Monte Carlo Calculation of Compression and Pion Multiplicity in Relativistic Central Heavy-Ion Collisions.

J. CUGNON, T. MIZUTANI and J. VANDERMEULEN
University of Liège, Physics Department - B-4000 Liège 1, Belgium

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Studies of nuclear matter in abnormal conditions have a fascinating prospect: with the advent of relativistic ion accelerators we now have an opportunity to investigate the properties of high-density nuclear matter, essentially in central collisions of two heavy ions.

Since the compressed matter formed during the collisions has a very short lifetime, one is forced to learn its characteristics only through the emitted products and, to relate the observed product particles to the nature of the highly compressed stage of nuclear matter, one needs some theoretical models.

We shall report here on a Monte Carlo calculation assuming successive independent baryon-baryon collisions to take place in cascade during the time of the nuclear process. We shall apply it to the study of central collisions between two ^{40}Ca nuclei at laboratory energies of 1 and 2 GeV/nucleon, which fall in the region of current experimental interest (1). Specifically, we shall concentrate on the time evolution of the nuclear-matter density during the collision and the average pion multiplicity.

The present calculation embodies three main features:

i) relativistic kinematics is adopted throughout;

ii) experimental energy-dependent cross-sections (both elastic and inelastic) are used (2,3) for the input nucleon-nucleon interactions;

iii) the excitation of the meson degrees of freedom is taken into account by assuming that, in the energy range of our interest, pion productions go via the formation of the $\Delta(1232)$ resonances: $\mathcal{N}+\mathcal{N} \rightarrow \mathcal{N}+\Delta$.

The initial conditions are assigned as follows. In their respective rest system nuclei are considered as spheres of radius $R = 1.12 A^{\frac{1}{3}}$ fm ($A = 40$). In each nucleus we

(1) For a review, *Proceedings of the Fourth High-Energy Heavy Ion Summer Study, July 24-28, 1978*, LBL-7766, Berkeley.
(2) PARTICLE DATA GROUP: *$\mathcal{N}\mathcal{N}$ and $\mathcal{N}$D interactions (above* 0.5 GeV/c). A compilation, UCRL-20000 NN (1970).
(3) G. J. IGO: *Rev. Mod. Phys.*, **50**, 523 (1978).

generate randomly the positions and momenta of constituent nucleons, the latter being in accordance with the free-Fermi-gas law (the nuclear binding is neglected). Then the nuclei are given back their original momenta by Lorentz transformations. The actual calculation starts when the Lorentz-contracted spheres touch each other. The constituent nucleons are assumed to move freely (*i.e.* along straight-line trajectories) between individual collisions. The numbers of any pair of nucleons are subject to scatter, provided that the square of the distance of closest approach d in their c.m. system (4) is smaller than σ_{tot}/π (5), where σ_{tot} is the total $\mathcal{N}\mathcal{N}$ cross-section corresponding to the c.m. energy of the nucleons. Once a collision does take place whether it goes elastic or inelastic (Δ production in our model) is determined randomly according to the relative weight of the corresponding cross-sections. For an elastic case the momenta of the nucleons after the collision are chosen randomly according to the angular dependence of the form $\exp[At]$ (t is the momentum transfer), where A is an empirical function of the energy (2). For an inelastic scattering we assume an isotropic Δ production. This choice has been made due to the lack of experimental information, which, however, appears plausible for a production not very far from the threshold.

The behaviour of the produced Δ in nuclear matter, especially at high density, is poorly understood at present. We shall discuss this problem later, but for the moment let us take three extreme situations.

A) The Δ has a null lifetime: once formed, it immediately decays isotropically. The produced pion is assumed to leave the system without interaction.

B) The Δ has a sufficiently long lifetime, like in ref. (6), and has no interaction with the rest of the system.

C) Similar to *B*) except that the Δ scatters elastically against the nucleons, the cross-section being assumed to be the same as for the elastic $\mathcal{N}\mathcal{N}$ collision (at the same c.m. energy). For both *B*) and *C*) Δ's are made to decay at the end of the nuclear collisions.

Figures 1*a*) and *b*) present the time evolution of the mean density for the case of collisions with zero impact parameter: more precisely the mean baryon number density inside the sphere of radius 2 fm around the origin of the total centre of mass. The full curve gives the result for noninteracting nucleons, *viz.* the nuclei are assumed transparent to each other; its almost symmetric shape indicates that the spurious evaporation entailed by the Fermi motion and by the neglect of binding effects is quite limited during the relevant time span.

For all other cases, *i.e.* *A*), *B*) and *C*), the curves have rather similar profiles. At 1 GeV/A, one observes a shoulder at roughly $(4\div6)$ fm/c. We may interpret this as follows: in the initial stage of the process, two nuclei dive into each other almost freely until a large number of $\mathcal{N}\mathcal{N}$ collisions starts, which slows down the motion of the nuclear matter, then there follows the compression. Finally the resulting high-density matter explodes leading to a fast decompression.

At 2 GeV/A, the diving phase has disappeared: very likely the reflection of the average higher inelasticity in $\mathcal{N}\mathcal{N}$ collisions at this energy. The result is the increase

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(4) The distance of closest approach for two moving nucleons is not a relativistic invariant. This is a general feature for any one-time theory. This point will be discussed extensively in a forthcoming publication.

(5) Below a relative kinetic energy of 50 MeV, the nucleons do not scatter. These soft collisions, most of which being inhibited by the Pauli principle, are not expected to affect the comprensional properties significantly.

(6) H. J. PIRNER and B. SCHÜRMANN: *Nucl. Phys. A*, **316**, 461 (1979).

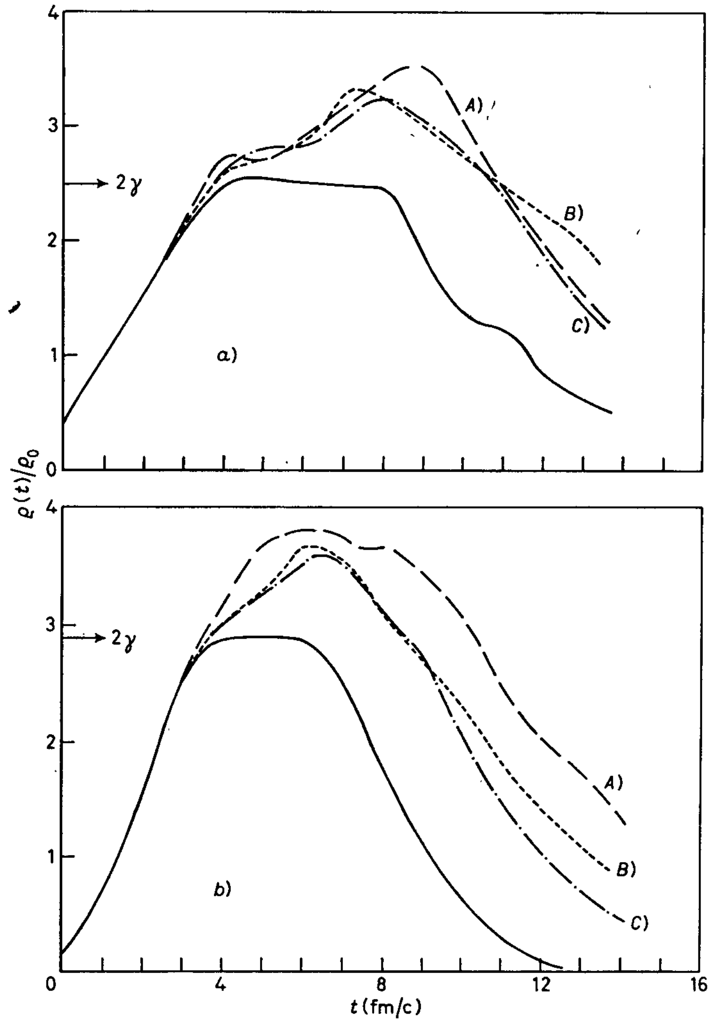

Fig. 1. – Time evolution of the baryon number density at the c.m. of the ^{40}Ca+^{40}Ca system: *a*) 1 GeV/A, *b*) 2 GeV/A. The density is evaluated in the c.m. rest frame. Full curve: transparency limit. Curves labelled *A*), *B*), *C*) refer to models described in the text. γ is the Lorentz factor for the motion of two nuclei in the total c.m. frame.

in the maximum compression (with increase in the collisional energy). The compression stage lasts roughly 5 fm/c (or $1.7\cdot 10^{-23}$ s), a very short time. The time scale for the decompression turns out to be also short: ~ 6 fm/c (or $2\cdot 10^{-23}$ s). This fast compression-decompression process may lead to the propagation of blast waves [7]. By observing the behaviour of the compressed nuclear matter within the sphere of radius 4 fm, we estimated the blast wave velocity to be roughly $\frac{2}{3}$ c.

The maximum compression rises above the transparency limit ($2\gamma\varrho_0$: the superposition of two Lorentz-contracted spheres) only by $(25\div 40)\%$. The compressed stage,

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(7) P. J. Siemens and J. O. Rasmussen: *Phys. Rev. Lett.*, **42**, 880 (1979).

however, cannot be viewed as roughly two bulks of matter running in opposite directions. During this stage ($(6\div10)$ fm/c at 2 GeV/A), the rapidity spectrum of the nucleons changes shape by spreading around zero rapidity (the proportion of nucleons with $|y| < 0.4$ increases from 37 to 52%). Hence the high compression is due to a drastic slowing down of the particles.

Model A) provides a slightly higher baryon number density than models B) and C). This may be attributed to the fact that in this case the isotropic decay of Δ's increases the transverse momentum of the nucleons, thus slows down more efficiently their overall momentum in the beam direction.

As one could intuitively guess, the introduction of the Δ degrees of freedom in describing relativistic heavy-ion processes is important. In our present study one may expect that the Δ's make the compression easier: in other words they make the equation of state softer if one is allowed to use the languages found in hydrodynamical approaches. This, in fact, we have observed by performing a calculation assuming that the experimental total $\mathcal{N}\mathcal{N}$ cross-section leads to elastic scattering only and found a 20% reduction in the maximum density reached.

With respect to the compression of nuclear matter in relativistic central collisions of heavy ions, several calculations have been reported so far [8-14]. One finds that, in general, approaches based upon relativistic hydrodynamics predict higher compression: sometimes twice as much as what we have found in our present work. We note that in hydrodynamic pictures the effect of collisions is implemented both in the equation of state and in the viscosity term. This latter term has eventually been dropped in all the calculations along this line, leading to a transfer of more energy to the compression degrees of freedom to give higher compression than in the case in which the viscosity term is retained.

Now we would want to know which model may be most realistic: A), B) or C), by first discussing the behaviour of the Δ in nuclear matter and then studying the mean pion multiplicity obtained from the above three models.

The behaviour of the Δ in the nuclear-medium has been studied in connection with π-nucleus scattering at medium energies [15,16]. For normal nuclear matter density the collisional broadening (due to multiple scattering of the pion through Δ-resonances) dominates the total width (more than twice the natural Δ width), whereas the Pauli effect quenches the natural width by 30 to 50%. This clearly shows that, at least for the case of normal nuclear density, model A) is not appropriate. The dominance of collisional width manifests the importance of the interaction of the produced pions with nucleons. This interaction may be taken care of in part by the effective $\mathcal{N}\Delta$ interactions in model C), thus this model seems to be preferred.

At higher nuclear-matter densities the Δ behaviour has not been studied fully. However, one could guess that both the collisional width and the Pauli quenching effect increase in such a case, thus model C) likely stays more appropriate than others.

Table I shows that model A) predicts higher (mean) pion multiplicity than models B) and C). On the other hand, these latter two give roughly the same numbers. This may be understood by recalling that C) differs from B) only in the presence of the elastic $\Delta\mathcal{N}$ scattering which could change significantly the spectrum of produced pions, but not the multiplicity.

(8) A. R. BODMER and C. N. PANOS: *Phys. Rev. C*, **15**, 1342 (1977).
(9) G. F. CHAPLINE, M. H. JOHNSON, E. TELLER and M. S. WEISS: *Phys. Rev. D*, **8**, 4302 (1973).
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(11) A. A. AMSDEN, F. H. HARLOW and J. R. NIX: *Phys. Rev. C*, **15**, 2059 (1977).
(12) J. HOFMANN, H. STÖCKER, U. HEINZ, W. SCHEID and W. GREINER: *Phys. Rev. Lett.*, **36**, 88 (1976).
(13) J. P. BONDORF, H. T. FELDMEIER, S. GARPMANN and E. C. HALBERT: *Phys. Lett. B*, **65**, 217 (1976).
(14) M. SOBEL, P. J. SIEMENS, J. P. BONDORF and H. A. BETHE: *Nucl. Phys. A*, **251**, 502 (1975).

It should be remembered that in our present work we have not included the effect of Δ absorption through the process $\mathcal{N}+\Delta \rightarrow \mathcal{N}+\mathcal{N}$. Its inclusion will certainly reduce the pion multiplicity listed in table I. A rough estimate based upon the decay mean

TABLE I. – *Average pion multiplicities.*

	A)	*B*)	*C*)
1 GeV/*A*	32	25.3	24.1
2 GeV/*A*	65	38.6	38.2

free path of Δ via the above-mentioned process at normal nuclear matter density using the $\sigma(\mathcal{N}+\mathcal{N} \leftarrow \mathcal{N}+\Delta)$ of ref. ([15]) indicates a 40% reduction. This, however, is obviously an over-estimate, since 1) subsequent recombinations $\mathcal{N}+\mathcal{N} \rightarrow \mathcal{N}+\Delta$ and 2) the Pauli blocking effect must be taken into account. Borrowing again the results of ref. ([15,16]) and taking the ratio of the total width of the Δ (in nuclear medium at normal density) and its spreading width (whose appreciable part comes from the above-mentioned Δ absorption process at low-to-medium energies), we find the reduction to be about 20%. Even at higher nuclear density we could expect this ratio to be about the same.

To compare our result with the experimental data of SANDOVAL *et al.* ([17]), we assume an exact isospin symmetry and interpolate linearly the numbers in table I to get the mean π^- multiplicity n_{π^-} in the $^{40}Ca+^{40}Ca$ reaction at 1.8 GeV/*A*. Correcting for the Δ absorption effect discussed above, we find $n_{\pi^-} = 9$ for models *B*) and *C*), and $n_{\pi^-} = 15$ for *A*). Reference ([17]) reported that for central collisions of Ar+KCl $n_{\pi^-} = 5.7$. Since this experimental result should be viewed as an average over the values of impact parameter between 0 and 2 fm, it is not too far from the prediction based upon models *B*) and *C*), but model *A*) seems to be ruled out.

We are currently working with a better treatment of Δ including the effect of the $\Delta+\mathcal{N} \rightarrow \mathcal{N}\mathcal{N}$ process.

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([16]) M. HIRATA, J. H. KOCH, F. LENZ and E. J. MONIZ: *Phys. Lett. B*, **70**, 281 (1977); and MIT preprint (1978).

([17]) A. SANDOVAL *et al.*: to be published. We thank Drs. SANDOVAL for providing us with the data prior to publication.