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Uncertainty quantification of Antarctic contribution to sea-level rise using the fast Elementary Thermomechanical Ice Sheet (f.ETISh) model

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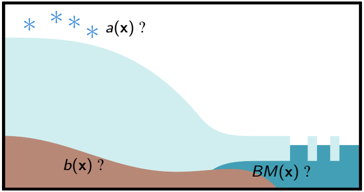
Motivation

- Uncertainties in Antarctic ice-sheet predictions [IPCC, 2013] have been identified as a major source of uncertainty in sea-level rise projections.
- Sources of uncertainty:
  - Sub-shelf melting;
  - Basal friction;
  - Bedrock topography;
  - Climate forcing;
  - Instability mechanisms.



- Robust predictions of future sea-level rise require efficient uncertainty quantification tools and ice-sheet models to assess the influence and importance of various sources of uncertainty.

New perspectives for ice-sheet modelling

- Uncertainty quantification in glaciology has been restrained by the high computational cost of ice-sheet models.

  New efficient ice-sheet models such as the hybrid thermomechanical f.ETISh model [Pattyn, 2017] can run a large number of simulations.

- Ensemble modelling methods [Bindschadler et al., 2013, Pollard et al., 2015] have been applied to parameter sensitivity in ice-sheet models.

  Stochastic methods [Le Maître and Knio, 2010] have been developed and applied with success to uncertainty quantification in science and engineering [SIAM UQ Group].

- In this presentation, we apply stochastic methods to the f.ETISh model to show how these methods can deal with various sources of uncertainty in ice-sheet models and to clarify the impact of uncertainty in sub-shelf melting underneath Antarctic ice shelves.

Outline

- Motivation.
- Stochastic methods for uncertainty quantification.
- Application to uncertainty in basal melting.
    - Application 1: Uncertainty in global basal melting.
    - Application 2: Uncertainty in regional basal melting.
- Conclusion and outlook.
- References.

Uncertainties in ice-sheet models

Bedrock elevation

Geothermal heat flux

Surface temperature

Surface accumulation

Sub-shelf melting

Atmospheric forcing

Basal sliding coefficient

...

+ uncertainty

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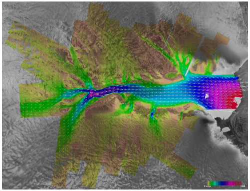

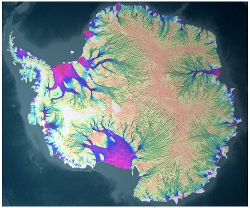

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Change in volume

Change in area

Grounding line position

Ice velocity

Instability analysis

...

Input variables

$(x_1, x_2, \ldots, x_m)$

Model

$y = g(x_1, x_2, \ldots, x_m)$

Output variable

y

Stochastic methods: Methodology

(1) Characterisation of uncertainties:

Available information		Probability distribution
Satellite observations In-situ measurements	$\longrightarrow$ Statistics Bayesian inference	$\rho_{(x_1,\ldots,x_m)}$

(2) Propagation of uncertainties:

Probability distribution		Probability distribution
$\rho_{(x_1,\ldots,x_m)}$	$\longrightarrow$ Monte Carlo Stochastic expansion	ρ_y

If the model is computationally expensive, propagation is performed using a surrogate model i.e. a low-cost model that mimics the original model.

(3) Sensitivity analysis: It aims at ranking the input uncertainties in terms of the order of significance of their contribution to output uncertainty.

Propagation of uncertainty: polynomial expansion

- Due to their low convergence rate, Monte-Carlo methods require a large number of simulations to achieve a given level of accuracy.
- A surrogate model acts as a substitute for $g(\mathbf{x})$ with a lower computational cost. One can approximate $g(\mathbf{x})$ as a polynomial regression model i.e.

$$g(\mathbf{x}) \approx g^p(\mathbf{x}) = \sum_{|\boldsymbol{\alpha}|=0}^{p} g_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}(\mathbf{x}),$$

  where $\psi_{\boldsymbol{\alpha}}(\mathbf{x})$ is a polynomial of order $|\boldsymbol{\alpha}| = \alpha_1 + \ldots + \alpha_m$ and the regression coefficients $g_{\boldsymbol{\alpha}}$ are estimated from a limited set of training points using quadrature rules or least-squares fitting [Le Maître and Knio, 2010].
- Polynomial regression models are efficient surrogate models for models with smooth response and low-dimensional parameter space.

Sensitivity analysis

- Sensitivity analysis aims at ranking the significance of the contribution of each input variable to the uncertainty in the output variable.
- Using a high-dimensional model representation [Saltelli et al., 2008] with orthogonal components, the variance σ_Y^2 is decomposed as

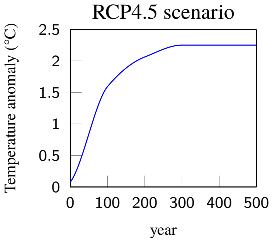
$$\underbrace{\sigma_Y^2}_{\text{variance of } Y} = \underbrace{s_{X_1}}_{\text{contribution from } X_1} + \ldots + \underbrace{s_{X_m}}_{\text{contribution from } X_m} + \underbrace{\text{remainder}}_{\substack{\text{contribution from} \\ \text{interaction of} \\ X_1,\ldots,X_m}}$$

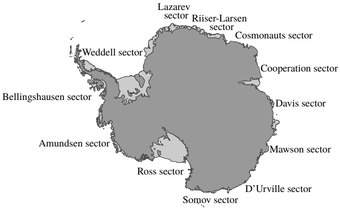
  where the s_{X_i} are the sensitivity descriptors.
- Sensitivity descriptors can be estimated from Monte-Carlo methods or orthonormal polynomial expansions i.e.

$$g^p(\mathbf{x}) = \sum_{|\boldsymbol{\alpha}|=0}^{p} g_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}(\mathbf{x}) \Rightarrow s_{X_i} = \sum_{\substack{|\boldsymbol{\alpha}|=1 \\ \alpha_j=0,\, j\neq i}}^{p} g_{\boldsymbol{\alpha}}^2.$$

Application: Problem setting

- Objective: Assess the influence of uncertain sub-shelf melting on the Antarctic contribution to sea-level rise under a schematic RCP4.5 scenario.
- Output quantity: Contribution to sea level after a 500-year climate forcing.
- Numerical model: f.ETISh model [Pattyn, 2017] applied to Antarctica.
- Application 1: Illustration of propagation methods with a spatially uniform sub-shelf melting (1 parameter).
- Application 2: Illustration of propagation methods and sensitivity analysis with a spatially non-uniform sub-shelf melting (multi-parameter problem).





UQ methodology

(i) Characterisation of uncertainty: Consider a uniform sub-shelf melting rate varying between 0 and 20 [m/yr] i.e. $BM \sim \mathcal{U}(0, 20)$. Statistical descriptors are $\mu_{BM} = 10$ [m/yr], $\sigma_{BM} = 1.67$ [m/yr] and $\sigma_{BM}/\mu_{BM} = 16.7\%$.

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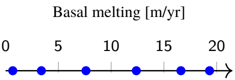
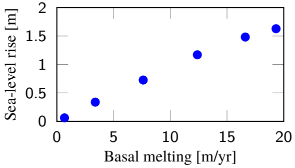
(ii) Evaluation of the model for a limited number of points :



→ f.ETISh →



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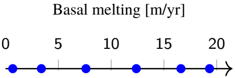

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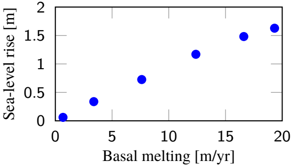
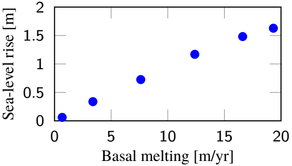
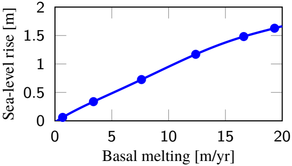


(ii) Construction of a surrogate model using Legendre polynomials:

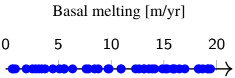


$$\longrightarrow \quad g \approx g^p = \sum_{|\alpha|=0}^{p} g_\alpha \Psi_\alpha \quad \longrightarrow$$

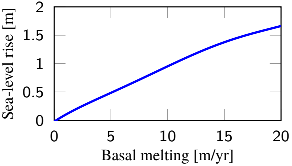


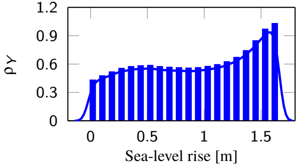
UQ methodology

(iv) Propagation of uncertainty through the surrogate model using Monte-Carlo:



Large number of samples from $\mathcal{U}(0,20)$

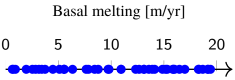




+ computation of statistical descriptors of the output: $\mu_Y = 0.92$ [m], $\sigma_Y = 0.49$ [m] and $\sigma_Y/\mu_Y = 54\%$.

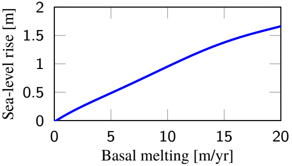
UQ methodology

(iv) Propagation of uncertainty through the surrogate model using Monte-Carlo:

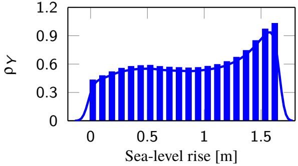


Large number of samples from $\mathcal{U}(0,20)$

$\longrightarrow$

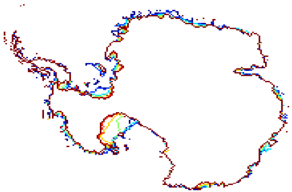


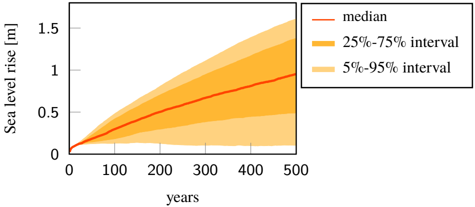
$\longrightarrow$



+ computation of statistical descriptors of the output: $\mu_Y = 0.92$ [m], $\sigma_Y = 0.49$ [m] and $\sigma_Y/\mu_Y = 54\%$.

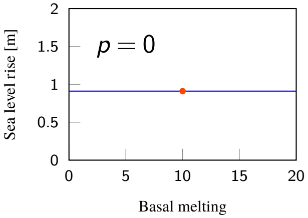
(v) Result interpretation.





Convergence analysis of polynomial expansion

- Graphical convergence:



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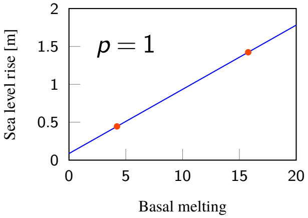

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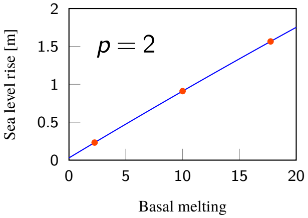

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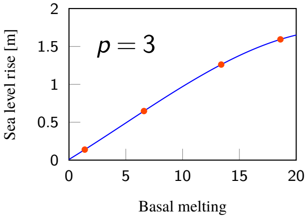

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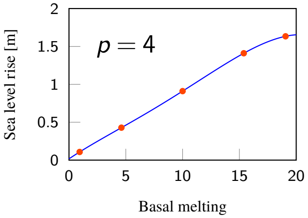

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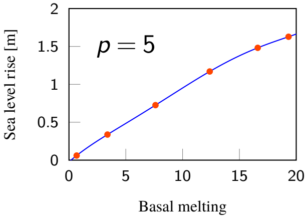

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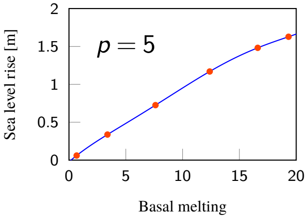
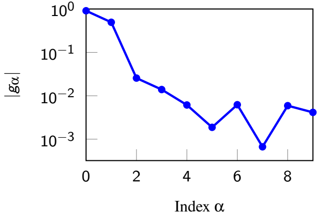


- Convergence of coefficients:



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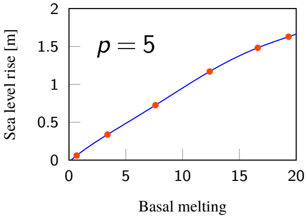

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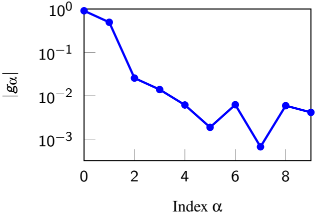
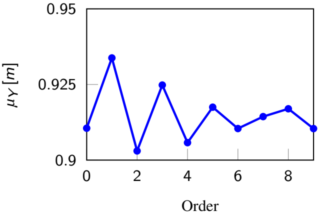
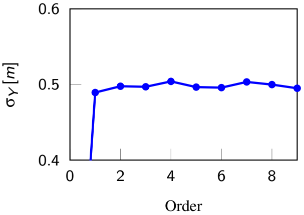


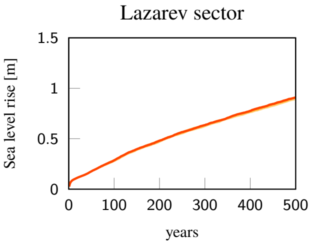
- Convergence of statistical descriptors:

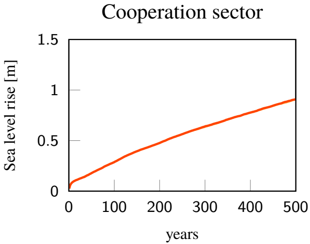


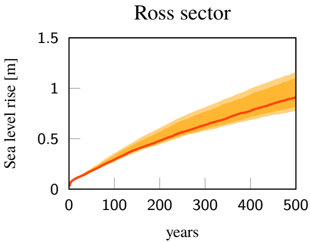


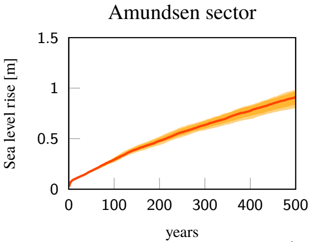
Univariate analysis

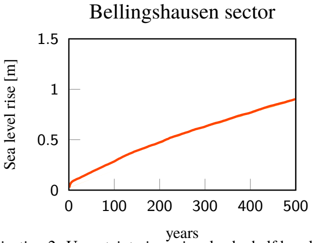
- As a first analysis, we can consider each parameter separately ($BM_i \sim \mathcal{U}(0, 20)$ and $BM_j = 10$ for $j \neq i$).

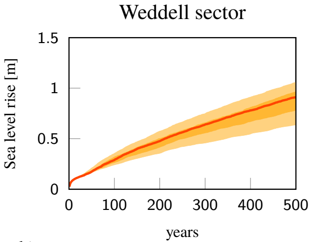






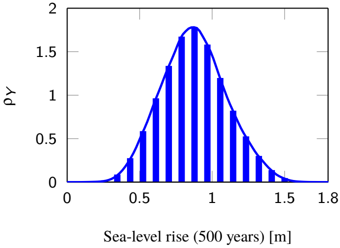


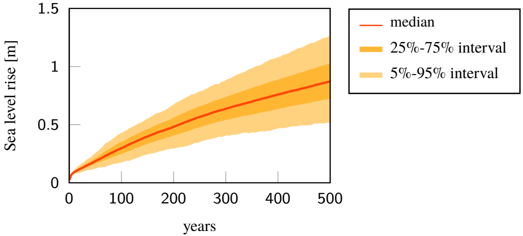




Multivariate analysis: Input and output distributions

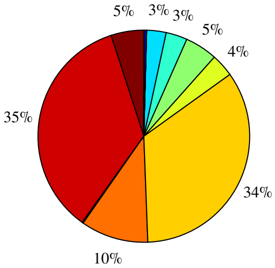
- Characterisation of input uncertainty: Sub-shelf melting rates in marine sectors are modelled as independent and identically distributed random variables ($BM_i \sim \mathcal{U}(0,20)$, $i = 1, \ldots, 12$). Statistical descriptors are $\mu_{BM_i} = 10$ [m/yr], $\sigma_{BM_i} = 1.67$ [m/yr] and $\sigma_{BM_i}/\mu_{BM_i} = 16.7\%$.
- Characterisation of output uncertainty: Statistical descriptors for sea-level rise contribution after 500 years: $\mu_Y = 0.88$ [m], $\sigma_Y = 0.22$ [m] and $\sigma_Y/\mu_Y = 26\%$.



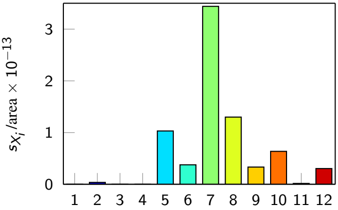


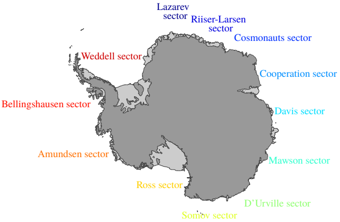
Multivariate analysis: Sensitivity analysis

- Sensitivity descriptors:



- Sensitivity descriptors/shelf area:





Conclusion and outlook

- New efficient ice-sheet models provide new opportunities for ice-sheet modelling as they can run a large number of simulations to perform uncertainty quantification.
- We have shown that stochastic methods can provide efficient probabilistic tools for uncertainty quantification in glaciology.
- We aim at studying a more complete set of uncertainties (including uncertainty in physical and model parameters) to identify and rank the most influential sources of uncertainty in sea-level predictions.
- We aim at combining uncertainty quantification methods with a stability analysis to identify critical thresholds in ice-sheet behaviour.

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FREEDOM TO RESEARCH