The $\mathcal{N}\Delta \rightarrow \mathcal{N}\mathcal{N}$ Cross-Section from Detailed Balance.

J. CUGNON and J. VANDERMEULEN
Institut de Physique B5, Université de Liège - Sart Tilman, B-4000 *Liège* 1, *Belgium*

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Summary. – Models describing nucleus-nucleus collisions at high energy need the value of the $\mathcal{N}\Delta \rightarrow \mathcal{N}\mathcal{N}$ cross-section. The latter may be obtained from the $\mathcal{N}\mathcal{N} \rightarrow \mathcal{N}\Delta$ cross-section by detailed balance. We point out the relation between the two cross-sections, relevant to models which do not discriminate the charges.

The collision between two nuclei in the relativistic regime faces the theorist with a very complicated quantum many-body situation, and simplifications have to be made to tract the problem. One possible schematization leads to the model of intranuclear cascade (1), in which the nuclear collision is built up as a sequence of binary collisions between hadrons. The particles are assumed to move freely between zero-duration collisions which occur when the minimum distance of approach for a pair is smaller than the square root of the cross-section divided by π.

The multiplicity of negative pions produced in central nucleus-nucleus collisions has recently received very much attention. In particular, the discrepancy between the observed pion yield and the predictions of the intranuclear cascade, the most successful model in every other respect, has been interpreted (2) as a manifestation of the compressional energy of nuclear matter, a quantity of considerable interest for nuclear physics.

In the intranuclear cascade model, pion production is assumed to take place via the occurence of Δ isobars during the process. The Δ's are allowed to decay and to interact in the course of the process. The following reactions are used:

$$a)\quad \mathcal{N}\mathcal{N} \rightarrow \mathcal{N}\Delta\,, \qquad b)\quad \mathcal{N}\Delta \rightarrow \mathcal{N}\mathcal{N}\,,$$

$$c)\quad \Delta \rightarrow \pi\mathcal{N}\,, \qquad d)\quad \pi\mathcal{N} \rightarrow \Delta\,.$$

(1) For a review, see J. CUGNON: *Nucl. Phys. A*, **387**, 191c (1982).
(2) R. STOCK, R. BOCK, R. BROCKMAN, J. W. HARRIS, A. SANDOVAL, H. STROEBELE, K. L. WOLF, H. G. PUGH, L. S. SCHOEDER, M. MAIER, R. E. RENFORDT, A. DACAL and M. E. ORTIZ: *Phys. Rev. Lett.*, **49**, 1236 (1982).

The Δ-isobars have a mass spectrum. In reaction *a*) the mass is chosen at random according to a distribution which closely follows the observed Δ-distribution; decay *c*) is governed by the natural width of the Δ.

The cross-sections for reactions *a*) and *d*) are taken from experiment, whereas the cross-section for reaction *b*) is computed as explained below.

The discrepancy between cascade calculation and experiment for pion production may arise from the inadequacy of the pion production model sketched above. In particular, the Δ having a small lifetime, off-energy shell effects may be important. But the discussion about the compression energy, by reference to the output of the cascade model, demands that this classical model uses correct ingredients. It appears, in particular, that a definite point has to be clarified. The derivation of the (integrated) cross-section for the $\mathcal{N}\Delta \to \mathcal{N}\mathcal{N}$ reaction, by detailed balance, is in principle straightforward, but has given rise to controversies in some circles using models of heavy-ion collisions (intranuclear cascade, chemical kinetics, ...). More precisely we point out hereafter the precise numerical factor to be used in the formula when the average over isospin states is made. Of course, in general, one studies the population of the particles in their various isospin states. But, sometimes, one may be interested in quantities summed over the charges. Moreover, in a symmetric system, one needs not discriminate between isospin states (see below), provided one concentrates on average quantities (*e.g.* the average number of negative pions). Furthermore, several intranuclear cascade codes used in the past did not distinguish between isospin states. In order to compare with new versions making this distinction, it is desirable to know how to handle the detailed balance (this is the crucial point) when isospin average is introduced.

As a usual procedure, we consider the isospin as an internal degree of freedom, similarly to the treatment of the spin. One has (with the nonrelativistic kinematics), for the A+B → C+D process ([3]),

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = (2\pi)^4 \mu_i \mu_f \frac{p_f}{p_i} \frac{1}{g_A g_B} \times \sum_{\substack{S_A S_B \\ T_A T_B}} \sum_{\substack{T_C T_D \\ S_C S_D}} |\langle \boldsymbol{k}_f, S_C S_D T_C T_D | T | \boldsymbol{k}_i, S_A S_B T_A T_B \rangle|^2 , \tag{1}$$

where $g_i = (2S_i + 1)(2T_i + 1)$. The other notations are quite standard.

This cross-section corresponds to the hypothetical situation of a mixed beam of protons and neutrons, in equal proportion, bombarding a target, also composed in equal proportions of neutrons and protons, and of a detector which does not discriminate between the members of an isospin multiplet. As usual, we have assumed an unpolarized beam and an unpolarized target, with respect to the spin. This cross-section is very well suited to a cascade code which does not distinguish the isospin states inside a given multiplet. The T-matrix in (1) involves antisymmetrized wave functions (see ref. ([3]), ch. 5, p. 242, and ref. ([4])).

For the reverse process, one has

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_2 = (2\pi)^4 \mu_i \mu_f \frac{p_i}{p_f} \frac{1}{g_C g_D} \sum_{\substack{S_C, S_D \\ T_C, T_D}} \sum_{\substack{S_A, S_B \\ T_A, T_B}} |\langle -\boldsymbol{k}_i, S_A S_B T_A T_B | T | -\boldsymbol{k}_f, S_C S_D T_C T_D \rangle|^2 . \tag{2}$$

([3]) M. L. GOLDBERGER and K. M. WATSON: *Collision Theory*, 2nd edition (R. E. Krieges Publ. Co. Inc., New York, N. Y., 1975).

([4]) G. PINSKI, A. J. MACFARLANE, E. C. G. SUDARSHAN: *Phys. Rev.*, **140**, 1045 (1965).

Time reversal invariance implies that (see ref. [3], ch. 4, p. 171)

$$(3)\qquad |\langle -\boldsymbol{k}_{\rm i}, S_{\rm A} S_{\rm B} T_{\rm A} T_{\rm B}|T|-\boldsymbol{k}_{\rm f}, S_{\rm C} S_{\rm D} T_{\rm C} T_{\rm D}\rangle|^2 = |\langle \boldsymbol{k}_{\rm f} S_{\rm A} S_{\rm B} T_{\rm C} T_{\rm D}|T|\boldsymbol{k}_{\rm i} S_{\rm A} S_{\rm B} T_{\rm A} T_{\rm B}\rangle|^2 .$$

Therefore,

$$(4)\qquad \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_2 g_{\rm C} g_{\rm D} \frac{p_{\rm f}}{p_{\rm i}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 g_{\rm A} g_{\rm B} \frac{p_{\rm i}}{p_{\rm f}} .$$

The important point is that eqs. (1) and (2) have been obtained by defining the differential cross-section as the ratio of the scattered to the incident flux even if the particles are identical (see, once again, ref. [3] for more detail). Therefore, the integrated cross-section is obtained by integrating over 2π units of solid angle, if the outgoing particles are identical (two nucleons, for instance, in our representation). We thus have

$$(5)\qquad \frac{\sigma_2}{\sigma_1} = \frac{p_{\rm i}^2}{p_{\rm f}^2} \frac{g_{\rm A} g_{\rm B}}{g_{\rm C} g_{\rm D}} \frac{1+\delta_{\rm CD}}{1+\delta_{\rm AB}} .$$

For the $\mathcal{N}\mathcal{N} \to \mathcal{N}\Delta$ reaction (we concentrate here and below on a definite Δ-mass) eq. (5) reduces to

$$(6)\qquad \sigma_{\mathcal{N}\Delta\to\mathcal{N}\mathcal{N}} = \frac{1}{8} \frac{p_{\rm i}^2}{p_{\rm f}^2} \sigma_{\mathcal{N}\mathcal{N}\to\mathcal{N}\Delta} .$$

Therefore, there is an extra factor $\frac{1}{2}$, coming from the identity of the nucleons, when spin *and* isospin are averaged over.

To make the argument more convincing, we show that this formula can be obtained from detailed balance, without explicitly introducing the isospin representation. We have to consider the following reactions:

$$(7)\qquad \begin{cases} \mathrm{pp} \to \mathrm{n}\Delta^{++},\ \mathrm{p}\Delta^{+} , \\ \mathrm{pn} \to \mathrm{n}\Delta^{+},\ \ \mathrm{p}\Delta^{0} , \\ \mathrm{nn} \to \mathrm{n}\Delta^{0},\ \ \mathrm{p}\Delta^{-} . \end{cases}$$

The branching ratios are $\frac{3}{4}$ and $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$, respectively. Detailed balance allows us to write

$$(8)\qquad \sigma_{\mathrm{n}\Delta^{++}\to\mathrm{pp}} = \sigma_{\mathrm{pp}\to\mathrm{n}\Delta^{++}} \frac{p_{\rm i}^2}{p_{\rm f}^2} \frac{1}{2} \frac{1}{2} ,$$

where the first $\frac{1}{2}$ factor comes from the spin and the second one from the identity of the two particles in the final states. The latter factor will not appear for pn final states. By summing equations similar to (8) for all the reaction channels (7), we easily get

$$(9)\qquad \sum_{\mathrm{x=n,p}} \sum_{\mathrm{y}=\Delta^{-},\ldots,\Delta^{++}} \sigma_{\mathrm{xy}\to\mathrm{ab}} = \frac{1}{2} \frac{p_{\rm i}^2}{p_{\rm f}^2} \left\{ \frac{1}{2} \sum_{\mathrm{xy}} \sigma_{\mathrm{pp}\to\mathrm{xy}} + \sum_{\mathrm{xy}} \sigma_{\mathrm{pn}\to\mathrm{xy}} + \frac{1}{2} \sum_{\mathrm{xy}} \sigma_{\mathrm{nn}\to\mathrm{xy}} \right\} ,$$

where ab represent the final nucleon-nucleon states accessible to the (x, y) pair. One may write

$$(10)\qquad \sum_{x=p,n}\ \sum_{y=\Delta^-,\ldots,\Delta^{++}}\ \sum_{a=n,p}\ \sum_{b=n,p}\sigma_{xy\to ab}=\frac{1}{2}\frac{p_i^2}{p_f^2}\left\{\frac{1}{2}\sum_{x,y}\sum_{a,b}\sigma_{ab\to xy}\right\},$$

where the cross-sections for reactions not allowed by charge conservation are conventionally put equal to zero. Hence,

$$(11)\qquad \left(\frac{1}{8}\sum_{x,y}\sum_{a,b}\sigma_{xy\to ab}\right)=\frac{1}{8}\frac{p_i^2}{p_f^2}\left(\frac{1}{4}\sum_{x,y}\sum_{a,b}\sigma_{ab\to xy}\right).$$

The parentheses are precisely the isospin-averaged cross-sections considered above. We thus recover the factor $\frac{1}{8}$ of eq. (6). We stress that this result is obtained without relying on a particular model for the Δ-production: for instance, it is independent of the branching ratios mentioned above.

We finally want to discuss the implications of relation (6) on usual models. Let us turn first to the so-called hadrochemical model and consider a mixture of nucleons and Δ-particles. We again do not discriminate between isospin. We thus may write the following evolution equations for the particle densities:

$$(12)\qquad \frac{d\varrho_{\mathcal{N}}}{dt}=-\frac{d\varrho_\Delta}{dt}=-\frac{1}{2}\varrho_{\mathcal{N}}^2\langle\sigma_{\mathcal{N}\mathcal{N}\to\mathcal{N}\Delta}v_{\mathcal{N}\mathcal{N}}\rangle+\varrho_{\mathcal{N}}\varrho_\Delta\langle\sigma_{\mathcal{N}\Delta\to\mathcal{N}\mathcal{N}}v_{\mathcal{N}\Delta}\rangle.$$

The factor $\frac{1}{2}$ arises from the identity of the reacting particles. It occurs, because, if we have N nucleons, we have only $N(N-1)/2\approx N^2/2$ reacting pairs, in contrast with the case in which the reactants AB belong to different species. Then the loss term would be simply proportional to $\varrho_A\varrho_B$.

The brackets in (12) indicate an average over the relative velocity distribution. For a thermal distribution, one has [5-7]

$$(13)\qquad \frac{\langle\sigma_{\mathcal{N}\mathcal{N}\to\mathcal{N}\Delta}v_{\mathcal{N}\mathcal{N}}\rangle}{\langle\sigma_{\mathcal{N}\Delta\to\mathcal{N}\mathcal{N}}v_{\mathcal{N}\Delta}\rangle}=2\frac{g_{\mathcal{N}}g_\Delta}{g_{\mathcal{N}}^2}\left(\frac{m_\Delta}{m_{\mathcal{N}}}\right)^{\frac{3}{2}}\exp\left[-(m_\Delta-m_{\mathcal{N}})/kT\right],$$

where the factor 2 comes once again from the identity of the nucleons through detailed balance. This factor 2 combines with the factor $\frac{1}{2}$ in eq. (12) to give an equilibrium ratio

$$(14)\qquad \left(\frac{\varrho_\Delta}{\varrho_{\mathcal{N}}}\right)_{eq}=\frac{g_\Delta}{g_{\mathcal{N}}}\left(\frac{m_\Delta}{m_{\mathcal{N}}}\right)^{\frac{3}{2}}\exp\left[-(m_\Delta-m_{\mathcal{N}})/kT\right],$$

a result which can be obtained from an equilibrium calculation [8], without reference to the cross-sections.

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[5] I. Montvay and J. Zimanyi: *Nucl. Phys. A*, **316**, 490 (1979).
[6] D. D. Clayton: *Principles of Stellar Evolution and Nucleosynthesis* (McGraw-Hill Book Co., New York, N. Y., 1968).
[7] C. A. Barnes: International School of Nuclear Physics at Erice, 1980.
[8] J. I. Kapusta: *Phys. Rev. C*, **16**, 1493 (1977).

Equation (12) is exact in the case of a $N = Z$ symmetric system. It is easy to show that equations similar to (12) written for every species (n, p, Δ^-, Δ^0, Δ^+, Δ^{++}) can be summed up to give eq. (12), when isospin average cross-sections are introduced. Moreover, the branching ratios for reactions (7) and the isospin conservation law, which implies that $\sigma(\mathrm{nn} \to \mathrm{n}\Delta^0 + \mathrm{p}\Delta^-) = \sigma(\mathrm{pp} \to \mathrm{n}\Delta^{++} + \mathrm{p}\Delta^+) = 2\sigma(\mathrm{pn} \to \mathrm{n}\Delta^+ + \mathrm{p}\Delta^0)$, guarantee an equal population for the members of a given isospin multiplet ($\varrho_\mathrm{n} = \varrho_\mathrm{p}$, $\varrho_{\Delta^-} = \ldots = \varrho_{\Delta^{++}}$) at any moment.

The same considerations apply to a standard cascade model. We remind, for instance, that the factor $\frac{1}{2}$ is indeed included in the machinery of a code like the one of ref. ([9]) since nucleon-nucleon *pairs* are checked to see whether they produce a collision. Of course, in a cascade calculation, the averages in eq. (12) are taken on the actual distributions and not on a thermal one. Furthermore, fluctuations are allowed, which may make, at a given time, the n/p ratio different from unity, even if one starts with a symmetric system.

([9]) J. Cugnon, D. Kinet and J. Vandermeulen: *Nucl. Phys. A*, **379**, 553 (1982).