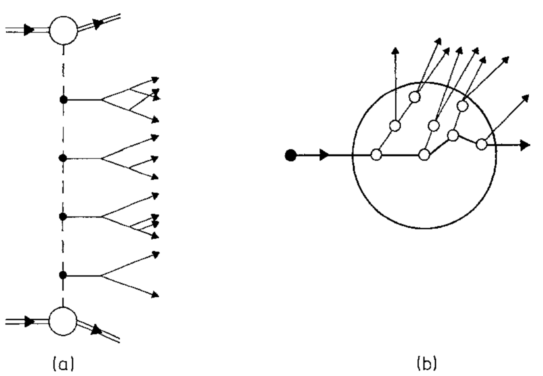

Fig. 1. a Cascade event in a high energy proton-proton or antiproton-proton interaction, showing the clan structure of the particle production. **b** Similar diagram for proton-nucleus interaction. See text for detail

ry particle (or pseudo-particle or parton, according to the desired terminology), named ancestor. All the particles with a common ancestor form a clan. The clans have no mutual interaction. Of course, the dynamics may be such that all the produced particles are correlated. This, however, fits into the above description in the sense that this particular case corresponds to a single clan.

According to [5], the NB distribution is ensured when the clans are produced independently, i.e. when the number N of clans follows a P law

$$f(N)=e^{-\bar{N}}\frac{(\bar{N})^N}{N!} \tag{2.3}$$

and when the number n_c of produced particles within any of the clans follows a logarithmic (LO) distribution

$$P_c(n_c)=\frac{(1-b)\,\bar{n}_c}{b}\,\frac{b^{n_c}}{n_c} \tag{2.4}$$

for $n_c \geq 1$ only ($P_c(0)=0$). Actually, it is known in statistics that (2.1) is mathematically equivalent to the combination of (2.3) and (2.4). Of course, $\bar{N}$, b and $\bar{n}_c$ are related to $\bar{n}$ and k. The average clan multiplicity $\bar{n}_c$ is given by

$$\bar{n}_c=\frac{\bar{n}}{k\ln\left(1+\frac{\bar{n}}{k}\right)}. \tag{2.5}$$

The average number $\bar{N}$ of clans is obviously equal to $\bar{n}/\bar{n}_c$ and b is the quantity

$$b=\frac{\bar{n}}{\bar{n}+k}. \tag{2.6}$$

The physical meaning of k^{-1} is explained in [7]. It is proportional to the ratio of the probability of finding a multiplicity n with N clans to the probability of finding a multiplicity n with $N+1$ clans. It therefore represents the measure of aggregation in clans. It also determines the "contamination process" in particle production. This means that the probability of producing $(n+1)$ particles compared to the one of producing n particles is enhanced with respect to an independent production as embodied by the P law.

In the following, we are going to study the nuclear analogs of the dynamical scheme of Fig. 1a and examine whether NB distributions are relevant to these cases.

3. Nuclear Cascades

3.1. Proton-Nucleus Interaction

The most obvious analog of the cascade scheme considered in [5] is provided by the interaction between a nucleus and an impinging proton of a few GeV as depicted schematically in Fig. 1b. Indeed, it is generally believed that in this energy range the incident proton travels more or less in straight line, hits a

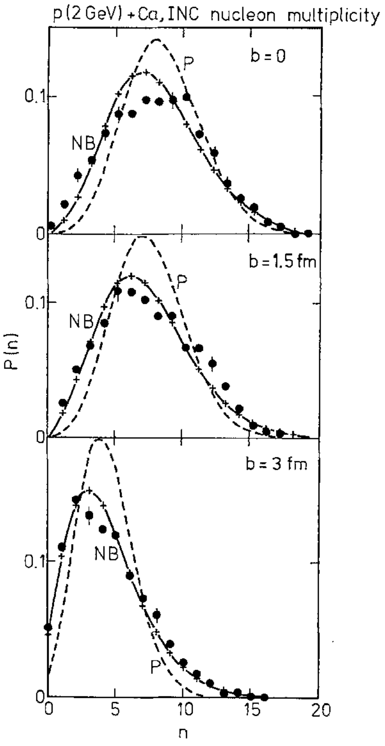

Fig. 2. Nucleon multiplicity distribution as predicted by the INC model [8] (dots) for reactions for 2 GeV protons with Ca nuclei at three values of the impact parameter b. In each case the best fits by a NB (crosses and full line to guide the eye) and a P distribution (dashed curve), respectively, are shown. Typical uncertainty of the INC model (due to statistics) are shown by the error bars

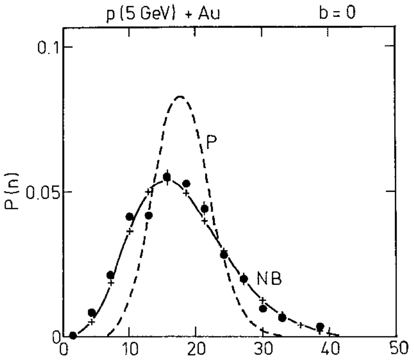

Fig. 3. Same as Fig. 2 for central collisions of 5 GeV protons with Au nuclei

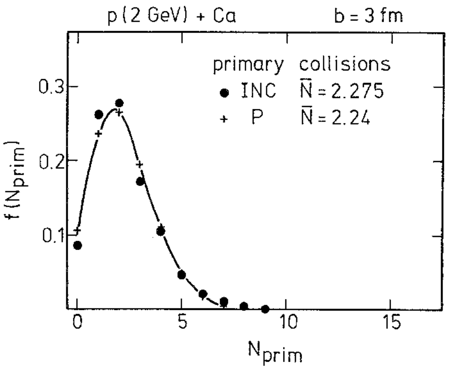

Fig. 4. The dots represent the distribution of the number of primary collisions N_{prim} in interactions of 2 GeV protons with Ca nuclei at impact parameter = 3 fm, as evaluated by the INC model [8]. The crosses give the best fit by a P distribution. The corresponding mean value is indicated on the figure. The full line is just to guide the eye

Table 1. The quantities $\bar{n}$, k, $\bar{n}_c$, $\bar{N}$ are the parameters of the best NB fits. The quantities $\bar{n}_{\text{INC}}$ and $\bar{N}_{\text{prim}}$ are the average multiplicity and the average number of primary collisions, respectively, as given by the INC calculation. See text for detail

System	Energy (GeV)	b (fm)	$\bar{n}_{\text{INC}}$	$\bar{n}$	k	$\bar{n}_c$	$\bar{N}$	$\bar{N}_{\text{prim}}$
$p+$Ca		0	7.83	7.81	13.68	1.26	6.17	3.95
	2	1.5	6.88	7.19	10.01	1.32	5.42	3.59
		3	4.40	4.42	4.61	1.42	3.10	2.24
		sum	4.88	4.82	3.86	–	–	–
$p+$Au	5	0	18.28	18.50	8.3	1.90	9.73	6.67
$p+$Ca	0.5	0	2.745	2.75	61	1.022	2.67	1.53
$b=0$	1	0	5.037	5.05	250	1.01	5.00	2.87
	2	0	7.83	7.81	13.68	1.26	6.18	3.95
	5	0	5.84	5.86	5.62	1.46	4.014	4.14
	10	0	5.58	5.59	4.27	1.56	3.57	3.39

few nucleons (that we call the primary nucleons), which in turn hit a few other nucleons. In a recent work [8], we verified that the INC model supports this physical picture. Here we analyze the multiplicity distribution for the ejected nucleons, as predicted by the INC model. It is shown in Fig. 2 for a typical example. In the same figure, we show the best fit by an NB distribution (full lines) along with the best fit by a P distribution (dashed lines) for the sake of comparison. Although the fits by the NB shape are not perfect (especially for $b=0$), they are sufficiently good to state that multiplicity distributions can be meaningfully characterized by a NB. In Fig. 3, we show another example, namely central collisions on Au target, which also exhibits a good NB fit. Actually, we can say that in more or less the energy range considered in [8], i.e. between ~0.5 GeV to ~10 GeV, the multiplicity distributions (for a given impact parameter) are closely described by NB.

To keep on with the clan analysis proposed in Ref. [5] and reviewed in Sect. 2, one would be inclined to consider that each collision made by the incident proton gives birth to a clan, all the more as the number of these (primary) collisions is expected to follow a P distribution. This is actually the case inside the INC model, as indicated in [9] and illustrated by Fig. 4 for a particular example. However, the use of (2.3)–(2.6) in conjunction with the NB fits can provide the associated values of $\bar{n}_c$ and of $\bar{N}$, the average number of clans*. They are given in Table 1 for a few cases. In most of them, the extracted value of $\bar{N}$ is larger than the (average) number $\bar{N}_{\text{prim}}$ of primary collisions. In our opinion, there are two reasons for that. First, the independent formation of clans is violated by the fact that the incident proton loses energy along its path. This seems to be the principal reason since, when the *relative* energy loss is small, like for 5–10 GeV protons on Ca [8], the quantities $\bar{N}$ and $\bar{N}_{\text{prim}}$ are then very close to each other. Second, at low energy, when the mutual deviation of $\bar{N}$ and $\bar{N}_{\text{prim}}$ is the largest, the secondary particles are produced more or less isotropically. As a consequence, connection between the secondary cascades is established more easily. Then the notion of clans should be revised and the ancestors should be defined at a secondary level. This second aspect seems to be important for the impact parameter dependence reported in Table 1. At large impact parameter, there is less chance for connection between the growing clans, for obvious geometrical reasons.

---

* The clan analysis of the cascade events is able to give directly the INC values of these quantities. For technical reasons, it was not possible for us to perform such an analysis in this case contrarily to the heavy ion case. See later

3.2. *Antiproton-Nucleus Interaction*

This system may be a good candidate for observing NB distributions, especially annihilation at rest. The latter liberates a few pions which are expected to cascade more or less independently inside the nucleus. To check this point, we have re-analysed the numerical results of [9], which applied the INC model to $\bar{p}$-annihilation at rest on several targets. In Fig. 5, we show the ejected nucleon multiplicity, as predicted in [9], for ^{98}Mo target, and the best fit by a NB distribution. The agreement is strikingly good. Good agreement is also achieved for in flight annihilation (for a given impact parameter) on ^{20}Ne, ^{98}Mo and ^{108}Ag targets in the LEAR energy domain. In the annihilation at rest, it is expected, at first sight, that the (average) number of clans $\bar{N}$ will be given by the (average) number of interacting pions $\bar{n}_{\pi,i}$. About half of the pions issued from the annihilation are escaping outwards without entering the nuclear volume. In the particular case shown in Fig. 5, the INC model predicts $\bar{n}_{\pi,i}=2.23$. Similarly to the proton-nucleus case, the extracted value of $\bar{N}$, equal to ~ 10, is larger than $\bar{n}_{\pi,i}$. This is not really surprising to us, since two nucleons hit successively by the same pion are likely to belong to different clans. Thus, each interacting pion is not the head of a single clan (which in our analysis is constituted of *nucleons* only), but rather gives birth to several clans, very much alike the incident proton in Fig. 1 b.

A surprising feature of our analysis is that when the energy of the antiproton increases up to the upper end of the LEAR energy domain, the nucleon distribution tends to a Poisson distribution.

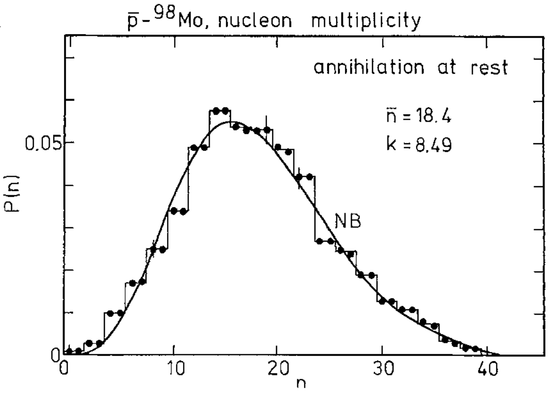

Fig. 5. The histogram gives the probability distribution for nucleon multiplicity n, as predicted by the INC calculation of [9]. The shape of the histograms results from a binning by pairs of values of n. The full curve is the best fit by a NB distribution

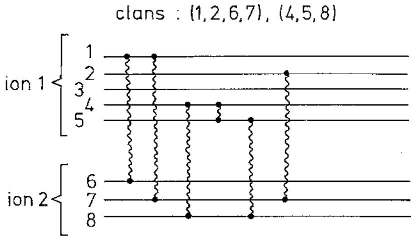

Fig. 6. Illustration of the clan decomposition in heavy ion reactions as proposed in [13]. The horizontal lines are the nucleon lines and the wavy vertical lines represent two-body scatterings between the two nucleons corresponding to the horizontal lines at which they are attached. The composition of the clans are indicated above. See text for detail

3.3. *Heavy-Ion Case*

Although the participants' multiplicity for heavy ion reactions at a given impact parameter are not expected to follow neither a NB nor a P distribution, since they are rather narrow [10–14], this case is nonetheless interesting to analyze. Indeed, in Ref. 13, the concept of clans was introduced in relation with heavy ion collisions in the GeV/A range. The clans, therein referred as "interacting clusters" are defined by diagrams like the one of Fig. 6, which describes a possible interaction between a system of A particles with pair-wise interactions*. The whole class of these diagrams can be obtained by drawing any number of interaction lines (wavy lines) between particle lines. The participants, i.e. the nucleons which have collided at least once, can be subjected to a clan decomposition. Two particles belong to the same clan if they have interacted with each other, i.e. if they are connected by a wavy line in Fig. 6. With this concept, the whole collision process can be decomposed into a number of sub-processes, each taking place within a clan. In comparison with the two previous cases, the clans do not really possess an ancestor, but they are clans in the sense that they have independent or disconnected dynamics (the "intimacy" of the clan). An obvious consequence is that there are at least two particles in any of these clans.

In Ref. 13, INC events are analyzed in terms of clans** with emphasis on average quantities. Here the same events are re-analyzed to establish distributions. The participant multiplicity distributions are rather narrow as shown in Fig. 7 (first column). They are narrower than a P distribution and do certainly not follow NB distributions. In fact, the present cas-

---

* The diagrams are "classical", however, since they neglect interferences between two pairwise interactions

** Therein, the soft collisions, i.e. those which correspond to a small momentum transfer are neglected, in order to avoid the spectators. See [13] for more detail

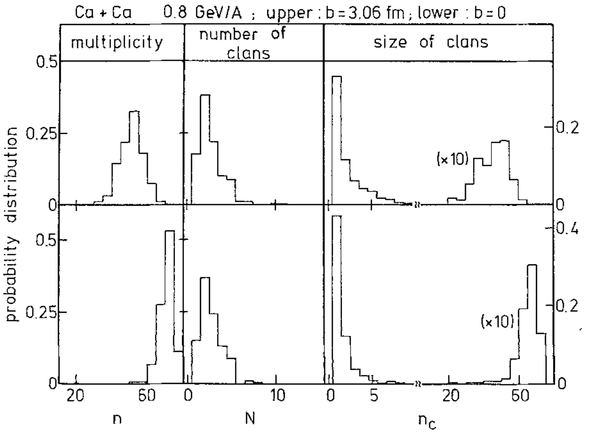

Fig. 7. Clan analysis of Ca+Ca collisions at 0.8 GeV/A for two impact parameters: $b=0$ (lower part), $b=3.06$ fm (upper part). The probability distributions, as extracted from the INC calculation, for the nucleon multiplicity n, for the number N of clans (scale on the left for both) and for the size of the clans (scale on the right) are displayed by the histograms

cade dynamics does not resemble at all to what is required for a NB distribution. First, the number of clans does not follow a P law, although the deviation is not dramatic (see Fig. 7). Second, and more importantly, the size of the clans has a very broad distribution, not at all akin to a logarithmic distribution. Actually this distribution has two components: one for small clans and one for large clans. Strangely enough, the small clans component is very well fitted by a logarithmic distribution starting at $n_c=2$, or better, by a shifted logarithmic distribution

$$\begin{aligned} P(n_c) &= \frac{(1-b)}{b}(\bar{n}_c-1)\frac{b^{n_c-1}}{n_c-1}, && n_c=2,3,\ldots \\ P(n_c) &= 0, && n_c=0,1 \end{aligned} \tag{3.1}$$

with the parameters shown in Table 2, for the two impact parameters shown in Fig. 6. Obviously, the small clans occur close to the geometrical frontier of the participant zone, whereas the bulk of this zone is divided in a small number of big clans. It is interesting to remind that the clan analysis of [13] was performed to check the conjecture made by many authors [15, 16] of the possible existence of many small clans. This conjecture was motivated by the supposedly forward propagation of the nucleons. The INC model reveals however that the perpendicular dispersion is quite important. Actually, at the beginning of the whole collision process, there are many small clans. As time proceeds, clans start to grow and finally they merge into large clans except for the clans at the surface of the interaction zone, because they are more isolated. Moreover, within a cluster, the energy is rather quickly randomised though not completely [13]. Therefore, as they evolve, the clans are more and more subject to the combined constraints of energy and baryon number conservation. This leads us to believe that NB could be the signature of an INC dynamics without efficient constraints and without strong dynamical correlations.

Table 2. The quantities $\bar{n}_c$, b are those appearing in (3.1) and χ^2/N is the χ^2 per degree of freedom of the fit

System	Impact parameter	χ^2/N	$\bar{n}_c$	b
Ca+Ca	0	0.49	2.607	0.56
0.8 GeV/A	3.06	0.40	3.32	0.78

4. Discussion. Conclusion

We have shown that the INC model gives rise to NB multiplicities for two physical processes. This raises several questions: *(a)* What characteristic properties of the cascade dynamics are required for having NB distributions? *(b)* Are not there other possible dynamical schemes which are compatible with NB? *(c)* How to detect NB distributions experimentally? *(d)* What are the similarities and the differences of the nuclear and high-energy cascades?

The answer to the first question has been given mathematically in [5]. But, the physical requirements for the occurrence of such a situation are not easy to point out with precision. From our observations, it seems that the cascade process should not be strongly influenced by some constraints, like energy and/or baryon number conservation. Furthermore, in proton-nucleus interactions, we have seen that there is a partial fusion between clans first initiated by the primary collisions of the incident proton. This does not seem to destroy the ability to produce NB distributions. The merging of clans should not be too important, however. The heavy ion case which is the place of birth of large clans is not at all compatible with NB distributions.

The second question is partially answered by [5], in the case of high-energy cascade. In the nuclear case, it is untimely to risk any answer, although pion multiplicities could be analyzed from this point of view. Let us turn to the third question. In nuclear physics, the experimental search for NB seems to be quite a hard job. Indeed, we have shown that NB distributions are very likely for a single impact parameter. But, usually, experiment sums over many impact parameters and the sum of NB's is not, in general, a

NB. Surprisingly enough, this seems to be the case anyway within the INC as Table 1 indicates (for the $p+\mathrm{Ca}$ system at 2 GeV). Of course, the distribution is larger than for any value of the impact parameter and the parameter k is correlatively smaller. This fact emphasizes the necessity of a careful analysis, which, in any case, will strongly require a good measure of the impact parameter in order to be physically significant. This measure could perhaps be provided by the energy loss. We are presently investigating this possibility. But, to our opinion, the only clear case at this stage is provided by the $\bar{p}$-annihilation at rest. Considerations of Sect. 3.2 are directly applicable. To close this point, let us mention that some problems are still pending. For instance, what is the best multiplicity to look at among all the multiplicities one can measure: nucleons, protons, all charged particles, pion multiplicities?

The fourth question raised above is vast and goes beyond the scope of this paper. The common feature to high energy and nuclear cascades is of course the existence of clans. The main difference lies in the elementary interaction. In the high-energy case, the basic interaction is a parton-parton interaction which *produces* ancestors. Incidentally, this makes the impact parameter question much less crucial in the high-energy case. In the nuclear case, the elementary interaction is the nucleon-nucleon (or baryon-baryon) interaction. Therefore, we believe that in some intermediate region (probably between 10–100 GeV in the proton-nucleus case), the dynamics passes from the nuclear cascade to the parton-parton cascade. In this energy range, NB should be less useful.

We are grateful to Prof. Van Hove for an interesting correspondence.

References

1. UA5 Collaboration, Alpgård, K., et al.: Phys. Lett. **115**B, 71 (1982)
2. Åsman, B.: Ph.D. Thesis, Stockholm 1985
3. UA5 Collaboration, Alner, G.J., et al.: Phys. Lett. **160**B, 193 (1985)
4. HRS Collaboration, Derrick, M., et al.: Phys. Lett. **168**B, 299 (1986)
5. Giovannini, A., Van Hove, L.: Z. Phys. C – Particles and Fields **30**, 391 (1986)
6. Koba, Z., Nielsen, H.B., Olesen, P.: Nucl. Phys. B**40**, 317 (1972)
7. Van Hove, L., Giovannini, A.: Contributed paper to the XVII International Symposium on Multiparticle Dynamics, Seewinkel, Austria, 1986
8. Cugnon, J.: Nucl. Phys. A **462**, 751 (1987)
9. Cugnon, J., Vandermeulen, J.: Nucl. Phys. A**445**, 717 (1985)
10. Fai, G., Randrup, J.: Nucl. Phys. A**381**, 557 (1982)
11. Nagamiya, S., Gyulassy, M.: Adv. Nucl. Phys. **13**, 201 (1984)
12. Yariv, Y., Fraenkel, Z.: Phys. Rev. C**20**, 2227 (1979)
13. Cugnon, J., Knoll, J. Randrup, J.: Nucl. Phys. A**352**, 505 (1981)
14. Cugnon, J., L'Hôte, D.: Nucl. Phys. A**452**, 738 (1986)
15. Hüfner, J., Knoll, J.: Nucl. Phys. A**290**, 460 (1977)
16. Knoll, J., Randrup, J.: Nucl. Phys. A**324**, 445 (1979)

J. Cugnon
Institut de Physique B5
Université de Liège
Sart Tilman
B-4000 Liège 1
Belgium