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PION PRODUCTION IN CENTRAL HIGH ENERGY NUCLEAR COLLISIONS

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Abstract: A model for pion production is studied in the context of a cascade calculation. The pions are produced through Δ-resonances which are allowed to decay. The emitted pions are assumed to interact with the other nucleons by forming new Δ-resonances. The time evolution of the pion and Δ-population is studied; it is found that Δs are always more numerous than pions during the sequence of baryon–baryon collisions. The spectrum of the pions is in considerably better agreement with experiment than the one obtained with frozen Δ-isobars. The presence of Δ-resonances appears to be important for the cooling of the pion system. The pion multiplicity is found to deviate from a Poisson distribution. The pion yield is overestimated by at least 25%; this result is discussed within the framework of conventional dynamics.

1. Introduction

In one of the first theoretical papers [1]) on relativistic nucleus–nucleus collisions, it has been suggested that the emitted pions could bear important indications on the reaction mechanism as well as on the nuclear equation of state. The first decade or so of experimental work at the Bevalac (and at Dubna) has brought an already important amount of data; for a review, see for instance ref. [2]). Although some theoretical analyses have included pion production, the main emphasis has been put on proton data and pion creation has often been treated in a rather crude way. We believe that this question should be investigated more carefully and our work aims at that goal. Before proceeding further, however, we want to make a short review of the available data, at the exclusion of the 0° and 180° pions for which the production mechanism is probably different from the one which dominates at large angles.

High energy π^+ and π^- data in the mid-rapidity region show an almost isotropic production and an exponential fall-off [3,4]). The associated "temperature" is lower than the one extracted from the proton spectra.

Low energy pion spectra in the mid-rapidity region show strong deviations from a pure thermal law [5–7]). An extensive measurement of negative pions down to ~20 MeV in the Ne+NaF system at 800 MeV/A exhibits however an almost perfect exponential decrease.

Streamer chamber experiments have revealed two important results. The average pion yield has been studied as a function of the incident energy, for the Ar+KCl system [8]). For this system [8]) and for the $Ar+Pb_3O_4$ system [9]), the π-multiplicity distribution has been extracted; it seems to follow a Poisson distribution.

Let us mention for the sake of completeness that two-pion correlations measurements have been performed. According to several authors [10,11]), these experiments could bring information about the space–time evolution of the pion source. As far as we know, the size of the pion source determined in this way is comparable with the nuclear size.

The models which have been used to analyze or explain the data may be organized in three main categories:

(*i*) *The hard scattering model.* The basic mechanism is the production of a pion in an inelastic nucleon–nucleon collision. In its extreme form [12]), this mechanism is not iterated. The model is probably quite realistic for forward production [13,14]) (the so-called fragmentation region), but it can hardly pretend to describe large angle production, where multiple scattering effects are probably important [15]);

(*ii*) *Thermal models* [16–19]). These models state that the collision creates a pion gas in thermal and chemical equilibrium with the nuclear gas. The main merit of these models is to correctly reproduce the shape of the high energy pion spectra ($\gtrsim$100 MeV). Also, if one considers an expanding fireball [18]), the difference between pion and proton temperatures may be accounted for. However, these models systematically overestimate, by a factor two or so, the production yield [16,19,20]), unless a freeze-out density of several times the normal nuclear matter density is used [19]);

(*iii*) *The isobar model.* In this model, the main pion source is provided by Δ-isobars produced during the sequence of nucleon–nucleon collisions and subsequently decaying. The model has been used in conjunction with cascade calculations [21–24]). It also offers [22]) an alternative explanation for the difference between pion and proton temperatures; it gives a pion yield in much better agreement with the data, although still too high, especially at low energy [22]). However, the calculated pion spectra at 90° c.m. exhibit some shoulder which is not seen in the data, at least in the extensive π^- data. The reason is easy to understand: a Δ-isobar at rest in the c.m. cannot produce a low-energy pion and thermal motion can only partly cure this situation.

We seem to face a dilemma where thermal models can reproduce the spectrum but not the pion yield and where isobar models can reproduce the pion yield but not the spectrum. Here, our purpose is to take over the cascade calculation of ref. [22]) with refined assumptions for pion production. In ref. [22]), the Δ-isobars were given a definite mass and a lifetime longer than the interaction time. Here, we introduce a spectrum for the isobars and a finite lifetime. Pions are then present in the system even during the interaction process.

The same production model is used in refs. [25,26]), but the spirit of the cascade calculation is different. In these works, the target is mainly considered as a medium which provides the cascade particles with a mean free path, while in our case all the sequence of two-body collisions is followed. Obviously, compression effects are neglected in one case and not in the other. In ref. [26]), the emphasis is put on the low energy positive pion cross section. In the present work we want to study several questions: can we reproduce at the same time the pion yield and the pion spectrum? Are there already many pions at the end of the interaction process, besides the remaining Δ-source? Do these two components in the pion yield have different properties?

In ref. [27]), the same model is investigated to study equilibration within a system composed of two infinite pieces of nuclear matter initially moving in opposite directions. Our calculation, however, includes compression and finite size effects.

The rest of the paper is organized as follows. In sect. 2, we describe the main features of the calculation. Sect. 3 is devoted to the analysis of the time evolution of the chemical composition of the system; the correlation between this evolution and the matter density of the system is studied. Sect. 4 contains our results concerning pion yield and pion multiplicity; the properties of the two pion components are investigated. Finally, our conclusion is presented in sect. 5.

2. Description of the calculation

We have refined the computing code already used in refs. [15,21,22]) and extensively described in ref. [22]). It suffices here to say that the collision process is treated as a succession of on-shell relativistic classical baryon-baryon collisions. The pion production is accounted for by allowing production and destruction of Δ-isobars. Contrarily to what was done previously, we now give the Δ-isobar a finite lifetime and a non-zero width. More precisely, we explicitly take the following reaction into account:

$$\mathrm{N}+\mathrm{N} \rightleftarrows \mathrm{N}+\mathrm{N}\,, \qquad \mathrm{N}+\mathrm{N} \rightleftarrows \mathrm{N}+\Delta\,, \qquad \Delta \rightleftarrows \pi+\mathrm{N}\,. \tag{2.1}$$

We neglect the $\mathrm{N}\Delta \rightleftarrows \Delta\Delta$ reaction because of the lack of experimental information on the cross section. We will come back to this point in sect. 4. Let us now give more detail on the handling of the reactions. In the cascade process any pair of nucleons within interaction distance is allowed to scatter elastically or inelastically: this is chosen at random with weights proportional to the experimental elastic and inelastic cross sections at the total c.m. energy of the pair. When an inelastic collision occurs, the mass of the Δ is chosen randomly according to a lorentzian distribution, subject to the following requirement: if the mass of the Δ is kinematically forbidden, a new random choice is performed and repeated if necessary until the value is consistent with the kinematical constraints. The lorentzian distribution is centered on $E_0 = 1232$ MeV and the width is taken as the natural width $\Gamma = 112$ MeV; it is also cut on the low energy side at the pion threshold. Let us finally mention that the angular distribution for the Δ-production has been taken isotropic, which is reasonable because of the proximity of the threshold. Furthermore, it has been shown in ref. [22]) that an acceptable anisotropic angular distribution does not practically change results.

The recombination $N+\Delta \rightarrow N+N$ cross section is determined by detailed balance.

Once a Δ is formed, its effective lifetime is chosen randomly according to the exponential law $e^{-\Gamma\tau}$, where τ is the proper time of the particle. After this duration, the Δ is forced to decay isotropically (in its c.m.) into a pion and a nucleon. The pion is assumed to propagate freely until it comes close to a nucleon: if the minimum distance squared (and multiplied by π) is smaller than the formation cross section, the pion and the nucleon disappear and give birth to a Δ-isobar at their c.m. The mass of the Δ is uniquely determined by the kinematics. The formation cross section is chosen as

$$\sigma = \frac{126.2}{1+4(E-E_0)/\Gamma)^2}\ \text{mb}\,, \tag{2.2}$$

where E is the c.m. energy and $E_0 = 1232$ MeV. It fits the (isospin averaged) experimental elastic π-nucleon cross section [27]) between $E=E_0-\Gamma$ and $E=E_0+\Gamma$; the non-resonant scattering outside these boundaries is neglected, as well as the relativity weak energy dependence of the width Γ. Both effects are expected to change the pion yield, on which we will focus our attention, by a few percent at the most.

The collision process is followed until the rate of the reactions (2.1) becomes negligible.

Let us finally mention that we do not distinguish between members of the same isospin multiplet (neutron and proton; negative, neutral, and positive pions). Since we will be considering symmetrical ($N \simeq Z$) systems, all members inside a multiplet are equally represented.

3. Time evolution of the chemical composition

We will concentrate here on central collisions of two ^{40}Ca nuclei. Many experimental data exist [8]) for the neighbouring system Ar+KCl at incident kinetic energies per nucleon E_{beam}/A ranging from 0.4 to 1.8 GeV. In particular, negative pion kinetic energy and multiplicity has been carefully investigated at 1.8 GeV.

Fig. 1 shows the time evolution of the numbers of Δ's and of the number of pions. For practical reasons, the nuclei are not exactly touching each other at the origin of the time scale, but rather at $t \simeq 1.5$ fm/c. The top of the figure shows the cumulated number of baryon-baryon collisions. We can see that this quantity does not practically change at times larger than $t_0 \simeq 12$ fm/c. Hence, only Δ-decay and pion reabsorption are going on after this time.

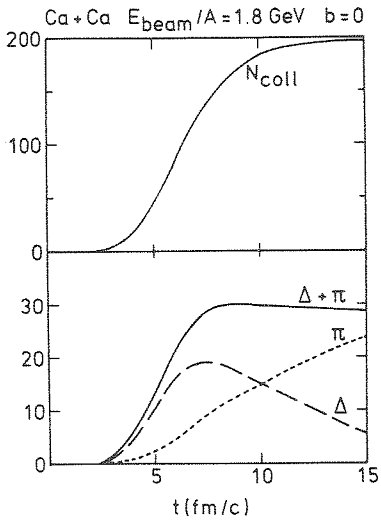

Fig. 1. Time evolution of the number of pions and Δ-resonances in central $^{40}Ca+^{40}Ca$ collisions at a beam energy of 1.8 GeV per nucleon. The cumulated number of binary collisions is shown for comparison.

Fig. 2 illustrates this situation in greater detail. The time variation of Δ's and pions due to the different elementary processes is displayed. The variation of the central density and the rate of baryon-baryon collisions are also shown. We may split the entire collision process in three stages:

(*a*) *The compression stage.* It lasts until $t \sim 7$ fm/c, for which time the number of binary collisions (the so-called luminosity) is the largest. During this stage, Δ-isobars are created abundantly, owing to the large NN inelastic cross section. They partially disappear by decay. Their destruction by collisions with nucleons sets in a little bit late, because the corresponding cross section is large when the Δ-nucleon relative velocity is small: one needs an appreciable population of Δ's and nucleons in the mid-rapidity region; as shown in ref. [22]), this happens around $t \sim 7$–8 fm/c.

(*b*) *The decompression stage*, which lasts until $t_0 \sim 12$ fm/c. It is characterized by a very fast expansion of the system: in 5 fm/c, the central density has fallen from 3.5 to 1 in units of normal nuclear matter density. The binary collisions, including Δ-production and recombination ($N\Delta \to NN$) are practically extinct at the end of this stage.

(*c*) *The pion-nuclear matter interaction stage.* After $t_0 \sim 12$ fm/c, the state of the system is modified through Δ-decays and subsequent pion reabsorption. It lasts until $t \sim 20$ fm/c.

Stages (a) and (b) are characterized by the presence of many Δ-resonances (compared to free pions). During stage (c), the Δ-particles are releasing pions,

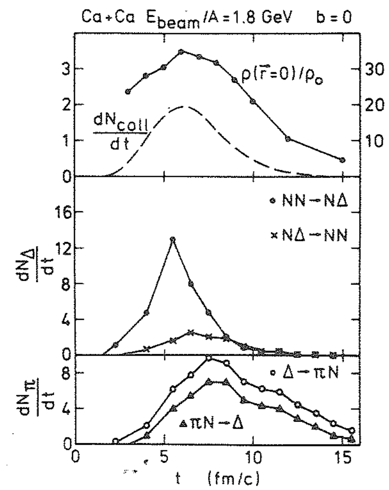

Fig. 2. Gain and loss terms contributing to the time-variation of Δ-resonances and pions in central $^{40}Ca+^{40}Ca$ collisions at a beam energy of 1.8 GeV per nucleon. The corresponding processes are indicated. The rate of binary collisions (scale on the right) and the central density (scale on the left) are given at the top of the figure, for comparison. The density is in fact the average baryon density in a sphere of 2 fm around the center of mass of the system. The quantity ρ_0 is the normal nuclear matter density ($=0.17\ fm^{-3}$).

which are partly reabsorbed. The pion reabsorption lasts longer than the other binary reactions because of a much larger cross section.

From a theoretical point of view, it can be of interest to distinguish between

(*i*) *Direct pions*: those present at the end of stage (b) and which have interacted with the compressed strongly interacting nuclear matter.

(*ii*) *Non-direct pions*, which appear later and which interact with a more or less dilute nuclear system. Of course, the separation is not sharply defined, since t_0 is not precisely determined. Later on, we will investigate a little bit the possibly different properties of the two kinds of pions.

It is interesting at this point to compare our results with the ones obtained by Montvay and Zimanyi [29]) in a kind of thermal model. The main difference lies in the rate of expansion of the system. They assumed an even faster expansion than the one obtained in our calculation; their expansion time is $\sim 3\ fm/c$, which renders their approach questionable: off-equilibrium effects are expected to be large. As a consequence, they obtained a tremendously fast cooling of the system; nevertheless, the delta and pion populations are quite similar in both calculations: N_π/N_Δ is around 0.3 at the time of maximum density and reaches unity $\sim 3\ fm/c$ later. A common feature of the two calculations is the fact that the sum of pions (free+contained in Δ's) is roughly constant beyond the time of maximum compression.

4. Properties of the final pions

We will concentrate here on the properties of the final pions, i.e. after the decay of all Δ-resonances. Occasionally, we will discuss the properties of the pions present at the end of the decomposition stage (b) (time t_0).

Table 1 shows the average number of pions and deltas present at t_0 and at 20 fm/c. Interestingly enough the percentage of pions present at t_0 (the direct pions) compared to the total number of pions (free + those enclosed in the Δ's) is fairly constant in energy. At 20 fm/c, a huge fraction of the pions are free. This fraction increases with the beam energy, because the expansion becomes faster and faster.

TABLE 1

Average number of Δ's and pions at time $t = t_0$ (see text) and time $t = 20$ fm/c (last two columns)

E_{beam}/A (GeV)	t_0 fm/c	N_Δ	N_π	N_Δ/N_π	$(N_\Delta+N_\pi)_{20}$	$(N_\Delta/N_\pi)_{20}$
0.4	~15	1.9	3.65	0.52	5.3	0.20
0.6	~15	3.5	6.4	0.54	9.7	0.13
1.0	~13.5	6.5	10.9	0.59	16.65	0.08
1.8	~12	10.7	18.4	0.58	28.85	0.06

Let us now look at the spectrum of the pions. In fig. 3, we have plotted the calculated invariant cross section for π^- production at 90° c.m. in central collisions at $E_{beam}/A = 1.8$ GeV. We have compared the total cross section predicted by the model of sect. 2 with the one predicted by a model in which the Δ's have a fixed mass of 1232 GeV and are stable during the cascade process. The latter calculation (open boxes in fig. 3) have more fluctuations because of a poorer statistics (20 runs per impact parameter instead of 80 runs), but it definitely shows a dip at small energy and a rather sharp fall-off at high energy. As we explained previously, this is the typical feature of the decay of a Δ-resonance partly smoothed by random motion. On the opposite, the spectrum calculated in the framework of the model of sect. 2 is pretty thermal-like, within the uncertainties of the calculation. The temperature parameter that can be extracted is $T = 102$ MeV, in good agreement with experiment [4]), considering the fact that we are dealing with a heavier system (Ar + KCl instead of Ne + NaF in this reference) and the fact that higher impact parameters steepen the curve in general. In fig. 3, we also give the spectrum of the direct pions, i.e. the spectrum of the free pions at $t = t_0$. The last curve (open circles) shows the spectrum of the pions which are trapped into the Δ-resonances at $t = t_0$ (the non-direct pions), as it would appear if these particles decayed at t_0. We give this result for the sake of comparison with the predictions of a frozen Δ-model. None of these two spectra shows a dip at low energy. Hence, we have to conclude that the dip obtained in ref. [21]) mainly comes from the fixed mass of the Δ and not from the freezing of the Δ-resonances with respect to their decay (see also below).

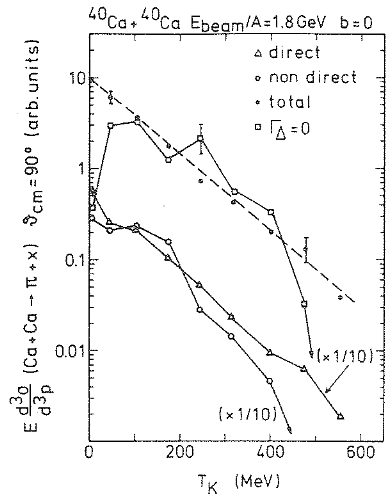

Fig. 3. Invariant inclusive cross section at 90° c.m. for the direct (triangles), non-direct (open circles) pions in central $^{40}Ca+^{40}Ca$ collisions at a beam energy of 1.8 GeV per nucleon. The total cross section (final pions) corresponds to the black points and is compared with the results obtained with a model (open squares) where the Δ-resonances are frozen. The long dashes are an exponential function fit to the total cross sections. Error bars indicate typical uncertainties in the calculation.

The similarity between the final pion spectrum and the direct pions is remarkable. Both can be roughly characterized by the same slope (or temperature) parameter: $T \simeq 102$ MeV, which turns to be smaller, by ~55 MeV, than the calculated proton temperature parameter. (All the temperatures mentioned in this discussion refer to central collisions only. They are expected to be slightly larger than the temperatures extracted from cross sections integrated over all impact parameters. For the grazing collisions, more direct processes give contribution to small transverse momenta, decreasing the temperature.) According to ref. [18]), this is a signature of an expanding fireball, which contains nucleons and pions in thermal equilibrium. Such a picture is probably a rough approximation of the physical reality, but is qualitatively supported by our calculation. However, the non-direct pion spectrum can be regarded as the primordial spectrum of the pions which are created after t_0. It has not an exponential shape, but can tentatively be characterized by a slope parameter of $T \simeq 85$ MeV. Hence, we may conclude that the cooling of the pions is not due to an expansion of the fireball only, but also partly to the trace left by the Δ-resonances in the primordial spectrum. This property is also illustrated in table 2, which presents the average pion energy in the c.m. system. The direct pions are on the average more energetic than the non-direct ones. Also, the average energy of final pions is 311 MeV, which agrees well with the experimental value of 300 MeV for the negative pions in the Ar+KCl system [8]), considering the fact that the calculated average pion energy slightly decreases with the impact parameter.

TABLE 2

Average energy per pion (in MeV) in central Ca+Ca collisions for $E_{beam}/A = 1.8$ GeV

	Direct	Non-direct	Total
$\Gamma_\Delta \neq 0$	333	300	311
$\Gamma_\Delta = 0$			329

Our calculations (table 3) also show that, as observed in the experiment [8]), the average pion energy does not depend upon the pion multiplicity.

We give a short comparison with the model of ref. [22]), in which the Δ's are frozen. In this model, the pions have more energy (329 MeV instead of 320 MeV), but the total energy contained in the pion system is lower: 8.21 GeV instead of 8.97 GeV. This, of course, implies that the total number of pions is smaller (see below). We interpret this situation as due to the width of the mass distribution of the Δ-resonances in the present model. Production of Δ-resonances with small masses is allowed and increases the possibility of removing energy from the nucleon system. In turn, it produces pions with less kinetic energy.

TABLE 3

Average energy per pion (in MeV) in central collisions as a function of the pion multiplicity (Ca+Ca, $E_{beam}/A = 1.8$ GeV)

M_π	23	24	25	26	27	28	29	30	31	32
$\langle E_\pi \rangle$	334	318	293	315	309	298	303	310	312	326

More insight into the mass-distribution of Δ-particles which is actually involved is gained by looking at fig. 4. We have plotted the distribution of the mass of the Δ-resonances which generate the pions. For the direct pions, it is the mass of the Δ-resonances giving rise to these pions. The distribution of fig. 4 may be regarded as the mass-distribution required in a pure Δ-resonance model to generate the spectrum of fig. 3. We see that the collision process considerably broadens the initial distribution (indicated by the full curve in fig. 4), distorts it (small masses are enhanced and high masses are depleted) and shifts it towards the lower masses. We note that fig. 4 provides another answer to the problem raised in ref. [19]), namely: what is the Δ-mass spectrum required to explain the pion energy spectrum? It is claimed there that the distribution contains a huge peak at $m_\Delta \simeq 1120$ MeV flanked by a smaller one at 1230 MeV. We show here that a broader peak centered at ~1200 MeV might do the job as well and, in addition, we provide the physical basis for such a broad peak, namely the collision broadening.

Our model is able to make some prediction for the multiplicity distribution. In fig. 5, we show the latter for central ($b = 0$) collisions. Our model yields a distribution

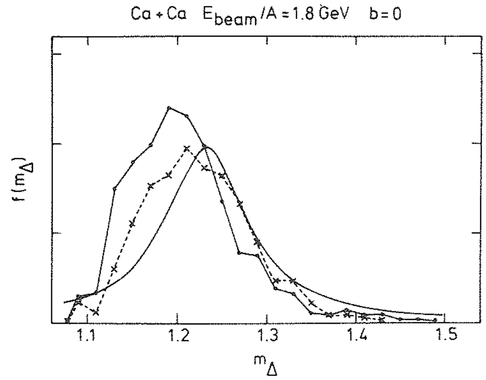

Fig. 4. Mass distribution of the Δ-resonances which have produced direct (crosses) and final pions (black dots). For comparison, the usual lorentzian shape of the (3, 3) resonance (full curve) is shown.

somehow different from a Poisson distribution. [When one averages over the small impact parameters, say from 0 to 2 fm, the distribution is closer to the Poisson distribution, as in the experimental one [8]).] This implies that, in our model, the pions are not produced fully randomly and independently from a system in equilibrium, but rather that they are produced with some constraint. It has been shown that energy momentum conservation [28]) narrows the distribution of pions emitted from an excited two-baryon system. In our case, this constraint is not strict, but our results seem to indicate that its effect remains sensitive. Of course refined criteria are needed for the selection of a narrow range of impact parameters before checking this property.

Let us now come to the important question of the total pion yield in central collisions. In fig. 6, the measured π^- yield [8]) is plotted as a function of the beam energy, together with our results. The latter are obtained by calculating with a good statistics (80 runs) for $b = 0$ and a twice less good statistics for $b = 2.5$ fm.

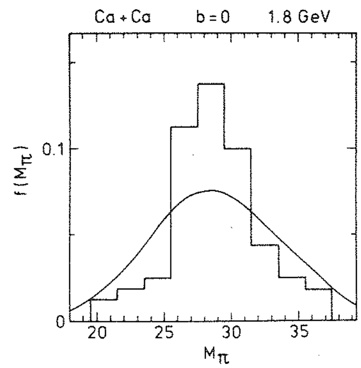

Fig. 5. Total pion multiplicity distribution for ($b = 0$) central $^{40}Ca + ^{40}Ca$ collisions at an incident kinetic energy of 1.8 GeV per nucleon (histogram) compared with a Poisson distribution of same mean.

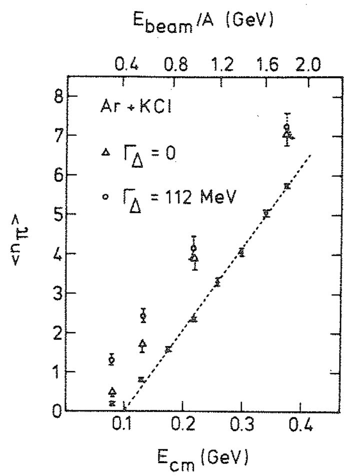

Fig. 6. Negative pion yield as predicted by the model (open circles) compared to the streamer chamber measurements of ref. [8]) for central trigger and Ar+KCl collisions. The numerical uncertainties in the calculation are shown. The results obtained within a model with frozen Δ-isobars are indicated by open triangles.

We interpolate between the two values and make an integration up to 2.5 fm covering the range estimated by the authors of ref. [8]) for their "central trigger". The uncertainty in the calculation comes from the statistics and the interpolation. We overpredict the π^- yield by a substantial fraction, although our results come off better than usual thermal models which are in general out by a factor of two at all energies. Also, the effective threshold seems to be shifted to lower energy as compared to experiment. It is worth mentioning that the pure Δ-model of ref. [22]) gives better results and this observation is in keeping with the discussion above. Let us recall, however, that the improvement was obtained at the expense of a bad pion spectrum at low energy.

We have addressed to ourselves the question of what could be changed reasonably in our model in order to improve our results. The relevant quantities in that respect are the Δ-formation cross section σ_F, the lifetime of the Δ-particle τ_Δ and the $\pi + N \rightarrow \Delta$ cross section σ_R.

The crucial quantity would seem to be σ_F, since it controls the path by which energy is removed from the nucleon system and eventually appears in the form of pions. However, through the principle of detailed balance, the same quantity controls the rate at which Δ's are destroyed by collisions with the nucleons. We have made the calculation with a modified shape of σ_F in order to illustrate the sensitivity of the pion yield with respect to this quantity. First we have cut the inelastic cross section below a c.m. energy of 2.17 GeV, which roughly corresponds to the sum of the nucleon mass and the average Δ-mass. We still get a pion yield linear in energy but with a threshold which is shifted towards the high energies by ~50 MeV in the total c.m. frame. Actually, this result is close to the prediction of the frozen Δ-model. Second, we have diminished σ_F to two thirds of its initial value. At $E_{beam}/A = 1.8$ GeV, the average π^- yield is reduced by 15%. These changes, which are not yet sufficient, are beyond the experimental uncertainties on the inelastic NN cross section. They can be justified by very strong medium effects, if the latter exist.

The other two quantities τ_Δ and σ_R are less important since they mainly influence the relative importance of the direct and non-direct pions. They influence the pion yield indirectly only. If more Δ's are present during the decompression stage for a fixed cumulated number of Δ's and π's, the final number of pions can be reduced by the Δ-recombination at the end of the interaction process. We have tried to quantify this consideration by multiplying the Δ-lifetime by a factor five. The pion yield is lowered half-way between the prediction of our model and the Δ-resonance model of ref. [22]). As for σ_R, it is not reasonable to change it drastically. Hence, it does not provide a clue for reaching an agreement with the data.

4. Discussion – conclusion

We have studied a model for pion production in relativistic heavy ion reactions, which appears as the most realistic model that could reasonably be constructed within the framework of conventional physics. The main properties of the produced pions are reproduced, but the total yield is overestimated by 20–30% at least.

We want to discuss what is left over in our treatment. Within conventional physics, very few effects are neglected. For example, we have neglected the processes $N + \Delta \rightleftarrows \Delta + \Delta$. The direct process would in fact *increase* even more the pion yield and the reverse one is quite unimportant (compared to $N + \Delta \to N + N$ which also destroys Δ-resonances) because the Δ-density is smaller than the nucleon density. Also, we have assumed that the Δ-decay is isotropic while it is not in the real world because of the spin of the Δ. But this is unlikely to change the pion yield, since the pions will essentially keep the same energy.

The model is based on gas dynamics which disregards composite fragments. For central collisions at $E_{beam}/A = 1.8$ GeV, the yield of composites is rather small [8]). Furthermore, this effect could not help very much in reducing the pion yield. Indeed, what really matters is how much energy is removed from the nucleons and appears under pions. The success of the coalescence model [31]) suggests that composites are formed when nucleons are close to each other in phase space. This does not perturb very much the momentum distribution of the nucleons and, hence, the kinematics associated with the pion production.

Another uncertainty of the model is connected with its lack of Lorentz invariance. This was already discussed in ref. [22]). Relativistic kinematics is used, of course, but the one-time nature of the model violates Lorentz invariance. However, it has been checked [22]), this gives rise to an uncertainty of a few percent for the observables.

We cannot preclude the possibility that all these effects cooperate to bring the calculated pion yield closer to the experimental values. But we cannot preclude either that, owing to size of the discrepancy, some room is left for unconventional physics in pion production. We list four effects in this perspective; at the same time, this will serve to delineate the boundary between conventional and unconventional physics.

(i) The Pauli blocking of Δ-resonance: when the latter is embedded in a nuclear medium, the decay is disfavoured. But this effect is partly compensated by the boson nature of the pions: when a pion state is occupied, the production probability to this state is enhanced. The boson enhancement is certainly smaller than the fermion reduction. Note, however, that both effects are probably unimportant because the occupation of the system in phase space is low, in the mid-rapidity region [32]), which is the relevant region for the Δ-resonances.

(ii) When the pions are decoupled from the nucleon system, i.e. when they leave the interaction region which is responsible for their creation and their destruction, they are experiencing an overall potential field; in terms of statistical mechanics, the pions feel a chemical potential. In the simplest form, the latter would introduce a factor $\exp(\mu/T)$ in the pion yield, if one can consider a thermal equilibrium situation. In Ar + KCl, at 1.8 GeV/A, the "temperature" is of the order of 100 MeV. We need a correction factor of ~ 0.80, which yields $\mu \simeq -22$ MeV. The negative sign is in keeping with the attractive interaction between a pion and a nuclear medium. But it is hard to attach any meaning to the precise value since, first, once again, thermal and especially chemical equilibrium is not reached; secondly, we have no precise idea to which density it corresponds.

(iii) Coherence effects may be present. By this, we mean that the pion birth place may not be localized precisely and implies several nucleons as a whole.

(iv) The Δ-production cross section may be changed because of the surrounding medium.

To summarize our work, we have included in a cascade calculation a mechanism which allows for production of pions during the collision process. The main results are:

(a) Half of the pions remain trapped into the Δ-resonances until the baryon-baryon collison rate vanishes;

(b) The spectrum of the pions is correctly reproduced;

(c) The average energy of direct pions is larger than the one of the non-direct ones. The latter are thus important for the observable cooling of the pions compared to the protons. In fact, they seem to be as important as the expansion mechanism proposed by Siemens and Rasmussen;

(d) The pion multiplicity distribution for a central collision deviates from a Poisson distribution;

(e) The π^- yield is overestimated over the range of E_{beam}/A from 0.4 to 1.8 GeV/A.

We have developed a model of classical type for pion production, including a number of refinements for the treatment of the particle collisions and decays. The model predictions come out reasonably close to experiment but some discrepancies remain. The latter call for some further refinements of the model, such as the inclusion of composite particles and of finer details of the Δ-behavior. They indicate that pion production is sensitive to many effects, in contradistinction with the proton case, where the simple gas dynamics gives right away a satisfactory description.

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