POSSIBLE FRAGMENTATION INSTABILITY IN A NUCLEAR EXPANDING FIREBALL

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The isoentropes of symmetric nuclear matter are calculated in the Hartree–Fock–Skyrme approximation. They suggest the possibility of a fragmentation instability during the expansion of an excited compressed nuclear fireball. Characteristic expansion and instability times are discussed quantitatively.

The properties of nuclear matter at finite temperature have been studied for several years already. It has been shown for the first time in refs. [1,2],that, in the Hartree–Fock approximation, the nuclear matter behaves like a van der Waals fluid. More precisely, calculations assuming a uniform density and using either phenomenological or more realistic [3] interactions yield isothermal pressure curves typical of a van der Waals fluid. The critical point is located around $\rho_c \approx 0.3\,\rho_0$ (ρ_0 = normal nuclear matter density) and $T_c \approx 20$ MeV. This question has recently received much attention since new accelerators are operating now in this parameter range.

The possibility of a liquid–gas transition in nuclei has been pointed out recently [4]. We want to discuss the problem here in relation with the expansion of a nuclear fireball. We first discuss the main properties of this expansion. Second, we present the isoentropic curves of nuclear matter calculated in the Hartree–Fock–Skyrme approximation. Third, we calculate the expansion in a simple approximate model. Finally, we discuss the possibility of the onset of a fragmentation instability.

The expansion of a fireball can be viewed as the transformation of the internal energy to the macroscopic outward flow. If this flow is not affected by friction forces, the internal degrees of freedom and the macroscopic flow degrees of freedom can be considered as two coupled systems which exchange work but no heat. Therefore the entropy of the (internal) system is constant. This important property has been conjectured by Sobel et al. [5] and recently discussed by Bertsch and Siemens [6]. It is substantiated by the cascade calculation of ref. [7] (see the remark below, however). In such a perspective, the isothermal curves are not relevant: the temperature of the system will decrease during the expansion. We have calculated the isoentropes for nuclear matter using the so-called Skyrme III interaction. The formalism is exactly the same as in ref. [1] and the same definition of the entropy is used. The results are given in fig. 1. For large values of the entropy per nucleon S/A, the isoentrope are increasing functions of the density. But for $S/A \lesssim 2$, the curves present a negative slope for some values of ρ. In other words, in this region, the (isoentrop compression modulus is negative. The system may become unstable in this region.

During a nucleus–nucleus collision, the system ho fully reaches a state represented by a point in the upper part of fig. 1, where the pressure is positive. The latter makes the system to expand along an isoentrop For S/A larger than 2, the pressure remains positive and the expansion will proceed unceasingly. The system may be viewed as a gas all the time. Occasionally the nucleons may form composites in this expansion, just by nuclear reactions as is well understood in the

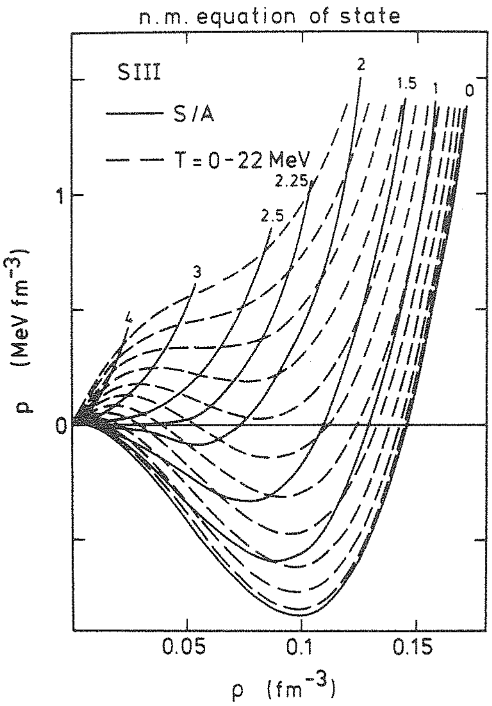

Fig. 1. Isotherms (dashed curves) and isoentropes (full curves) for symmetric nuclear matter in the Hartree–Fock–Skyrme interaction. They give the pressure p as a function of the baryon density ρ. The so-called Skyrme III force is used. For further detail, see refs. [1,2]. The values of the entropy per nucleon is indicated for each of the isoentropes. The isotherms are given for temperatures from $T = 2$ (bottom) to $T = 22$ MeV (top) by steps of 2 MeV. The $T = 0$ isotherm coincides with the $S/A = 0$ isoentrope.

dilute gas limit. On the other hand, if S/A is less than 2 the system may reach the region where the pressure is negative. The latter develops a force which tends to reverse the outward motion. If the initial pressure is not very important the system will oscillate around the equilibrium point ($p = 0$). If the initial pressure is sufficiently large, the system may eventually reach the instability zone. What may happen there depends upon the different characteristic times: equilibration time, expansion time, typical time for the onset of the instability. We first try to elucidate the question of the expansion time.

The expansion of a compressed nuclear system may be described quantitatively in the following simple model. We assume the system may be represented by a sphere of uniform particle density ρ, which is a function of time t. Let $\dot{\rho}$ be the time derivative of ρ. The following velocity field

$$v(r) = -\tfrac{1}{3}(\dot{\rho}/\rho)\, r \tag{1}$$

guarantees the conservation of mass at any time. One can thus calculate the kinetic energy associated with the macroscopic flow (in a non-relativistic approach):

$$K = \frac{M}{2}\int \rho v^2\, d^3r = 2\pi\rho M \int_0^R v^2(r)\, r^2\, dr, \tag{2}$$

where R is the fireball radius. If A is the mass number and M the nucleon mass, we can write

$$K = \tfrac{1}{30}\,(3/4\pi)^{2/3} M A^{5/3}\, \dot{\rho}^2/\rho^{8/3}. \tag{3}$$

The internal energy U is assumed to come from the bulk energy only. It can be written as $U(S, \rho)$ where S and ρ are the instantaneous entropy and density. The time evolution of the system is governed by the conservation law:

$$d(K + U)/dt = 0. \tag{4}$$

Along an isoentrope, one has

$$dU/dt = -p\, dV/dt, \tag{5}$$

where V is the volume. Using eq. (3) and after a little algebra, one finds the following equation for the evolution along an isoentrope

$$\ddot{\rho} - \tfrac{4}{3}\dot{\rho}^2/\rho = -\,Cp(\rho)\,\rho^{2/3}, \tag{6}$$

with

$$C = 1/[\tfrac{1}{15}\,(3/4\pi)^{2/3} M A^{2/3}]. \tag{7}$$

In eq. (6), $p(\rho)$ represents the pressure along the isoentrope (see fig. 1). This equation clearly shows that at equilibrium, the pressure vanishes. In this model, the surface contribution to the energy is left out. In the actual free isoentropic expansion of a fluid, the density may be uniform all the time, but not the pressure, nor the temperature [8]. However, besides these local variations, the evolution of the average quantities is solely determined by conservation laws. Therefore, in eqs. (5), (6), p is to be interpreted as the average pressure. We present in figs. 2 and 3 the results of the integration of eq. (6) for different values of S/A. We choose $A = 200$ to minimize the effect of the surface energy. At the initial time the system is compressed and left at rest [$\dot{\rho}(t = 0) = 0$].

For $S/A = 1$, the system oscillates harmonically

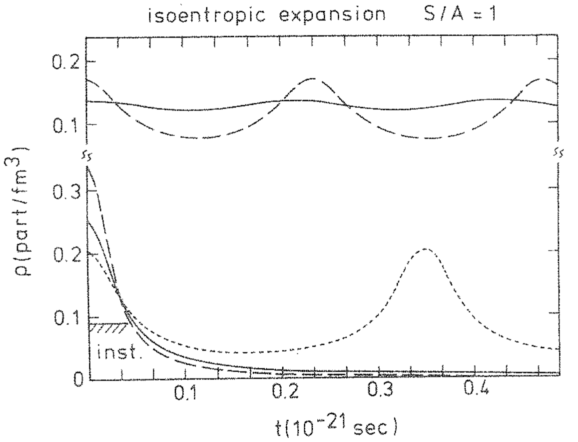

Fig. 2. Time evolution of the density of an expanding fireball, according to eq. (6) and, for different values of the initial density. At the initial time $t = 0$, the time derivative of the density ρ vanishes. The system evolves along the isoentrope $S/A = 1$ (fig. 1). The dashes at the lower left corner indicate the upper density of the instability zone for the isoentrope.

around the equilibrium density $\rho_{eq} = 0.129\ fm^{-3}$, provided the starting density is not far from this value. The period of oscillation ($\tau \approx$ fm/$c \approx 0.23 \times 10^{-21}$ s) is, as it should, given by

$$\tau = 2\pi/[C\rho_{eq}^{2/3}(\mathrm{d}p/\mathrm{d}\rho)_{\rho=\rho_{eq}}]^{1/2},$$

obtained by making a linear approximation of $p(\rho)$

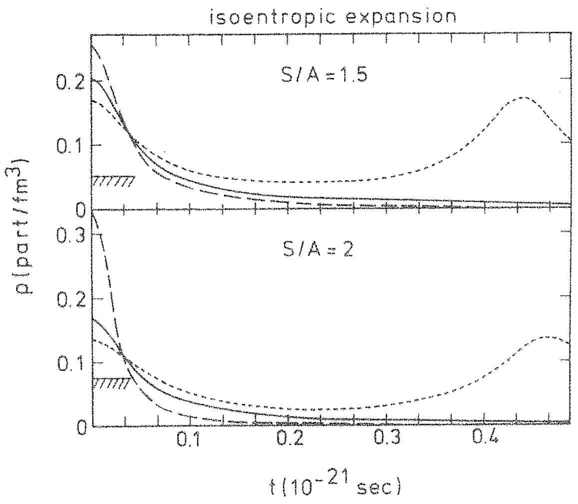

Fig. 3. Same as fig. 2, for $S/A = 1.5$ and 2.

around the equilibrium point. For larger values of $\rho(t = 0)$ the oscillation starts to be anharmonic and period increases because of the non-linearity of eq. Finally, for $\rho(t = 0) \approx 0.25\ fm^{-3}$, the system expa for ever. For times later than $\approx 0.5 \times 10^{-22}$ s, it sta in the instability zone. For $S/A = 1.5$ and 2, the evc tion pattern is qualitatively the same. Smaller value: $\rho(t = 0)$ are however required to obtain a continuoι expansion of the fireball: $\sim 0.2\ fm^{-3}$ for $S/A = 1.5$ $\sim 0.16\ fm^{-3}$ for $S/A = 2$ respectively. Note that fo $S/A = 1$, the system penetrates the instability zone in the oscillatory regime [see fig. 1 and $\rho(t = 0) \approx 0$ fm^{-3}]. The results of figs. 2, 3 determine the time scale for the non-oscillatory expansion. This time m be considered as the time necessary to reach $\rho \approx 0.$ fm^{-3}, for which the interactions do not seem to pl role any more. We can consider that $t_{ex} \approx 60$ fm/c ($\sim 0.2 \times 10^{-21}$ s) or even smaller. It is smaller than oscillation period around the equilibrium point.

Let us discuss the conditions for reaching the ins bility zone. First, one has to build a system with no too large an entropy ($S/A \lesssim 2$). Such an instability seems thus to be ruled out in the composite produc in the heavy ion collisions around 1 GeV/A, due to the large entropy production [7,9]. A second condi tion is that the maximum density should be slightly larger than normal nuclear matter density. Looking fig. 1, one can find out the corresponding temperatı of the fireball prior to the expansion. One finally ca say that these conditions are expectedly encountere in symmetric collisions around $E_{lab}/A \approx 100$ MeV. The precise value cannot be stated without a model for the compression phase. The developments at hig energy indicate that too much entropy would be cr ed by shock waves [10]. This would imply that a ve asymmetric system would be preferable to a symme ric one.

A second condition is that the (thermal) equilibr tion time is small compared to t_{ex}. As an estimate w can consider

$$t_{eq} \approx 1/\rho v \langle\sigma\rangle, \qquad ($$

where v is the average velocity and where $\langle\sigma\rangle$ is the average NN cross section corrected for the Pauli pri ciple [11]. Using $v \approx v_F$ and $\langle\sigma\rangle \approx 20$ mb, we find t $t_{eq} \ll t_{ex}$ for a large range of values of ρ.

Another condition to be met is that the characte tic time scale t_{ins} for the growth of the instability should be smaller than the time required by the system to cross the instability, i.e. t_{ex} for practical purposes. The time t_{ins} may be estimated in the following way. Let us consider an extended system with uniform density $\bar{\rho}$ and let us investigate the stability against an oscillatory modification of the type

$$\rho(r) = \bar{\rho} + a \sin(k \cdot r). \quad (10)$$

Again, at this density modification is associated a velocity field

$$v(r) = (\dot{a}/\bar{\rho})\,(k/k^2) \cos(k \cdot r), \quad (11)$$

which guarantees the continuity equation. This introduces a modification in the kinetic energy, which in lowest order, is

$$\delta K = (M/4\bar{\rho})\, V \dot{a}^2/k^2, \quad (12)$$

where V is the volume of the system (once again, surface effects are neglected). If H is the bulk energy density, calculated as for fig. 1, the modification of U is given by

$$\delta U = \int \{(\partial H/\partial \rho)_S\, [\rho(r) - \bar{\rho}] + \tfrac{1}{2}\,(\partial^2 H/\partial \rho^2)_S\, [\rho(r) - \bar{\rho}]^2 + \ldots\}\, d^3r. \quad (13)$$

Neglecting surface effects, one has in lowest (i.e. in second) order

$$\delta U = (1/4\bar{\rho})\,(\partial p/\partial \rho)_S\, a^2\, V. \quad (14)$$

Eqs. (12), (14) provide the basis for a small amplitude theory. Writing $a(t) = a_0 \exp(i\omega t)$, one readily sees that the system is stable against oscillatory perturbations as long as $(dp/d\rho)_S$ is positive. If $(dp/d\rho)_S$ is negative, the amplitude $a(t)$ will grow exponentially [$\sim \exp(\Gamma t)$] with

$$\Gamma = k\,[-M^{-1}(\partial p/\partial \rho)_S]^{1/2}. \quad (15)$$

For typical values ($k \approx 0.5$ fm^{-1}, $\bar{\rho} \approx 0.05$ fm^{-3}, $S/A \approx 1.5$) of the instability zone, one has $t_{ins} \approx \Gamma^{-1} \approx 20$ fm/c, smaller than t_{ex}. The long wavelength ($k \ll$) instabilities will set in much slower. The small wavelength ($k \gg$) instabilities are probably very sensitive to surface energy effects, i.e. to terms in $\nabla\rho$ in the energy density functional H. Consequently, the size of the fragments arising from the instability are sensitive to surface effects and will be treated in a later work.

Still other conditions are required for the existence of a fragmentation instability. First, the system should not evaporate too much nucleons before reaching the instability zone. Estimates by Curtin et al. [12] indicate that for sufficiently low entropy (or temperature) the characteristic time of the evaporation is comparable with t_{ex}. The question may be more intricate, however. The gross lines of the scheme proposed here may be preserved, in the presence of evaporation, provided the residual system keeps roughly the same density and the same entropy. A last point concerns the nuclear viscosity. If the latter is important, the system does not follow an isoentrope: entropy is produced. The giant monopole resonance, which may be viewed as a small oscillation around an equilibrium point (p=0 for small entropy, is damped, indicating a viscous motion. The damping is not very important however ($\Gamma/\hbar\omega \approx 0.3$) and, in fact, half of the width only may be attributed to the viscosity, the rest is due to the coupling to the continuum [13].

In conclusion, we have presented a scheme where an excited compressed system can develop a fragmentation instability during its isoentropic expansion. The time scale for the expansion, for the thermal equilibration and for the onset of the instability are consistent with this possibility. The entropy per nucleon should be smaller than 2 units, and probably even smaller, because of possible viscosity and evaporation effects, which are not yet well evaluated.

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