THE PARTICLE–VIBRATION COUPLING FORM FACTOR

J. CUGNON [1]

Kellog Radiation Laboratory, California Institute of Technology,
Pasadena, CA 91125, USA

and

O. HAROUNA

Université de Liège, Institut de Physique, B4000 Liège 1, Belgium

In the framework of a simple model, it is shown that the particle–vibration coupling form factor is intermediate betwee[n] the so-called surface and volume forms. A simple parametrization of our results is also given.

The coupling of a valence (or incoming) nucleon with the vibrations of the core (or of the whole nucleus) has been studied extensively. This coupling plays an important role in many nuclear phenomena, like the contraction of the valence orbits, which has recently received attention [1], and the radiative capture of 10–20 MeV nucleons, which is also of current interest. The theoretical approach to this coupling has nowadays reached a high level of sophistication, where both the valence nucleon motion and the vibration are treated microscopically [2,3]. However, for many purposes, the vibration can be treated macroscopically as a phonon field. This is especially true for the investigation of the radiative capture, and also of the deep inelastic collisions within the model elaborated by Broglia et al. [4]. In this simplified picture of the particle–vibration interaction, one has to define a form factor, which in a sense describes how vibration is excited when the distance between the nucleon and the core is varied. For E1, $T = 1$ vibration, this form factor is in general taken as the derivative of the core density. One then speaks of a surface peaked form factor. This choice relies on a simple hydrodynamical or macroscopic model [5,6]. However, it has recently

been argued [7,8], in relation with the semi-direct model of the fast neutron capture, that a volume form factor would be preferable. The experimental test of the two forms is obscured by the introduction of an imaginary particle–vibration coupling, whose theoretical foundation is not yet established. It is our purpose to investigate this form factor in the frame of a simple model which had been introduced in ref. [5]. The idea is to write the interaction in a microscopic and in a macroscopic form and to identify the matrix elements of the two forms between the ground state and the giant resonance state, the latter being interpreted as a one-phonon state.

To be more precise, the particle–vibration interaction has the following microscopic form:

$$H_{int} = \sum_i v(x, x_i), \tag{1}$$

where x represents the coordinates, including spin and isospin, of the valence (or incoming nucleon), and x_i those of the core nucleon i. For simplicity, we assume a static, zero-range interaction:

$$v(x, x_i) = \delta(r - r_i)$$
$$\times (P_0 + P_S \sigma \cdot \sigma_i + P_T \tau \cdot \tau_i + P_{TS} \sigma \cdot \sigma_i \tau \cdot \tau_i). \tag{2}$$

---

[1] On leave of absence from University of Liège, Belgium.

The interaction hamiltonian H_{int} is a scalar product of irreducible tensors relative to the valence nucleon, $\mathcal{M}_\mu$, and of irreducible tensors relative to the core, $A_\mu(r)$:

$$H_{int} = \sum_{\mu=LMTM_TSM_S} P^\mu A_\mu(r) \mathcal{M}_\mu, \tag{3a}$$

where

$$\mathcal{M}_\mu = Y^*_{LM}(\Omega) \otimes \xi_{SM_S} \otimes \eta_{TM_T}, \tag{3b}$$

$$\xi_{SM_S} = \eta_{TM_T} = 1, \quad \text{for } S=0, T=0, \tag{3c}$$

$$\xi_{SM_S} = \sigma_{M_S}, \quad \text{for } S=1,$$
$$\eta_{TM_T} = \tau_{M_T} \quad \text{for } T=1. \tag{3d}$$

The quantity P^μ depends only on the indices S, M_S, T, M_T of the set μ and $A_\mu(r)$ is given by:

$$A_\mu(r) = \sum_i \frac{\delta(r-r_i)}{r^2} \xi_{SM_S}(i) \eta_{TM_T}(i) Y_{LM}(\Omega_i). \tag{4}$$

We assume that for a given multipole μ, we can relate the giant resonance state $|G_\mu\rangle$ and the ground state by

$$|G_\mu\rangle = Q_\mu|0\rangle/\langle0|Q^\dagger_\mu Q_\mu|0\rangle^{1/2}, \tag{5}$$

where Q_μ is a one-body operator of multipolarity μ:

$$Q_\mu = \sum_i f(r_i) \xi_{SM_S}(i) \eta_{TM_T}(i) Y_{LM}(\Omega_i). \tag{6}$$

Approximation (5) amounts to assuming that the state $|G_\mu\rangle$ exhausts the non-energy-weighted sum rule for the operator Q_μ.

The macroscopic form of H_{int} is

$$H'_{int} = \sum_\mu k_\mu h_\mu(r) \mathcal{M}_\mu \alpha_\mu, \tag{7}$$

where α_μ is the collective coordinate associated with the vibration of the core, and where k_μ is a coupling constant. We make the assumption that the zero-phonon state is the ground state and the one-phonon state $|1\mu\rangle$ can be identified with the giant resonance state $|G_\mu\rangle$. Then, we require that

$$\langle G_\mu|H_{int}|0\rangle = \langle1\mu|H'_{int}|0\rangle, \tag{8}$$

or for the multipole μ

$$P^\mu\langle G_\mu|A_\mu(r)|0\rangle = k_\mu h_\mu(r)\langle1\mu|\alpha_\mu|0\rangle. \tag{9}$$

The next step is to assume that α_μ and Q_μ have the same matrix elements. If we choose k_μ equal to P^μ,

we then get the following expression for the form factor:

$$h_\mu(r) = \langle G_\mu|A_\mu(r)|0\rangle/\langle G_\mu|Q_\mu|0\rangle, \tag{10}$$

or, using eq. (5):

$$h_\mu(r) = \langle0|Q^\dagger_\mu A_\mu(r)|0\rangle/\langle0|Q^\dagger_\mu Q_\mu|0\rangle. \tag{11}$$

Noting that, according to eq. (4)

$$\int r^2 f(r) A_\mu(r) \, dr = Q_\mu, \tag{12}$$

we have the following normalization:

$$\int f(r) h_\mu(r) r^2 dr = 1. \tag{13}$$

We have evaluated (11) with a Slater determinant for the ground state wave function. As usual, we have taken Q_μ proportional to the electric multipole operator, $f(r) = r^L$. We give the final result only, the details of the calculation being of less interest:

$$h_\mu(r) = \frac{r^L \rho(r) - F_L(r)}{A\langle r^{2L}\rangle/4\pi - H_L}. \tag{14}$$

Here, $\rho(r)$ is the nucleon density of the core, A is the number of nucleons in the core and $\langle r^{2L}\rangle$ is the $2L$th moment of the density distribution. The function F_L is given by:

$$F_L(r) = \sum_{[a]} \sum_{[b]} \delta_{\tau_{3a}\tau_{3b}} \langle r^L\rangle_{ab} \phi_a(r)\phi_b(r) \frac{1}{4\pi}$$

$$\times (2l_a + 1)(2l_b + 1)(2j_a + 1)(2j_b + 1) \tag{15}$$

$$\times \begin{Bmatrix} j_a & L & j_b \\ l_b & 1/2 & l_a \end{Bmatrix}^2 \begin{vmatrix} l_a & L & l_b \\ 0 & 0 & 0 \end{vmatrix}^2,$$

where

$$\langle r^L\rangle_{ab} = \int_0^\infty r^{L+2} \phi_a(r)\phi_b(r) \, dr. \tag{16}$$

The sum $[a]$ runs over the occupied orbitals, neglecting their magnetic quantum number and $\phi_a(r)$ is the radial function in state a. The quantity H_L is defined by
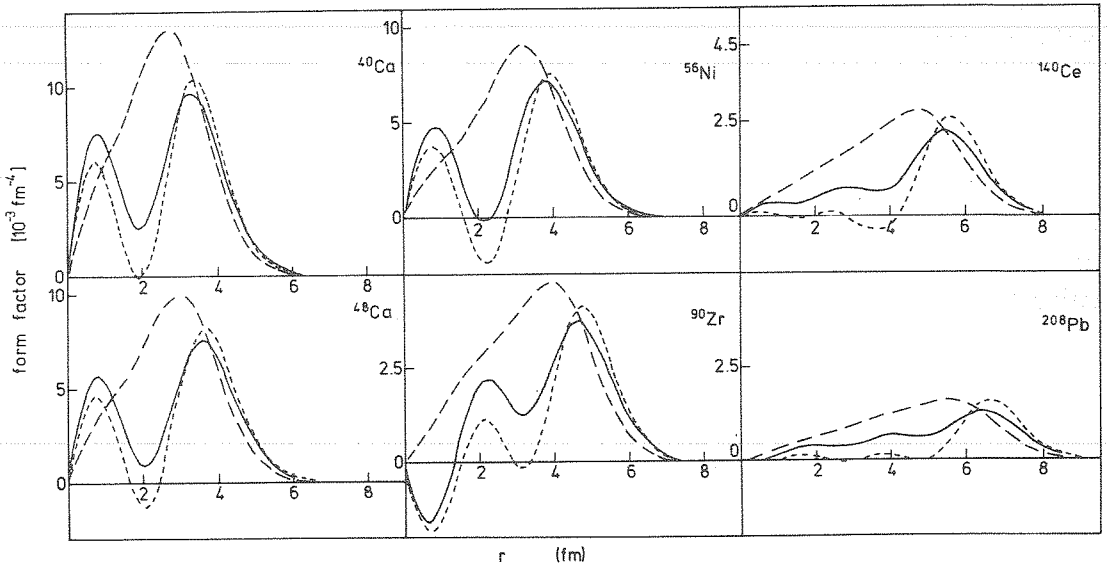
Fig. 1. Form factor for the $L = 1$, $T = 1$ multipoles. The full curve corresponds to our calculation. The long dashed curve is the vo ume form factor of ref. [6], while the short dashed curve is the surface form factor.

$$H_L = \sum_{[a]} \sum_{[b]} \delta_{\tau_{3a}\tau_{3b}} (\langle r^L \rangle_{ab})^2 \frac{1}{4\pi}$$

$$\times (2l_a + 1)(2l_b + 1)(2j_a + 1)(2j_b + 1) \qquad (17)$$

$$\times \begin{Bmatrix} j_a & L & j_b \\ l_b & 1/2 & l_a \end{Bmatrix}^2 \begin{vmatrix} l_a & L & l_b \\ 0 & 0 & 0 \end{vmatrix}^2 .$$

This formula is valid for $L = 1$ $(T = 1)$, $L = 2$ $(T = 0, 1)$, L odd $\geqslant 3$ $(T = 0, 1)$ multipoles; for all of these, $s = 0$. It can be easily checked that (14) is normalized as (13). We note that the volume form of ref. [6] is obtained by dropping the terms F_L and H_L.

In figs. 1 and 2 we show numerical results for the dipole and quadrupole operators, respectively, in the case of several magic nuclei. The single particle wave functions are calculated in a Woods–Saxon potential well whose parameters are taken from ref. [9]. We have neglected the energy dependence of the well depth for simplicity. We compare our calculated $h(r)$ with the volume form h_v of Longo and Saporetti [7] as well as with a surface form h_s proportional to r^{L-1} $\times \, d\rho/dr$. All the forms are normalized in the same way. One can see that our model yields curves which are in between the volume and the surface form factors. For
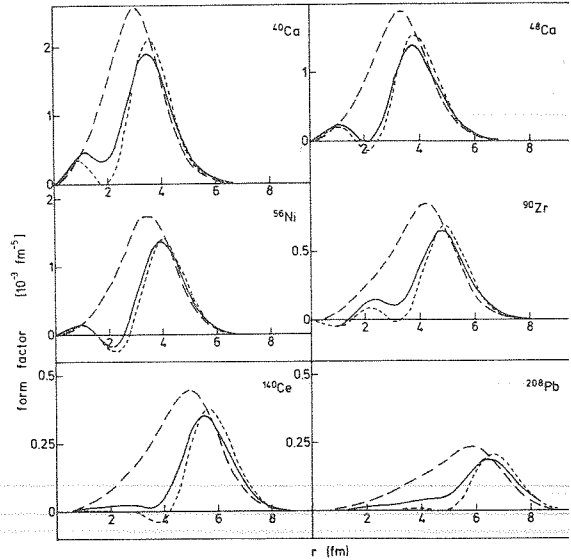

Fig. 2. Same as fig. 1 for the $L = 2$ $(T = 0$ or $1)$ multipole.

the lightest nuclei (^{40}Ca, ^{48}Ca, ^{56}Ni) our result lies closer to the surface form factor. This is even more true for ^{16}O (not shown). We observed that when ha monic oscillator wave functions are used for nuclei from ^{16}O to ^{56}Ni, one obtains $h(r) = h_s(r)$ for E1 mu

17

tipolarity. This property follows from the fact that in this case (E1, $f(r) = r$), the sum rule is exhausted by a single oscillator state. In other words, the state $|G_\mu\rangle$ (eq. (5)) is essentially a 0p state in the collective variable.

For the quadrupole operators, the situation is the same, the quantity $h(r)$ lying a little bit closer to h_s. This trend is even more pronounced for the $L = 3$ multipoles (results not shown).

It is interesting to parametrize our results in a manner simple enough to be introduced in the existing codes for calculating semi-direct capture cross sections. Since $h(r)$ enters in a matrix element weighted by r^2, we have sought an overall fit covering all the nuclei, the $L = 1$ and $L = 2$ operators and the domain where r is larger than one half of the half-density radius. We found that a good fit is provided by the arithmetic mean

$$h(r) = 0.5 h_s + 0.5 h_v. \tag{18}$$

For the $L = 3$ multipole, the surface form factor should be weighted by a factor 0.5.

We have examined the question of knowing whether our results are tied to the zero range forces. In fig. 3, we show the result of a calculation based on a Yukawa-type of interaction. We see that even a small range parameter $\mu = 2$ fm^{-1} (corresponding to a mass of ≈ 200 MeV) does not change the shape of $h(r)$ drastically (the normalization (13) is not valid for a Yukawa force). The appropriate range parameter is probably much larger. Indeed, for the nucleon–nucleon force, the pions do not contribute to the $\tau \cdot \tau_i$ on purely scalar parts. For the $\tau \cdot \tau_i$ part, the light boson that contributes is the ρ meson with a range parameter of $\mu = 3.83$ fm^{-1}. This range is likely to remain roughly the same for the effective nucleon–nucleon interactions in a nucleus. Fig. 3 shows that for such a value of μ, the delta interaction gives a sufficiently good approximation of $h(r)$.

In some cases, especially for $L = 2$ transitions, a substantial part of the non-energy-weighted sum rule lies outside the giant resonance state and hence, other states can be excited in the semi-direct capture. To take this into account, we propose to include a coupling of the
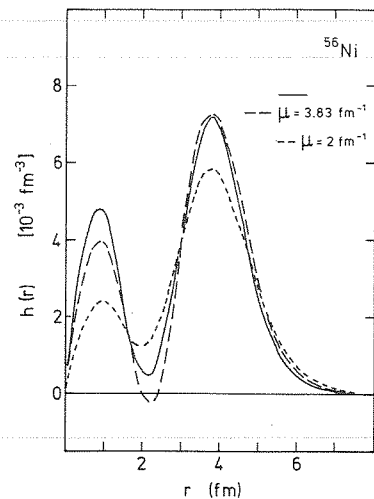

Fig. 3. Form factor for the $L = 1$, $T = 1$ multipole using a Yukawa-type interaction. The parameter μ is the inverse of the range. The full curve corresponds to the delta interaction ($\mu = \infty$).

form (7) for all the states and multiply the coupling constant by a factor $\sqrt{x}$, where x is the fraction of non-energy-weighted sum rule contained in the corresponding state. This approximation is reasonable but has a classical nature and some phases are neglected.

References

[1] G.F. Bertsch, P.F. Bortignon, R.A. Broglia and C.H. Dasso, Phys. Lett. 80B (1979) 161;
    G.E. Brown, J.S. Dehesa and J. Speth, to be published.
[2] V. Bernard and N. Van Giai, to be published.
[3] G.F. Bertsch and T.T.S. Kuo, Nucl. Phys. A112 (1968) 204.
[4] R.A. Broglia, Proc. Ettore Majorana School on Heavy ion interactions at high energy (Erice, March, 1979), to be published.
[5] C.F. Clement, A.M. Lane and J.R. Rook, Nucl. Phys. 66 (1965) 273.
[6] L.J. Tassie, Aust. J. Phys. 15 (1962) 135.
[7] G. Longo and F. Saporetti, Nucl. Phys. A199 (1973) 530.
[8] F. Saporetti, F. Fabbri and R. Guidotti, Nucl. Phys. A321 (1979) 354.
[9] K. Bear and P.E. Hodgson, J. Phys. G4 (1978) L287.