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INTRANUCLEAR CASCADE MODEL. A REVIEW

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Abstract : The results of the intranuclear cascade model for heavy ion collisions in the energy range from 250 MeV to 2 GeV per nucleon are reviewed. The emphasis is put on specific features of the model : off-equilibrium behaviour, large mean free path, fluctuations.

1. Introduction

In recent years, the intranuclear cascade model (INC) has been applied successfully to the study of heavy ion collisions with a beam energy ranging from 250 MeV to 2 GeV per nucleon. In fact, this model had been used previously in nuclear reaction physics, for the investigation of the interaction of high energy hadrons with nuclei[1]) (with a moderate success), and for the description of pre-equilibrium reactions[2]). Generally speaking, this model can be applied to situations where multiple scattering is important.

Among the host of models which have been proposed to explain the general features of the relativistic nucleus-nucleus collisions, *the INC has a special place for two reasons* : (i) it is the most *successful* model, with a very high predictive power. Generally, it does not include free parameters. (ii) It is a *microscopic* model. By this, we mean that it involves the coordinates of all the particles and admits basic nucleon-nucleon quantities (the cross sections) as input.

Though, it is a very peculiar approach based on *simulation* (Monte-Carlo techniques) and, therefore, its *connection with theory* (if it exists) or with any existing theoretical approach, *is still very loose*.

As a consequence of this situation, it is difficult to delineate precisely the domain of validity of the model. It is generally accepted that the latter covers the domain of the *classical collision regime*, which corresponds to a dynamics dominated by two-body collisions of particles whose de Broglie wavelength is small. This regime could be realized in heavy ion collisions. However, this question is still debated.

Nevertheless, the INC calculations of many observable quantities have met an impressive success and provide a coherent picture of the collision process. Some of the results can also be obtained by other (simpler) models, with less computational effort. Hence, it is interesting to determine which kind of observables require the specific properties of the INC, as e.g. off-equilibrium aspects.

Also, INC calculations can make predictions on nonobservable quantities, such as the maximum density reached during the collision process and may therefore be helpful in our understanding of the reaction mechanism.

Here, we want to make a (short) review of the numerous results accumulated by the INC calculations in the past few years and examine some of the questions mentioned above. In section 2, we briefly describe the nature of the INC approach and we make a (rough) comparison between various existing codes. In section 3, we go a little bit into the delicate question of the possible relation with existing theoretical frames. In section 4, we make a review of the principal observables (and of other quantities) and of their description by the INC. Section 5 is devoted to some aspects of the heavy ion physics which call for the specificity of the INC : off-equilibrium behaviour, finite mean free path, fluctuations. In section 6, we briefly examine the problems raised by a possible INC approach to nucleus-nucleus collision in the intermediate energy (E_{beam} = E/A = 20-200 MeV) range. Finally, section 7 contains our conclusion.

2. Description of the Intranuclear Cascade Model

As we shall soon see, there is not a single INC model, but a whole variety of them, corresponding to as many numerical codes. Some aspects are however common to all of them. The different codes may be classified in two main categories : the first one studies the interaction between two collections of nucleons, whereas the other puts the emphasis on the interaction of a collection of nucleons with a continuous medium characterized by a mean free path.

We describe the first category (refs.[3-8]) in more detail. Since there are some variations from one code to the other (see Table I), what follows can be considered as an average procedure. The nucleons are positioned initially at random inside spheres representing the target and the projectile . Their momenta are chosen randomly according to the Fermi gas law. The projectile is boosted against the target with the beam velocity. Lorentz contraction and relativistic kinematics are included. The fate of all the nucleons is followed. The latter are assumed to move along straight lines between collisions. When, for a pair of nucleons, the minimum relative distance of approach is reached, the nucleons are forced to scatter, if the minimum relative distance d_{min} is smaller than $\sqrt{\sigma_{tot}(s)/\pi}$, where $\sigma_{tot}(s)$ is the total cross section at the nucleon-nucleon centre of mass energy $\sqrt{s}$. The momenta are then changed according to the experimental differential cross section. At each interaction, energy and momentum are conserved, but not angular momentum. After the interaction, straight line motion is resumed until, for another pair of nucleons, the minimum distance of approach is realized. The sequence of two-body collisions is followed until the rate of collisions vanishes. The procedure is repeated, until a sufficient statistics is reached. The required number of runs may depend upon the quantity one is interested in.

Several features are (or are not) added to this simple scheme. First, the nucleons may be moving in potential wells, which are travelling with the ion velocities. Second, the Pauli principle may be accounted for by forbidding, in a generally approximate way, the final states already occupied in the associated Fermi seas. Third, the pion production may be introduced by considering the inelastic nucleon-nucleon cross section. Generally, the pion production proceeds through Δ-resonances created in nucleon-nucleon reaction. Reabsorption of the pions is introduced through the sequence $\pi N \to \Delta$, $N\Delta \to NN$.

The models of the second category are described in refs.[9-14]. In these codes, the cascading particles (projectile) interacting with a continuum (target) are made to scatter after a path whose length is chosen at random according to an exponential law with a mean equal to the mean free path inside the medium for the incident energy. Then a hole is "punched" in the medium and the nucleon which is assumed to raise from the hole is counted as a cascading particle and can thus interact with the medium. The procedure may be symmetrized between projectile and target. In refs.[9,12], the interaction between cascading particles is neglected. In ref.[10], this interaction is included, but in such a way that for a cascading particle, the others look like a continuum. The holes may be filled immediately by rearranging the density (the "fast rearrangement" option) or may be left over (the "slow rearrangement").

It is hard to compare the two categories without analyzing the details, but it seems to us that the approach based on the mean free path may neglect compression and two-particle correlation effects.

In Table I, we give a comparison between the most popular codes. This comparison is inevitably crude and incomplete and we refer the interested reader to the literature for finer details.

A special word should be mentioned for the SIMON code[6,7], which was originally based on billard balls dynamics only, but which now includes more realistic interactions, and for the code by Smith and Danos[15] where the interaction picture is closer to a field theoretical vertex interaction than to a classical hadron-hadron scattering.

Let us stress two important points :

(1) The INC models do not include any adjustable parameter.

(2) There are several different INC models and they do not form a single approach. Unfortunately, the different authors are generally interested in different

Table I. Comparison between several INC codes

	Stevenson ref.[5]	VEGAS refs.[9,10]	Toneev ref.[12]	SIMON refs.[6,7]	Smith-Danos ref.[15]	Liège ref.[3]
mode	d_{min}	λ	⁑ (λ)	d_{min}	⁑	d_{min}
Int.between cascading particles	yes	no/yes	no	yes	yes	yes
Potential well	for the target	yes (target)	yes/symmetrized	no	yes	no
Pauli principle	lab frame	yes	yes	no	yes	no/yes
Pion production	no	yes	yes	no	yes	yes
Cross sections	exp	exp	exp	billard balls/ + exp	vertex functions	exp
Δ-isobars	no	yes	⁑	no	⁑	yes
Relativistic kinematics	yes (?)	yes (?)	yes	no	yes	yes

⁑ = no clear information, d_{min} = minimum distance of approach,
λ = mean free path, exp = experimental

quantities, making the comparison difficult.

Several models are closely related to the INC. The rows on rows model[16,17] is very much in the same spirit. It retains the finite particle number effects (see section 5) but totally disregards the spatial evolution of the system. The clear knock-out model[18] includes the first hard scattering. On the opposite, the fireball model[19] embodies the thermal (large number of scatterings) limit and the clean-cut separation of participant and spectators (see section 7). The diffusion model[20] assumes the multiple scattering to be describable by a diffusion equation. The equation of motion approach[21-23] adopts a totally classical description, by solving Hamilton equations.

Let us describe in a few words the output of the INC calculations. We forget production processes to simplify the presentation. The output can be represented as a collection of 6 A quantities

$$\{\vec{r}_1,\ldots,\vec{r}_A,\vec{p}_1,\ldots,\vec{p}_A\},\quad \vec{r}_1=\vec{r}_1(t,\omega),\ldots,\vec{p}_A=\vec{p}_A(t,\omega) \quad , \tag{2.1}$$

whose physical meaning is obvious and which are all functions of time and of the stochastic variables, that we simply denote by ω . The latter labels the runs. The output is equivalently given by the quantities :

$$f_1(\vec{r},\vec{p},t) = N_\omega^{-1} \sum_\omega \sum_{i=1}^{A} \delta(\vec{r}-\vec{r}_i)\ \delta(\vec{p}-\vec{p}_i) \quad , \tag{2.2}$$

$$f_2(\vec{r},\vec{p},\vec{r}\,',\vec{p}\,',t) = N_\omega^{-1} \sum_\omega \sum_i \sum_{j\neq i} \delta(\vec{r}-\vec{r}_i)\ \delta(\vec{r}\,'-\vec{r}_j)\ \delta(\vec{p}-\vec{p}_i)\ \delta(\vec{p}\,'-\vec{p}_j) \quad , \tag{2.3}$$

$$f_A(\vec{r},\vec{p},\ldots,\vec{r}^{(A)},\vec{p}^{(A)},t) = N_\omega^{-1} \sum_\omega \sum_{i_1} \sum_{i_2\neq i_1} \sum \ldots\ \delta(\vec{r}-\vec{r}_{i_1})\ldots\delta(\vec{r}^{(A)}-\vec{r}_{i_A})\ \delta(\vec{p}-\vec{p}_{i_1})\ldots$$

$$\ldots \; \delta(\vec{p}^{(A)} - \vec{p}_{i_A}) \quad , \tag{2.4}$$

which are the familiar one-body, two-body, ..., A-body distribution functions. N_ω is the number of runs. For evident reasons, only the functions with the lowest indices can be evaluated reliably. In practice, the δ functions are replaced by step functions defined on a mesh in phase space.

3. Relation with theory

The INC is, strictly speaking, neither a model nor a theory. It is a *simulation*. Although this approach to knowledge is very fruitful, the INC cannot easily be replaced in a theoretical frame. A widely spread opinion is that INC calculations provide a numerical trick to solve the Boltzmann equation. To support this statement, one invokes the basic ingredients of the INC : classical scheme and free-space nucleon-nucleon cross section. Moreover, as in the Boltzmann approach, one disregards the modification of the one-body distribution function during the collision (or interaction) time τ_{int}. However, in the INC, two nucleons collide if they are close enough, an information which is basically contained in the *two-body* distribution function, a quantity which does not appear in the Boltzmann equation. Furthermore, the INC includes memory effects which are neglected in the Boltzmann equation. We briefly compare in Table II the assumptions made in the two approaches. We based our discussion on ref.[24]).

Table II. Comparison between Boltzmann Equation and INC

BE	INC
$\tau_{int} \lesssim \Delta t \ll \tau_{rel}$ $r_c^3 \rho \lesssim 1$	$\tau_{int} \lesssim \Delta t$ $r_c^3 \rho \lesssim 1$
Markovian	Non necessary
Local in space Homogeneous	Local in space Non necessary
No effect of $f_3, f_4 \ldots$ $f_2 = f_1 f_1$	No effect of $f_3, f_4 \ldots$ Yes (second category) No (first category)
τ_{rel} = relaxation time, r_c = interaction radius, ρ = density	

Like the Boltzmann equation, the INC introduces irreversibility (see ref.[3]) and below). Hence, very likely, it solves a *kinetic equation* which is more general than the Boltzmann equation. It would be desirable to know which one. Without too much optimism, we think that could be achieved in a way similar to the derivation of a transport equation in the deep inelastic collisions, as proposed by Weidenmüller and his collaborators[25]). Basically, they formally solve the equation for the density matrix and obtain an infinite series in the powers of the interaction, which they consider as a stochastic variable. They perform ensemble average analytically and get a kinetic equation. The same scheme would presumably be used in our problem. Performing ensemble averages numerically then appears as a way to solve the corresponding kinetic equation.

The INC may also include some quantal effects, which are undeterministic in nature[26]. This point, however, needs further clarification.

Let us briefly discuss in which domain the conditions of validity of the INC are satisfied. For the classical description to be valid, one has to require at least that an incident nucleon can resolve the particles in the target. In other words, the de Broglie wavelength should be smaller than the average internuclear distance r_{12} :

$$\lambda\!\!\!^{-} = \frac{\hbar}{p_{lab}} \ll r_{12} \quad . \tag{3.1}$$

For normal nuclear density, r_{12} is of the order ~ 2 fm . Hence, (3.1) is easily verified for incident energies of, say, $\gtrsim 250$ MeV per nucleon. Actually, condition (3.1) is verified at even lower energies, but one has to keep in mind that conditions are changing (slowing down of the particles, compression...) during the collision process. So, it is safe to choose the above mentioned limit. Furthermore, this energy roughly corresponds to having two disjoint Fermi spheres in the entrance channel, making the question of quantum statistics less critical.

The INC only treats time variation with characteristic intervals Δt at least larger than the interaction time τ_{int} , which requires the low density condition[24]

$$X = r_c^3 \rho \lesssim 1 \quad . \tag{3.2}$$

If we consider r_c as related to the total nucleon-nucleon cross section by the relation

$$\pi r_c^2 = \sigma_{tot} \quad , \tag{3.3}$$

we can evaluate X for an incident ion of $E_{beam} > 800$ MeV : $X \approx 0.24$ and condition (3.2) is satisfied. During the collision process, the parameters are changing and relation (3.2) may be marginally satisfied. At low energy ($E_{beam} \lesssim 200$ MeV) , condition (3.2) breaks down even in the initial stage.

We also want to say a few words about the connection with hydrodynamics. It is well known that the hydrodynamical equations deal with the first moments (in $\vec{p}$) of $f_1(\vec{r},\vec{p},t)$. Their conditions of validity are rather badly known. However, it is quite clear that if hydrodynamics is valid, it can only describe disturbances with characteristic variation length larger than the mean free path. The latter simply writes

$$\Lambda = \frac{1}{\rho\sigma} = \frac{1}{\rho\pi r_c^2} = X^{-1}\frac{r_c}{\pi} \quad . \tag{3.4}$$

With the above conditions ($E_{beam} > 800$ MeV , normal density), $\Lambda \approx 1.5$ fm . Moreover, if we consider a nucleon hitting a cold nucleus, what matters, is not Λ , which is the mean free path for making a collision, but Λ_s , which is the mean free path for bringing the nucleon momentum down to the mean target nucleon momentum. Λ_s may be much larger than Λ , as already emphasized in ref.[27]. Hydrodynamics is then hardly applicable in the considered energy range. Artifices like two fluids[28] or three fluids[29] to circumvent long mean free path situations may be of interest. Let us finally notice that hydrodynamics may be valid when condition (3.2) breaks down.

4. A review of the results

4.1. OBSERVABLE QUANTITIES

The invariant nucleon production cross section is related to the one-body distribution function by :

$$E\frac{d^3\sigma}{dp^3} = \lim_{t\to\infty} \int_0^{b_{max}} 2\pi b db \int d^3\vec{r}\; E\, f_1(\vec{r},\vec{p},b,t) \quad , \tag{4.1}$$

where we have made explicit the dependence upon the impact parameter b . If the INC code does not treat composites explicitly, the cross section refers to all the nucleons, including those lying in the composites. The results of the INC calculations for proton production compare remarkably well to experiment as testified by refs.[4,5,7,9,10]. Predictions are also quite good for neutrons[30] (provided that composite formation is taken into account) and moderately good for the pions[4,9,10,31].

Another largely investigated observable[32] is the correlation yield between two protons. It is related to the two-body distribution function by

$$C(\vec{p}_1,\vec{p}_2) = \lim_{t\to\infty} \frac{\int 2\pi b db \int d^3\vec{r}_1\, d^3\vec{r}_2\, f_2(\vec{r}_1,\vec{p}_1,\vec{r}_2,\vec{p}_2,b,t)}{\int 2\pi b db \int d^3\vec{r}_1\, d^3\vec{p}_1\, f_2(\vec{r}_1,\vec{p}_1,b,t)} \quad . \tag{4.2}$$

In general, not all the six variables are measured. The refs.[4,10] contain several examples, where the agreement between predictions and experimental data are quite satisfactory.

Let us finally mention the observables obtained by global event analysis, which are already measured in streamer chambers[33] and which are going to be measured extensively in new detectors like the plastic ball, Diogène, ... Generally speaking, they are defined in terms of a momentum tensor $Q_{\mu\nu}$, which, for an event, is :

$$Q_{\mu\nu}(\omega) = \sum_i \gamma(p_i)\, p_i^\mu\, p_i^\nu \quad , \tag{4.3}$$

where i runs over the fragments and μ over the Cartesian coordinates. We draw the attention to the special weight $\gamma = A_i^{-1}$, the inverse of the mass number of the fragments; it renders $Q_{\mu\nu}$ invariant under clusterization. In other words, whether a deuteron is counted as a deuteron or two nucleons does not matter.

Determination of $Q_{\mu\nu}$ and of other related quantities (like the thrust, see below) requires very exclusive measurements. We are going to show that, *in the average*, they are just related to the one-body distribution function. Indeed, eq. (4.3) can be rewritten as (assuming nucleons only) :

$$Q_{\mu\nu}(\omega) = \lim_{t\to\infty} \int d^3\vec{x}_1\, d^3\vec{q}_1 \ldots d^3\vec{x}_A\, d^3\vec{q}_A \sum_{i=1}^{A} \gamma(q_i)\, q_i^\mu\, q_i^\nu\, (A!)^{-1} \sum_{i_1=1}^{A} \sum_{i_2(\neq i_1)} \times$$

$$\sum_{i_3(\neq i_2,\neq i_1)} \sum \ldots\, \delta(\vec{x}_1-\vec{r}_{i_1})\, \delta(\vec{q}_1-\vec{p}_{i_1}) \ldots\, \delta(\vec{x}_A-\vec{r}_{i_A})\, \delta(\vec{q}_A-\vec{p}_{i_A}) \quad . \tag{4.4}$$

Of course, one has

$$<Q_{\mu\nu}> = N_\omega^{-1} \int Q_{\mu\nu}(\omega)\, d\omega = \lim_{t\to\infty} (A!)^{-1} \int d^3\vec{x}_1\, d^3\vec{q}_1 \ldots d^3\vec{x}_A\, d^3\vec{q}_A \sum_{i=1}^{A} \gamma(q_i)\, q_i^\mu\, q_i^\nu \times$$

$$f_A(\vec{x}_1,\vec{q}_1,\ldots,\vec{x}_A,\vec{q}_A,t) \quad , \tag{4.5}$$

or, more simply, because of relations (2.2)-(2.4)

$$<Q_{\mu\nu}> = \lim_{t\to\infty} \int d^3\vec{r}\, d^3\vec{p}\, \gamma(p)\, p^\mu p^\nu\, f_1(\vec{r},\vec{p},t) \quad . \tag{4.6}$$

In the average, quantities provided by the global analysis techniques (except for correlations) can also be given by inclusive measurements. But, their fluctuations which require event by event analysis may contain valuable information (see below).

Clusters are in general not included explicitly in the INC. Hence, cluster formation cross section can be evaluated with additional assumptions only. The simplest of them is to consider that clusterization is just a small perturbation to the state of the system (described by the distribution functions) at the end of the interaction process. Let us consider a deuteron for simplicity. If the latter can be considered as a proton and a neutron very close to each other in phase space, we may write

$$\frac{d^3\sigma}{d\,\vec{P}_d^3} = \int 2\pi b db \int d^3\vec{R}\, d^3\vec{r}\, d^3\vec{p}\; f_2(\vec{R}-\frac{\vec{r}}{2}, \vec{P}_d-\vec{p}, \vec{R}+\frac{\vec{r}}{2}, \vec{P}_d+\vec{p})\; g(\vec{r},\vec{p}) \quad , \tag{4.7}$$

where $g(\vec{r},\vec{p})$ is the probability for a neutron-proton pair to make a deuteron when they have a relative momentum $\vec{p}$ and when they are at a relative distance $\vec{r}$. It can be identified to the Wigner representation of the deuteron wave function. More simply, it can be taken as

$$g(\vec{r},\vec{p}) = \theta(r_o-r)\ \theta(p_o-p) \quad , \tag{4.8}$$

where r_o and p_o are related to known properties of the deuteron. They are linked by the condition

$$\int g(\vec{r},\vec{p})\ d^3\vec{r}\ d^3\vec{p} = 1 \quad . \tag{4.9}$$

More elaborate arguments may be used[34]. Results[34] are very promising. As for the protons, the predictions should be compared to the total (free + those contained in heavier composites) deuteron cross sections.

4.2. NON-OBSERVABLE QUANTITIES

The simplest of these quantities is the matter density

$$\rho(\vec{r},\vec{t}) = \int d^3\vec{p}\ f_1(\vec{r},\vec{p},t) \quad , \tag{4.10}$$

which is of interest in relation with the possible formation of exotic phases. Figure 1 shows the maximum baryon density reached by the $^{40}Ca + {}^{40}Ca$ system du-

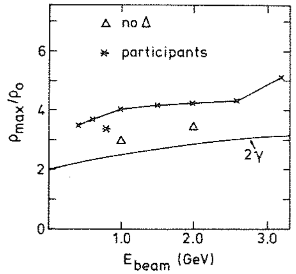

Fig. 1. Maximum baryon density (in the c.m. system) reached during central Ca+Ca collisions, according to INC calculations[3] (crosses). The triangles show the results when the isobar formation is neglected. The full line corresponds to a complete transparency of the nuclei. The star gives the maximum participant density.

ring central collisions, according to ref.[3]. The baryon density is not *a priori* an interesting quantity : the property of the matter may strongly depend upon the momentum content, i.e. the p-dependence of the one-body distribution function. Although there are sensible differences between INC calculations[4,12], they all predict strong compression (of the participants) with high "temperature" for a short time ($\sim 10^{-23}$ sec) . As illustrated by fig. 1, the production of Δ-isobars favours compression because kinetic energy is so transformed into mass energy. Yet, it is not enough to prevent a very fast decompression[3,12]. In a few (3-5) fm/c , the central density has dropped from ρ_{max} (see fig. 1) to normal or less than normal. This has important implications, as we will see later on.

INC calculations have also shown that the separation between spectators and participants is a reasonable picture of the process[4,35] and closely follows the clean-cut geometrical recipe, a feature which has been recently checked by experiment[36]. This suggests a rather small transverse momentum transfer, which probably comes from the forward-peaked nucleon-nucleon cross section σ_{NN} .

The concept of weak transverse communication may also be applied to the participants. In its extreme version, it is embodied by the rows on rows model : nucleons interact within tubes of cross section equal to σ_{NN} and the tubes do not interact with each other.

In an attempt to check this assumption, the INC calculations have been analyzed in terms of interacting clusters[37]). These entities are defined by fig. 2, which depicts an event as a succession of interactions within clusters which, on the other hand, have no interaction between each other. These clusters are composed of M nucleons of the projectile and of N nucleons of the target, which eventually exchange energy-momentum. With such a picture the total (also the inclusive) reaction cross section may be written as

$$\langle\nu\rangle\,\sigma^r = \sum_M \sum_N (M+N)\,\sigma_{MN} \quad , \tag{4.11}$$

where $\langle\nu\rangle$ is the average number of participants and where σ_{MN} is the cross section for having a cluster (M,N) . Note that a given event may contribute to several σ_{MN} . We may be interested to the mean cluster size

$$\langle M+N\rangle = \frac{\Sigma(M+N)\,\sigma_{MN}}{\Sigma\,\sigma_{MN}} \quad . \tag{4.12}$$

Not all the nucleon-nucleon collisions give rise to a large momentum transfer. A soft collision with small momentum transfer is not important for the equilibration process. Hence, expression (4.12) is meaningful if we only consider collisions with an energy momentum transfer t with $-t > q_c^2$, where q_c is some cut-off. Of course, expansion (4.11) depends upon the precise value of q_c . A reasonable value is the Fermi momentum $p_F = 270$ MeV/c . Figure 3 shows the results of such

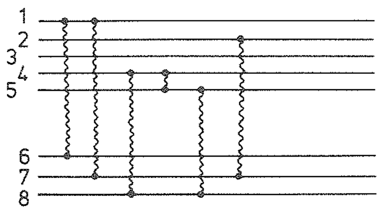

Fig. 2. Decomposition of a collision in two clusters (1,2,6,7) , (4,5,8) . The particle 3 is a spectator. See text.

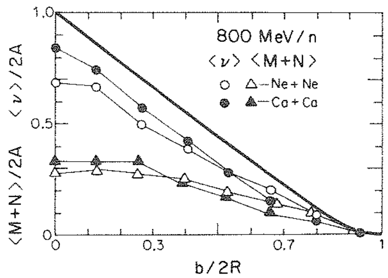

Fig. 3. Mean cluster size <M+N> and mean number of participants <ν> as functions of the impact parameter for Ne+Ne and Ca+Ca collisions. The full line indicates the value of <ν> resulting for the clean-cut separation (from ref. [37])).

an analysis for two systems at 800 MeV . The remarkable aspect is that the participants may be divided into 2 or 3 clusters with no interaction between each other.

Finally, the INC records the history of every nucleon and may thus provide the number of collisions it has undergone[5,38]). In the average, a nucleon makes ∿ 3 collisions in Ca + Ca at 800 MeV and ∿ 5 collisions in Ne + U at 250 MeV . This has important consequences on a possible equilibration of the system (see below). Let us mention that the number of collisions is widely spread and that single scattering has a non-negligible contribution.

5. Specific features of the INC

In this section, we want to point out some aspects which are contained in the INC and which are not present either in simplified models (e.g. thermal models), or in an hydrodynamical approach or in both.

5.1. DEPARTURE FROM GLOBAL EQUILIBRIUM

Grossly speaking, the participants form an equilibrated system, which can be characterized by a temperature parameter. However, measurements[39] show a departure from isotropy : a forward-backward peaking is observed even for high multiplicity events. This is consistent with a picture where the number of collisions undergone by the participants may be small. As shown in ref.[38], the single scattering component is still very forward-backward peaked (a trace left by the forward NN cross section), whereas multiple scattering component are more and more isotropic. The importance and the shape of the single scattering component also account for the shouldering of the 90° c.m. proton spectrum[40].

The two proton correlation measurements[32] are also indications of very direct (short) processes. There is however a discrepancy between the estimate of the clear knock-out contribution (two nucleons interact only once and do not interact with any other one) as extracted from these experiments[41] and the predictions for this process[37,42]. The explanation is perhaps that the correlation experiments count more than the clear knock-out process[38].

5.2. EFFECTIVE MEAN FREE PATH

Let us first emphasize that speaking of mean free path in the context of relativistic heavy ion collisions is rather inappropriate. Strictly speaking, the mean free path is a concept valid for a particle travelling in an homogeneous and extended medium, without perturbing it too much (f.i. without drastically changing the momentum distribution). This is certainly not the case in relativistic heavy ion collisions. The estimates of section 3 are relevant to the very initial stage of the process only. Perhaps it is more judicious to speak about effective mean free path λ_{eff} .

The latter quantity may be estimated from the spatial extension of the interacting clusters[37] (see section 4) (λ_{eff} = 3 fm for Ar + KCl at 800 MeV) , from the average number of collisions ($\lambda_{eff} \approx 2.5$ for the same system) or from the analysis of the correlation experiments[43] (λ_{eff} = 2.5 fm independent of the system at 800 MeV) . Here, we want to discuss this quantity in relation with two problems, which have been extensively studied in the last two years.

5.2.1. The entropy

Recently, Siemens and Kapusta[44] suggested that the entropy produced during the collision could be measured by the experimental deuteron to proton ratio R_{dp}. They found that the entropy determined by this way is too high compared to what is predicted by usual equations of state. In the meantime, their argument linking entropy and R_{dp} has been heavily criticized[45,46]. Nevertheless, it is interesting to investigate the question in the frame of the INC. The entropy is related to the one-body distribution function by

$$S/A = 1 - \frac{4}{A} \int d^3\vec{r}\, d^3\vec{p}\, f_1(\vec{r},\vec{p},t)\, \ell n[f_1(\vec{r},\vec{p},t)(2\pi\hbar)^3] \quad . \tag{5.1}$$

There is no time dependence after the last NN collisions. For central (b = 0) Ca + Ca collisions at E_{beam} = 800 MeV , an entropy S/A of 4.4 is found[47], much less than the "experimental value" 5.6 , but higher, by almost one unit, than the entropy of a free nucleon gas (with the same average temperature and density). The finite mean free path allows a larger spatial extension. If this interpretation is correct, the latter would be increasing with the impact parameter, since the mean free path is larger compared to the size of the participant system. This is corroborated by Table III.

Table III. Entropy in Ca + Ca , E_{beam} = 800 MeV (ref.[48])

b (fm)	0	2.05	4.1	6.15
S/A	4.4	4.75	5.2	6.3

5.2.2. The thrust

Jet analysis techniques can be used to study the momentum and/or mass flow after the collision. They may use the sphericity tensor $Q_{\mu\nu}$ (eq. (4.3)) or the thrust

$$T = \min_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} , \qquad (5.2)$$

where the sum runs over all the ejectiles. The thrust is invariant under clusterization. What we are going to discuss here is the direction of the thrust. In collisions of equal nuclei, it is expected to lie in the reaction plane (at least this is revealed by the calculations[48-50]) and is thus characterized by an angle θ, which gives the direction of the main flow. In fig. 4, we show the result of

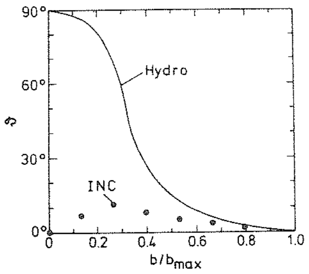

Fig. 4. Value of the thrust angle (see text) as given by the hydrodynamic calculation of ref.[49]) (full line) and by the INC (ref.[48])) (dots).

an INC calculation[48]) in comparison with an hydrodynamical approach[49]). In the latter, the vanishingly small mean free path forces the matter to escape at $90°$ in the c.m. system for central collisions. In a striking contrast, the INC predicts a thrust always pointing at forward angles, indicating an average main flow *in the beam direction*, something which requires a relatively long λ_{eff}. This prediction seems to agree with fragmentary results obtained with streamer chamber[51]).

5.3. COLLECTIVE FLOW

For asymmetric systems, the Frankfurt group suggested that during central collisions, the main flow is going sidewards[52]). High multiplicity Ne(E/A = 393 MeV) + U experiments indeed reveal a peak around $\theta_{lab} \approx 70°$ for low energy ($E < 100$ MeV) protons[53]). The INC model does not predict such an emission pattern[54]) with ordinary NN cross sections. INC results come closer to hydrodynamics if the cross section is increased by a factor ~ 3 [55]). However, it may appear that the depression of the proton cross section at small angles may be due to a higher clusterization[34]).

5.4. FLUCTUATIONS

The emission pattern may fluctuate very strongly from one event to the other, even for a given impact parameter, because of statistical and/or quantum fluctuations. We discuss here this point in relation with the direction of the thrust (section 5.2.2). For central collisions, INC predicts $\theta = 0$. This may cover two distinct situations, as depicted in fig. 5. In (a), two fragments are always poin-

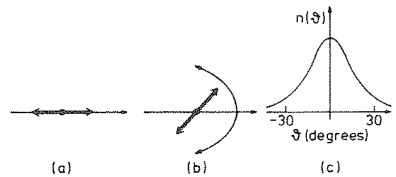

Fig. 5. (a),(b) : two different situations yielding a vanishing average thrust angle (see text). (c) : dispersion of the thrust angle in an INC calculation for central Ar+KCl collision at E_{beam} = 800 MeV .

ting exactly in the forward direction. In (b), two fragments are emitted back to back, but in a direction fluctuating randomly from $\theta = -90°$ to $\theta = 90°$. (Other interesting fluctuations may arise in the value of the thrust[55])). It is clear that the two patterns pertain to two different physical situations which, however, yield the same *average* thrust angle. The INC approach tends to indicate that the actual situation is neither (a) nor (b), as shown in fig. 5 (c) (aside from the fact that the big arrows in (a) and (b) must be replaced by a bunch of smaller ones with the same contribution). It is clear that the advent of 4π detectors will provide data about fluctuations, which cannot be handled by any approach, but the INC. Further theoretical investigations are needed before these fluctuations are interpreted in simple terms or with the help of a limited number of parameters, as it is the case for the deep inelastic collisions.

5.5. PRODUCTION OF COMPOSITES

This topic is related to the important question of the disassembly of hot nuclear matter. In a simple approach[57-59]), the composites are assumed to be in equilibrium with the nucleons at the freeze-out density. Calculations require knowledge of the (local) temperature. What we discuss above can hardly be compatible with such a mechanical equilibrium. Even if the latter is realized, the fast expansion of the system may prevent chemical equilibrium to settle. This is shown in fig. 6, which gives the results of a chemical evolution calculation assuming

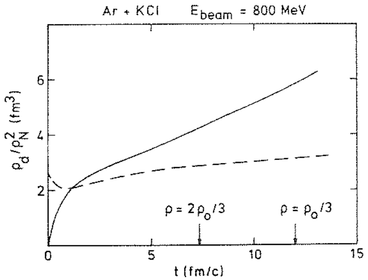

Fig. 6. Deuteron abundance as given by a chemical evolution calculation in an expanding fireball (full line, ref.[60])). The dashed curve gives the same quantity in an instantaneous equilibrium. The arrows indicate the range for the freeze-out density.

mechanical equilibrium at any moment[60]). The expansion rate is so fast that the deuteron abundance is significantly higher than the equilibrium value at the freeze-out density.

Off-equilibrium effects are probably important in composite production, as is suggested by the success of the coalescence model[61]) and of the model of section 4.1. The matter density of the system at the end of the interaction process strongly fluctuates from event to event. This aspect has to be included in a proper treatment of the disassembly of nuclear matter.

6. Limitations. Extension to low energy

The INC provides a good description of the bulk of the experimental data in the 250 MeV - 2 GeV range without introducing dense matter (equation of state) effects. But one has to admit that there is no real evidence for them, except perhaps in the pion yield close to the production threshold.

The INC approach is limited on the high energy side, because at some point the quark structure of the nucleons is expected to manifest itself in the process. Very exciting works have been done on the possible quark-gluon plasma[62]). It would be interesting to include these results in a scheme which manages possibly large off-equilibrium aspects.

The INC is limited on the low-energy side, because of the quantum motion of the nucleons, which make them able to interact coherently. In the extreme low-energy regime, TDHF seems to be the best approach (although it neglects fluctuations). Attempts to include collision terms[63,64]) seem promising. The connection with the INC is still nonexistant. To our opinion, the question is not to find where the first ceases to be valid and leaves the place to the other, but rather to create a larger scheme which will allow a gradual decrease of potential effect and the gradual manifestation of the on-shell collision regime.

Nevertheless, the INC may be useful for different purposes :

(1) the investigation of the low-energy limit for the validity of the spectator-participant picture. Some experimental information is already available[65].

Also, the fragmentation process may depend on the initial disturbance giving birth to a cold abraded pre-fragment[66]. The INC may provide useful information about this initial state.

(2) The investigation of the transition from explosion at high energy to fission at low energy. Apparently, in intermediate conditions (^{12}C beam of 96 MeV per particle), a few fast particles are emitted from a fusing system[67]. This process presumably presents a fluctuating pattern, a feature that is inherent to the INC appraoch. However, prior to application, numerical codes should be adapted to properly account for the Coulomb forces and for the Pauli principle.

(3) The study of the disassembly of hot matter. The intermediate energy range corresponds to a rather low temperature (a few MeV) and a high clusterization degree. This situation will provide a check of the ideas on the disassembly of nuclear matter, which are still in their native form.

7. Conclusion

The INC has been very fruitful in explaining the data in the 250 MeV - 2 GeV energy range. The bulk of the data is consistent with the collision regime. This, however, does not completely rule out dense matter effects. But the latter, if they exist, probably leave very small traces in the data.

Some of the experimental data are reproduced by other approaches, which have the appealing feature of requiring less computational efforts. However, according to us, the INC will remain a unique tool to study and analyze :

(1) off-equilibrium effects, which are especially related to finite particle number.

(2) Finite mean free path effects, as they are revealed in global variables.

(3) Fluctuations, which are probably important in the analysis and/or the prediction of some observables, among which is the clusterization degree.

Some of these effects are probably active in the intermediate energy domain. Therefore, the INC is called to play a certain role in the study of heavy ion experiments in this energy range, although the dynamics is less dominated by the collision regime.

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