Switching Costs in Two-sided Markets*

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Abstract

In many markets, there are switching costs and network effects. Yet the literature generally deals with them separately. This paper bridges the gap by analyzing their interaction (or "indirect bargain") in a dynamic two-sided market. It shows that in the symmetric equilibrium, the classic result that the first-period price is U-shaped in switching costs does not emerge, but instead switching costs always intensify the first-period price competition. Moreover, an increase in switching costs on one side decreases the first-period price on the other side. Policies that ignore these effects may overestimate the extent to which switching costs can reduce welfare.

Keywords: switching cost, two-sided market, network externality

JEL Classification: D43, L13, L96

1 Introduction

There are countless examples of markets in which there are switching costs and network effects.[1] In the existing literature, there is a wealth of works in the dual areas of switching costs and two-sided markets, which, for instance, finds that high switching costs cause firms to charge more to their locked-in customers (Klemperer, 1987b), whereas large network externalities cause platforms to charge less (Armstrong, 2006a). On the other hand, very little is known or understood about how markets react to the interaction between the two forces. This paper provides new insights on how switching costs and network externalities affect firms' pricing

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[1]See Levin (2011).

strategies. It shows that in the presence of both effects, welfare analyses that merely sum up the bargain effects of switching costs and network externalities in the introductory period are prone to error.

A useful example is the smartphone operating system market. Apple, Google and Windows are key players in the market. Each of them faces two groups of consumers: application users and application developers. While it is easy for consumers to migrate data from an older version of Windows Phone to a newer one, a consumer switching from Android to Windows Phone incurs the cost of migrating—if not re-purchasing—a set of apps, media files, as well as contacts, calendars, emails and messages.[2] As Hal Berenson suggests, one of the problems faced by Windows Phone is its weak app library.[3] Suppose now that Windows improves its library by introducing more Android apps. This not only raises the utility of users through the network externality but also lowers their switching costs in terms of data migration. For instance, making some Android movie or music streaming apps available also for Windows Phone enables users to migrate their media files across devices more easily without the hassle of moving the data manually, which results in lower switching costs. Such a change may seem to be welfare-improving because the extent to which platforms can exploit their locked-in customers is smaller. In a model incorporating both switching costs and network effects, however, I show that a decrease in switching costs of users leads to an increase in the price for developers. Since developers value the participation of users and a decrease in switching costs of users makes attracting users easier, the platform can price higher to extract the increased value to developers. As a consequence, lower switching costs may not improve overall consumer welfare. Identifying this cross-group effect of switching costs is one of the main contributions this paper as this effect does not emerge from the classic Armstrong's (2006a) two-sided model due to the model's static property or from the classic Klemperer's (1987b) switching cost model due to the one-sidedness of his model. Moreover, the existence of cross-group effects emphasizes that regulators need to consider the interaction between switching costs and network externalities carefully and avoid a mechanical analysis of them by simply adding up their effects, since the overall effect across all consumer groups, through feedback effects, can be larger than the sum of effects. The analysis also considers the implications from both regulatory (e.g., welfare concerns about switching costs) and managerial points of view (e.g., how switching costs and network effects affect the profits of platforms, which may lead to very different app/OS design strategies beyond pricing), and provides insight into other two-sided markets with switching costs, such as media, credit cards, video games, and search engines.

To gain these insights, I develop a two-period duopoly model, where platforms 0 and 1 sell their products to two groups of consumers. Each group is represented by a Hotelling line with unit length. Each consumer can purchase from either platform (single-homing). The

[2]In reality, there are also other types of switching costs (e.g., the cost of learning how to operate a new interface and the psychological inclination to stick with what we know). See Klemperer (1995) and National Economic Research Associates (2003).

[3]As discussed in his blog post about "Will Microsoft get the new Surface(s) right? Part 1," May 8 2014, available at `hal2020.com`, he said, "High price [and] lack of consumption apps... doomed the Surface. They could have broken through by pricing the Surface aggressively to drive sales volume that created a pull on app developers. But they didn't. Consumers stayed away."

penultimate section extends the analysis to cover the multi-homing case. There are both switching costs, i.e., consumers exhibit inertia in their product choice, and indirect network externalities, i.e., participation of one group increases the value of participating for the other group. Consumers are farsighted, which means that they make decisions based on their lifetime utility, and have independent preferences across periods. I consider the following game. In the first period consumers decide which platform to join, and in the second they learn their second-period preferences, and bear a switching cost if they decide to switch to another platform. Moreover, I focus on the symmetric equilibrium, in which, for each side, the prices charged by the two platforms are the same—that is, in the smartphone example the two platforms charge users the same price, and they charge developers the same price. This model is flexible enough to collapse to either a pure switching cost model or to a pure two-sided model for extreme parameter values. When both effects are at work, I show that conventional results will change: the overall bargain effect in this model can be larger than the sum of effects in pure switching cost and pure two-sided models.

The main results can be summarized as follows. I show that in equilibrium switching costs do not affect second-period prices, whereas the impact of switching costs on first-period prices depends on the strength of two effects. The first is the consumer anticipation effect: more patient consumers are less tempted by a temporary price cut because they understand that the price cut will later be followed by a price rise. Their demand therefore is less elastic, and platforms will respond by charging higher prices. The second effect is the firm anticipation effect: more patient platforms put more weight on future profits, hence both compete aggressively for market share. When network externalities are weak, the first-period price is U-shaped in switching costs: the firm anticipation effect dominates when switching costs are small, while the consumer anticipation effect dominates when switching costs are large. When externalities become sufficiently strong, however, the consumer anticipation effect is weakened because consumers value the platform for facilitating their interaction with the other side even though they anticipate that the platform might exploit their reluctance to switch later. Consequently, switching costs with strong network externalities overturn the standard U-shaped result and always intensify the first-period price competition. This effect is new in the literature because both the consumer and firm anticipation effects are absent from Armstrong's (2006a) model and the consumer anticipation effect goes in the opposite direction in Klemperer's (1987b) model. Furthermore, there is another new cross-group effect that is absent in both of these models: an increase in switching costs on one side unambiguously decreases the price on the other side. The reason is that platforms can build market share on one side either by directly lowering the price on this side or by indirectly increasing the participation on the other side. When switching costs on the first side are large, an easier way to build market share is to focus on the indirect channel; consequently the first-period competition is intensified on the other side. I call this new interaction between switching costs and network externalities an "indirect bargain effect", as opposed to the traditional "direct bargain effect" of switching costs in Klemperer's (1987b) model, where indirect network externalities are absent, see Proposition 3 for details.

One of the major contributions of this paper is that it provides some general guidance for understanding how markets react to switching costs and network externalities in the absence of Coasian bargaining.[4] While the existing literature has tended to focus on either switching costs or network externalities, I study the two concepts together and show that switching costs may work differently in two-sided markets as compared to one-sided ones.[5] The important policy rule is to recognize the role of the indirect bargain effect, and factor the effect into the overall assessment of the effects of switching costs. Regulators should not merely sum up the effects of switching costs and network externalities in traditional models because failing to account for the indirect bargain effect may overestimate the extent to which switching costs can reduce welfare (see Section 4). Furthermore, from a managerial perspective, the result that switching costs and network externalities reduce platforms' profits explains why strategies of lowering either or both of them are frequently employed in practice. To reduce switching costs, platforms have tried to deliver apps and services across a breadth of OS (e.g., between Windows and Android as well as between Windows and iOS). To reduce network externalities, platforms have tried to deliver services across a breadth of devices that belong to their own ecosystem only (see Section 3).

The indirect bargain effect holds even in richer analyses with other forms of consumer heterogeneity and price discrimination that are rarely studied in the two-sided market literature.[6] The main conclusion of this paper is that when there are network externalities, switching costs should be less of a concern to policymakers, because they not only have their own bargain effect directly on prices but also have an indirect bargain effect. In Section 5, I show that this policy conclusion remains valid even in a number of variations on the model, e.g., when consumers are heterogeneous with respect to their level of farsightedness and loyalty, and when third-degree price discrimination is feasible. Furthermore, this paper covers compatibility policy more broadly, as we can easily interpret switching costs as the difference between the cost of migration across platforms and the cost of backward compatibility when staying with one platform without changing the qualitative results. An interesting twist is then more compatibility between new and older versions of one platform's products can be more beneficial to consumers in a market with network externalities than in a market without network externalities, because in the alternative interpretation, an increase in switching costs may mean increasing the compatibility between the products of one platform.

1.1 Related Literature

There is a sizeable literature on switching costs which, broadly speaking, can be categorized into two main groups.[7] One group of papers assumes that firms cannot discriminate between

[4]The Coase (1960) Theorem does not apply in this model because I focus on simple spot price contracts, which cannot eliminate the inefficiencies caused by switching costs. Moreover, as defined by Rochet and Tirole (2006), a necessary condition for a market to be two-sided is that Coasian bargaining cannot take place.

[5]By one-sided markets I mean markets without indirect network externalities.

[6]Heterogeneity in consumers is unusual in the two-sided literature except for a few recent papers focusing on matching problem, e.g., Gomes and Pavan (2013). Also, poaching is largely an unexplored topic. One exception is Liu and Serfes (2013), who examine the effect of first-degree price discrimination in two-sided markets.

[7]Farrell and Klemperer (2007) and Klemperer (1995) provide excellent overviews on the literature of switching costs.

old and new consumers. As firms know that they can exercise market power in the second period over those consumers who are locked-in, they are willing to charge a lower price in the first period in order to acquire these valuable customers. This "bargains-then-ripoffs" pattern is the main result of the first-generation switching cost models (see, e.g., Klemperer 1987a, 1987b). A second group of works allows for price discrimination so firms can charge a price to its old customers and a different price to new ones. Chen (1997) analyzes a two-period duopoly with homogeneous goods. Under duopoly, consumers who leave their current supplier have only one firm to switch to. Since there is no competition for switchers, this allows the duopolist to earn positive profits in equilibrium. Taylor (2003) extends Chen's model to many periods and many firms. With three or more firms, there are at least two firms vying for switchers, and if products are undifferentiated, these firms will compete away all their future profits. More recent contributions include Biglaiser, Crémer and Dobos (2013), who study the consequence of heterogeneity of switching costs in an infinite horizon model with free entry.[8] They show that even low switching cost customers are valuable for the incumbent.

The design of pricing strategies to induce consumers on both sides to participate has occupied a central place in the research on two-sided markets.[9] The pioneering work is Caillaud and Jullien (2003), who analyze a model of imperfect price competition between undifferentiated intermediaries. In the case where all consumers must single-home, the only equilibrium involves one platform attracting all consumers and the platform making zero profit. In contrast, when consumers can multi-home, the pricing strategy is of a "divide-and-conquer" nature: the single-homing side is subsidized (divide), while the multi-homing side has its entire surplus extracted (conquer). Armstrong (2006a) advances the analysis by putting forward a model of competition between differentiated platforms by using the Hotelling specification. He finds that the equilibrium price is determined by the magnitude of cross-group externalities and whether consumers single-home or multi-home. His approach comes closest to mine. However, he focuses on a static model of a two-sided market without switching costs while here with switching costs and different degrees of sophistication the problem becomes a dynamic one. Another closely related paper is Rochet and Tirole (2006), who combine usage and membership externalities (as opposed to the pure-usage-externality model of Rochet and Tirole (2003), and the pure-membership-externality model of Armstrong (2006a)), and derive the optimal pricing formula. They, however, focus on the analysis of a monopoly platform.

This study apart, there is little literature that studies the interaction between switching costs and network externalities. Su and Zeng (2008) analyze a two-period model of two-sided competing platforms. Their focus is on the optimal pricing strategy when only one group of consumers has switching costs and their preferences are independent, and hence their model applies only to a limited subset of multi-sided markets, such as browsers, search engines, and shopping malls,[10] whereas this paper studies a richer setting in which both sides bear switching

[8] Other papers, such as Einav and Somaini (2013) and Rhodes (2014), also examine the effect of switching costs in a dynamic setting, but there are no network externalities.

[9] See Rysman (2009) and Weyl (2010) for excellent overviews on the literature of two-sided markets.

[10] In the market for browsers, users can switch relatively more easily between Internet Explorer, Chrome, and Firefox than content providers: whereas users only need to uninstall the old browser and install the new one, content providers need to rewrite their codes so that the codes are compatible with the new browser. For

costs. This seems a more natural feature of many markets, such as smartphone and video games. More importantly, ignoring the switching costs of one group of consumers and focusing on independent preferences sever the connections between the first-period and second-period utilities of these consumers, and hence the indirect bargain (which is the focus here) would vanish in their setting. Biglaiser and Crémer (2011, 2014, 2016) and Biglaiser, Crémer and Dobos (2013), in a series of papers, compare the effect of switching costs and network effects on entry in a one-sided market. They show that switching costs and network effects together can have complicated effects on the profits of the incumbent, which depends on the relative importance of switching costs versus network effects. However, this model differs from theirs in that I focus on the interaction between "two-sided" network effects and switching costs, whereas they focus more on "one-sided" network effects. Similarly, Suleymanova and Wey (2011) analyze how switching costs and network effects affect market structure, but in a static model and with one-sided network effects. This means that in their models switching costs of one type of consumers do not affect prices of the other type, which is one of the key elements I am studying here. This is also one of the key forces that lead to the whole effect of switching costs and network effects being greater than the sum of its parts in this model.

2 Model

Consider a two-sided market with two periods. Each side $i \in \{A, B\}$ of the market is characterized by a Hotelling line with unit length, and two platforms are located at the endpoints 0 and 1 on each side. That is, platform 0 is located at 0 and platform 1 at 1 on both sides of A and B.

On each side i, there is a unit mass of consumers, who are uniformly distributed on the Hotelling line. A consumer on side i can switch to the other platform at a cost s_i. Assume that all consumer preferences are independent across the two periods, which means that each consumer is randomly relocated on the Hotelling line in the second period. Independent preferences can be interpreted as consumers having changing tastes, or consumers being ignorant about their future tastes.[11] This assumption is needed for technical reason because it smoothes the demand function, which is a standard assumption in the switching costs literature (see, e.g., Klemperer 1987b; Einav and Somaini 2013). Moreover, assume that all consumers are far-sighted, which means that on each side consumers make decisions based on their lifetime utility,

search engines, switching costs are almost negligible for users as they can switch easily between Google, Bing and Yahoo in as little as one click, but switching costs are larger for publishers because they need to pay again a fee to another search engine so that their websites will appear at the top of the search results. And for shopping malls, shoppers are free to go to any shopping malls, but there are high transaction costs for shop owners in terminating the old contract and initiating a new one.

[11]A survey published by Consumer Intelligence Research Partners (CIRP) reveals that 20% of Apple's new iPhone customers were previous Android phone owners. The possibility of learning new information overtime could be one reason why consumers switch, as it is difficult for consumers to fully understand in advance their taste for apps and smartphones, which are constantly evolving. This quarterly survey was taken from data surveying 500 subjects in the U.S. who had purchased a new mobile phone in the previous 90 days over the last four quarters, between July 2012 and June 2013.

and discount the second-period utility at rate δ_i. Further, assume that consumers single-home, i.e., they choose to join only one platform in each period.

In Section 5, I show that the main results continue to hold when some consumers are loyal (i.e., their preferences do not change), when some consumers are myopic (i.e., they make decisions based on their first-period utility only), when multi-homing decisions are allowed, when price discrimination is feasible, and when there are some asymmetries between different groups of consumers, as well as between platforms.

Platforms and consumers have common knowledge about consumer preferences and the value of switching cost s_i. The timing of the game is as follows:

- At the beginning of the first period, consumers are unattached to any platforms. They learn their initial preferences. Platforms set the first-period prices. Consumers choose which platform to join.

- At the beginning of the second period, consumers learn their second-period preferences. Platforms set the second-period prices. Consumers decide to switch or not.

The solution concept for the game is subgame perfect equilibrium.

The utility of a consumer on side i at time t, who is located at x, is

$$v_i + e_i n^j_{k,t} - |x - k| - p^i_{k,t},$$

where $i, j \in \{A, B\}$ and $i \neq j$. Let v_i denote the intrinsic value of consumers on side i for using either platform. Assume that v_i is sufficiently large such that the market is fully covered. The benefit that a consumer from side i enjoys from interacting with any consumer on the other side is given by e_i, and $n^i_{k,t}$ denotes the market share of platform k on side i in period t, where $k \in \{0, 1\}$, $i \in \{A, B\}$ and $t \in \{1, 2\}$. Thus, $e_i n^j_{k,t}$ is the total external benefit from interacting with the other side of the market. The transport cost when a consumer purchases from platform k is given by $|x-k|$, where the unit cost is normalized to one. Platform k charges a uniform price $p^i_{k,t}$ on side i in period t. Assume for simplicity that consumer utility does not depend on the number of people on the same side in the current period, and the number of people on the other side in previous periods.

Assuming that the marginal cost of production is equal to zero, platform k's profit in period t is given by

$$\pi_{k,t} = p^A_{k,t} n^A_{k,t} + p^B_{k,t} n^B_{k,t}, \tag{1}$$

which is the sum of revenues from side A and side B. Both platforms discount the second-period profit at rate δ_F. Moreover, I assume the following: First, assume that $s_i \in [0, 1)$, where one is the unit transport cost, so that at least some consumers will switch. Second, assume $e_i \in [0, 1)$ in order to ensure that the profit function is well-defined, and the demand is decreasing in a platform's own price and increasing in its rival's price. Finally, platforms charge uniform prices and they cannot price discriminate among their previous customers and those who have bought the rival's product in the previous period (this assumption will be relaxed in Section 5.4).

2.1 Second Period: the mature market

I work backward from the second period, where each platform has already established a customer base. Given first-period market shares $n_{0,1}^A$ and $n_{0,1}^B$, a consumer on side i, located at θ_0^i on the unit interval, purchased from platform 0 in the first period is indifferent between continuing to buy from platform 0 and switching to platform 1 if

$$v_i + e_i n_{0,2}^j - \theta_0^i - p_{0,2}^i = v_i + e_i(1 - n_{0,2}^j) - (1 - \theta_0^i) - p_{1,2}^i - s_i.$$

The indifferent consumer is given by

$$\theta_0^i = \frac{1}{2} + \frac{1}{2}[e_i(2n_{0,2}^j - 1) + p_{1,2}^i - p_{0,2}^i + s_i].$$

Another consumer on side i, positioned at θ_1^i, previously purchased from platform 1 is indifferent between switching to platform 0 and continuing to purchase from platform 1 if

$$v_i + e_i n_{0,2}^j - \theta_1^i - p_{0,2}^i - s_i = v_i + e_i(1 - n_{0,2}^j) - (1 - \theta_1^i) - p_{1,2}^i.$$

The indifferent consumer is given by

$$\theta_1^i = \frac{1}{2} + \frac{1}{2}[e_i(2n_{0,2}^j - 1) + p_{1,2}^i - p_{0,2}^i - s_i].$$

The second-period demand for platform 0 on side i is then given by

$$n_{0,2}^i = n_{0,1}^i \theta_0^i + (1 - n_{0,1}^i)\theta_1^i, \tag{2}$$

which consists of its first-period customers, who do not switch in the second period (the first term on the right hand side), and the first-period customers of platform 1, who switch to platform 0 in the second period (the second term on the right hand side). The total demand for platform 1 is defined similarly.

Solving for the second-period market shares and substituting them into the profit functions, we obtain the following second-period prices:

Proposition 1. *Given first-period market shares $n_{k,1}^i$, $i \in \{A, B\}$, $k \in \{0, 1\}$, the second period prices are given by*

$$p_{0,2}^i = 1 - e_j + \frac{\eta_i(2n_{0,1}^i - 1)s_i + \epsilon_i(2n_{0,1}^j - 1)s_j}{\Delta},$$

and

$$p_{1,2}^i = 1 - e_j - \frac{\eta_i(2n_{0,1}^i - 1)s_i + \epsilon_i(2n_{0,1}^j - 1)s_j}{\Delta},$$

where $\Delta = 9 - (e_A + 2e_B)(e_B + 2e_A) > 0$, $\eta_i = 3 - e_j(e_j + 2e_i) > 0$, and $\epsilon_i = e_i - e_j$.

Proof. See Appendix A. □

The second-period prices consist of two parts: the first part, $1 - e_j$, is independent of switching costs, and is analogous to the result in the one-period model of Armstrong (2006a); on the other hand, the second term is related to switching costs. A closer examination of the second term shows the following.

Corollary 1. *Given first-period market shares, on side i, $i \in \{A, B\}$, the platform with a larger market share ($n^i_{k,1} > 1/2$), $k \in \{0, 1\}$, increases the second-period price $p^i_{k,2}$ as switching costs s_i increase; whereas the other platform with a smaller market share ($n^i_{k,1} < 1/2$) decreases the second-period price $p^i_{k,2}$ as switching costs s_i increase. When platforms have equal market shares in the first period ($n^i_{0,1} = n^i_{1,1}$), switching cost s_i does not affect second-period prices $p^i_{k,2}$.*

Proof. See Appendix A.1. □

This result is standard in the switching cost literature, where two effects are at play: On the one hand, the platform might want to exploit its locked-in customers with a high price due to its market power over these customers. On the other hand, the platform might want to poach its rival's customers with a low price. A larger market share means exploiting old customers is more profitable than attracting new consumers. Notice that if both platforms have equal market share in the first period, these two effects offset each other, which means that switching cost does not affect second period prices. This is indeed what happens in the symmetric equilibrium (see Proposition 2). However, analyzing second-period pricing strategy is important because it determines the intertemporal effect of first-period pricing: a first-period price change will lead to a change in the second-period profit and hence the second-period price.

Moreover, there is a new cross-group effect of switching cost, i.e., the pricing on one side also depends on the switching cost on the other side.

Corollary 2. *Given first-period market shares, the second-period price of platform 0, $p^i_{0,2}$, is increasing in switching costs on the other side s_j if*

(i) Consumers on side j are more valuable ($e_i > e_j$), and platform 0 has a larger market share on side j as compared to platform 1 ($n^j_{0,1} > 1/2 > n^j_{1,1}$), or

(ii) Consumers on side i are more valuable ($e_i < e_j$), and platform 0 has a smaller market share on side j as compared to platform 1 ($n^j_{0,1} < 1/2 < n^j_{1,1}$).

Proof. See Appendix A.2. □

More specifically, switching costs on side j create some market power for platforms, which is captured by the term $(2n^j_{0,1} - 1)s_j$. Such market power can be extended to side i in two ways. First, the value of attracting consumers on side i increases when they generate positive network externalities on side j. This tends to intensify competition on side i, and reduces the price by $-e_j(2n^j_{0,1} - 1)s_j/\Delta$. Second, the value of attracting consumers on side i goes down when side j generates positive network externalities on side i. This tends to weaken competition on side i, and increases the price by $e_i(2n^j_{0,1} - 1)s_j/\Delta$. The overall effect is then positive in two situations: (i) when a platform has more market power than its rival, meaning that it has a larger first-period customer base (that is, $n^j_{0,1} > 1/2$), and the effect of weakening competition is stronger than the intensifying effect (that is, $e_i > e_j$), which reinforces the market power created by switching costs; and (ii) when a platform has less market power than its rival (that is, $n^j_{0,1} < 1/2$), the price on side j decreases as s_j increases. This reduces the value of attracting consumers on side i, and reverses the sign of the two effects. That is, $-e_j(2n^j_{0,1}-1)s_j/\Delta$ now becomes positive, whereas $e_i(2n^j_{0,1}-1)s_j/\Delta$ becomes negative. Since $e_i < e_j$, the first effect dominates, and the price on side i increases. Note that what platform 1 will do is just the opposite of platform 0 because the market is fully covered.

2.2 First Period: the new market

I now turn to the first-period equilibrium outcomes when consumers are unattached. If a consumer on side i located at θ^i_R joins platform 0 in the first period, his expected second-period utility is given by

$$U^i_{0,2} = \int_0^{\theta^i_0}(v_i + e_i n^j_{0,2} - x - p^i_{0,2})dx + \int_{\theta^i_0}^1 (v_i + e_i(1-n^j_{0,2}) - (1-x) - p^i_{1,2} - s_i)dx.$$

The first term captures the situation in which the consumer's preference has changed a little (that is, $x < \theta^i_0$) and chooses to stay with platform 0, and the second term captures the situation in which the consumer's preference has changed a lot (that is, $x > \theta^i_0$) and chooses to switch to platform 1, despite there being some switching costs.

Similarly, if he joins platform 1 in the first period, his expected second-period utility is given by

$$U^i_{1,2} = \int_{\theta^i_1}^1 (v_i + e_i(1-n^j_{0,2}) - (1-x) - p^i_{1,2})dx + \int_0^{\theta^i_1}(v_i + e_i n^j_{0,2} - x - p^i_{0,2} - s_i)dx.$$

A consumer on side i is indifferent between purchasing from platform 0 and platform 1 if

$$v_i + e_i n^j_{0,1} - \theta^i_R - p^i_{0,1} + \delta_i U^i_{0,2} = v_i + e_i(1-n^j_{0,1}) - (1-\theta^i_R) - p^i_{1,1} + \delta_i U^i_{1,2}.$$

After some rearrangement, this gives

$$\theta^i_R = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,1}-1) + p^i_{1,1} - p^i_{0,1} + \delta_i(U^i_{0,2} - U^i_{1,2})],$$

and this is also the first-period market share of platform 0 on side i, that is, $n^i_{0,1} = \theta^i_R$.

Hence, we can derive the profit functions, and solve for the equilibrium prices. I focus on the platform-symmetric equilibrium, which means that both platforms charge the same price to each side (that is, $p^A_{0,1} = p^A_{1,1}$ and $p^B_{0,1} = p^B_{1,1}$).[12]

Proposition 2. *The single-homing model has a unique symmetric equilibrium. The equilibrium prices are given by*

$$p^i_{0,1} = 1 - e_j - \kappa_i s_i^2 - \sigma_j s_j s_i - \delta_F \xi_i s_i,$$

and

$$p^i_{0,2} = 1 - e_j,$$

where σ_i and ξ_i are positive, κ_i may be positive or negative, for $i, j \in \{A, B\}$, and $j \neq i$.

[12] When e_i and s_i are relatively small, I show in Appendix B that there is a unique symmetric equilibrium. However, analyzing the existence of other asymmetric equilibria is beyond the scope of this paper.

Proof. See Appendix B, where expressions for κ_i, σ_i, and ξ_i are also given. □

The derivation of the equilibrium is in Appendix B, and the intuitions are as follows: the existence and uniqueness of the symmetric equilibrium requires that platforms' profits are concave (see Equation B.1), and there is no profitable deviation from the equilibrium prices both on and off the equilibrium path (see Equation B.3 and B.4).[13]

Both Klemperer (1987b) and Armstrong (2006a) are special cases of this model. More particularly, when there are no switching costs ($s_i = 0$), or neither the consumers nor the firms care about the future ($\delta_i, \delta_F = 0$), $i \in \{A, B\}$, the first-period equilibrium price becomes

$$p^i = 1 - e_j,$$

which is the same as that in Proposition 2 of Armstrong (2006a). This equation shows that platforms compete fiercely for the more valuable group, whose external benefit exerted on the other group of consumers is larger.

On the other hand, when there are no indirect network externalities ($e_i = 0$), $i \in \{A, B\}$, the first-period equilibrium price becomes

$$p^i_{0,1} = 1 + \frac{2}{3}(\underbrace{\delta_i s_i^2}_{consumer\ anticipation} - \underbrace{\delta_F s_i}_{firm\ anticipation}),$$

which is equivalent to Equation (18) in Klemperer (1987b).[14]

Since the level of the first-period price is lower in a market with switching costs than without them, the literature calls it a "bargain". However, the extent of the bargain depends on switching costs. More specifically, Klemperer (1987b) shows that the first-period price is U-shaped in switching costs: whether the first-period price increases or decreases with switching costs depends on the relative strength of the consumer and the firm anticipation effects. On the one hand, farsighted consumers anticipate that if they are locked-in in the second period, the platform will raise its price. Thus, consumers are less responsive to a first-period price cut. This explains why consumer sophistication increases the first-period price through δ_i. On the other hand, forward-looking platforms have strong incentives to invest in market share because they anticipate the benefit of having a larger customer base in the future. Thus, platform sophistication decreases the first-period price through δ_F.

[13]Notice that although I focus on the analysis of symmetric equilibrium, its existence does not require all parameters on the two sides (e_A and e_B, s_A and s_B) to be completely symmetric, provided that consumers on each side view the platforms as symmetric. I briefly discuss the case of asymmetric platforms where it is more costly to switch from one platform to the other ($s_0 \neq s_1$) in Section 5.3.

[14]In Klemperer's equation,

$$p_1^A = p_1^B = c + t\left\{1 + \lambda\left[(1-\mu-\nu) + \frac{2}{3(\mu+\nu)}\left[(1-\mu-\nu) + \frac{\mu s}{t}\right]^2\right]\right\} - \frac{2\lambda}{3(\mu+\nu)}[(1-\mu-\nu)t + \mu s],$$

c is the marginal cost of production, which is equal to 0 here, ν represents new consumers, which is equal to 0 here, μ represents consumers with changing preferences, which is equal to 1 here, $1-\nu-\mu$ represents consumers with unchanged preferences, which is equal to 0 here, λ is the discount factor, which is equal to δ_i and δ_F here, and t and s are defined similarly as transport cost and switching cost.

2.3 The Indirect Bargain

The pattern of attractive introductory offers followed by higher prices to exploit locked-in customers—the "bargains-then-ripoffs" pricing—is well-known in the literature. However, this analysis is the first to decouple the "direct" bargain effect of switching costs in the first period from the "indirect" bargain effect.[15] More specifically,

- A direct bargain means that the first-period price is lower with switching costs than without, as defined in Klemperer (1987b).
- An indirect bargain means increasing participation on one side increases the value of the platform to the other side, and such indirect network effects leads to an even bigger bargain effect of switching costs for consumers on the other side, which is a new effect in the literature.

Let us now examine this indirect bargain more formally.

Proposition 3. *In the single-homing model, with all consumers and both platforms equally patient $\delta_i = \delta_F = \delta > 0$, and symmetric externalities $e_i = e > 0$, $i \in \{A, B\}$, the first-period price is given by*

$$p^i_{0,1} = 1 \underbrace{-e}_{\textit{Armstrong (2006a)}} + \underbrace{\frac{2\delta}{3}(s_i^2 - s_i)}_{\textit{Klemperer (1987b)}} - \underbrace{\frac{\delta}{3(1-e^2)}(e^2 s_i^2 + e s_i s_j)}_{\textit{Indirect bargain}}.$$

There exists a threshold $\bar{e} = (\sqrt{s_j^2 + 32} - s_j)/8 \in (0,1)$ such that

i. If network externalities are weak ($e < \bar{e}$), on each side the first-period price $p^i_{0,1}$ is U-shaped in switching costs s_i.

ii. If network externalities are strong ($e \geq \bar{e}$), on each side the first-period price $p^i_{0,1}$ is decreasing in switching costs s_i.

iii. The first-period price charged to side i, $p^i_{0,1}$, is decreasing in switching costs on side j, s_j.

iv. The first-period price charged to side i, $p^i_{0,1}$, is decreasing in network externalities, e.

Proof. See Appendix C. □

Proposition 3 shows that the extent of the bargain does not depend only on switching costs on one side (as in Klemperer) and the strength of network externalities (as in Armstrong), but also on switching costs on the other side, which operates through the indirect bargain.

More specifically, part (i) shows that when network externalities are weak, we attain the same result as Klemperer: the first-period price is U-shaped in switching costs. The reason

[15] Notice that although the "ripoff" effect in the literature, which means the second-period price paid by consumers is higher in a market with switching costs than in a market without switching costs, does not emerge in the basic model, under price discrimination the ripoff effect on a platform's existing customers can emerge, as shown in Section 5.4.

is that switching costs have two opposing effects on the first-period price. First, the firm anticipation effect tends to lower the first-period price: as platforms can charge higher prices to exploit locked-in customers in the second period, they will compete aggressively for first-period market shares. Second, the consumer anticipation effect tends to increase the first-period price: anticipating that platforms will take advantage of the locked-in customers in the second period, consumers are less responsive to first-period price cuts. Because the firm anticipation effect is stronger for low switching costs and the consumer anticipation effect is stronger for high switching costs, the U shape emerges.

Part (ii) describes the first term in the indirect bargain. It shows that strong externalities overturn the U-shaped result: the first-period price is always decreasing in switching costs, and the positive relationship between the first-period price and switching costs does not arise. The reason is that network externalities together with switching costs weaken the consumer anticipation effect because participation on the other side increases the value of the platform, even though consumers anticipate that the platform might exploit their reluctance to switch later. Consequently, switching costs always lead to more first-period price competition when externalities are strong. This effect differs from Armstrong (2006a): since he examines a static two-sided model, both the consumer and firm anticipation effects are absent. It also differs from Klemperer (1987): the consumer anticipation effect in his model *softens* price competition, while the effect in this model *intensifies* competition through the indirect bargain.

Part (iii) describes the second term in the indirect bargain. It shows that an increase in switching costs on one side unambiguously decreases the first-period price charged to the other side. The reason is that platforms can build market share on side j via two channels: directly through side j, and indirectly through side i. When switching costs on side j are large, consumers are less responsive to price cuts because they expect a price rise to follow in the second period. An easier way to build market share on side j is then to focus on the indirect channel, i.e., attracting side i. As a result, the first-period price competition is increased on side i.

Furthermore, because a larger s_j makes it more attractive for platforms to compete for consumers on side i, an increase in s_j decreases the threshold requirement of the level of network externalities for overturning the U-shaped pricing on side i, $\bar{e}$. Interestingly, all these cross-group effects of switching costs are absent from Armstrong's and Klemperer's models.

Notice that the indirect bargain works through consumers' anticipation about their second-period utility, but it does not change firms' anticipation. To see this, consider a variant of the basic model, where consumers and platforms have different discount factors, and network externalities are time-dependent, that is, $e_A = e_B = e_1$ in the first period and $e_A = e_B = e_2$ in the second period. Consequently, the first-period prices become

$$p^i_{0,1} = 1 \underbrace{-e_1}_{Armstrong\ (2006a)} + \underbrace{\frac{2}{3}(\delta_i s_i^2 - \delta_F s_i)}_{Klemperer\ (1987b)} - \underbrace{\frac{\delta_i}{3(1-e_2^2)}(e_2^2 s_i^2 + e_2 s_i s_j)}_{Indirect\ bargain}.$$

Clearly, the firm anticipation effect $-2\delta_F s_i/3$ does not depend on e. The reason is that given first-period market shares, the effect of switching costs on second-period prices do not depend on network externalities, which can be seen from the second-period equilibrium outcome in Proposition 1:

$$p_{0,2}^{i} = 1 - e_2 + \frac{s_i}{3}(2n_{0,1}^{i} - 1).$$

In addition, we can see that the indirect bargain depends only on e_2 but not on e_1.

Taken together, the results of (ii) and (iii) show that we cannot simply add up the bargain effects of switching costs and network externalities as combined together they will lead to an even bigger price reduction than the sum of price reductions in pure switching cost and pure two-sided models. This is complementary to the literature because it provides a formal explanation for the mechanism through which the interactions of es_i and es_is_j work.

Part (iv) shows that stronger network externalities not only intensify price competition directly as in Armstrong (2006a), but they also lead to an increase in the indirect bargain effect, which further pushes down the first-period prices. This also explains why there is a threshold value for e above which the standard U shape ceases to hold: since the indirect bargain is a monotone function of e, adding it to the standard U-shaped direct bargain initially flattens the U shape when externalities are weak, but as externalities become stronger, the indirect bargain effect eventually dominates and results in the first-period prices decreasing in s.

3 Profits and Managerial Implications

I now turn to the effect of switching costs on profits. In the platform-symmetric equilibrium, the two platforms share consumers on each side equally, that is, $n_{0,1}^{A} = n_{0,1}^{B} = 1/2$. Therefore, the expected profit of platform 0 is

$$\pi_0 = \frac{1}{2}p_{0,1}^{A} + \frac{1}{2}p_{0,1}^{B} + \delta\pi_{0,2},$$

where $\pi_{0,2}$ is the second-period profit.

Corollary 3. *There exists a threshold $\hat{e} = (\sqrt{s_j^2 + 8} - s_j)/4 \in (0,1)$ such that, if network externalities are weak ($e < \hat{e}$), the total profit of each platform π_0 is U-shaped in switching costs s_i, whereas if network externalities are strong ($e \geq \hat{e}$), π_0 decreases with s_i. Furthermore, π_0 always decreases with e.*

Proof. See Appendix D. □

Since the profit in the second period, $\pi_{0,2}$, is not affected by switching costs in equilibrium, it is clear that the comparative statics of profits with respect to switching costs and that of first-period prices with respect to switching costs analyzed in Proposition 3 work in the same direction. Similarly, with respect to network externalities, the comparative statics on profits is closely related to that on first-period prices performed in Proposition 3 except that network externalities have an additional effect on second-period prices. Given $p_{0,2}^{i} = 1 - e$ (as shown in Proposition 2), it is clear that stronger network externalities always reduce the total profits of a platform because they intensify price competition in both periods.

In the following discussion I focus on the case where e is large enough.[16] As in Klemperer (1987b), switching costs do not affect the second-period profit of the platform, but they lead to a decrease in overall profits because the presence of market power over locked-in customers intensifies price competition in the first period. More interestingly, I identify a new channel—the indirect bargain—through which switching costs can further reduce overall profits. In particular, there are three ways in which the indirect bargain can affect profits: First, network externalities together with switching costs weaken the consumer anticipation effect because consumers value participation on the other side. This effect distinguishes this model from that of Armstrong (2006a) because such an effect is absent from his model, and from that of Klemperer (1987b) because the consumer anticipation effect goes in the opposite direction. As a result, switching costs on side i intensify price competition on side i (see (ii) of Proposition 3). The second effect of the indirect bargain, which is absent from both Armstrong (2006a) and Klemperer (1987b), is that higher switching costs on side i also lead to more competitive behavior on side j because capturing more consumers on side j is a cheaper way to build market share on side i. Side i consumers are harder to attract as they have strong incentives to avoid being locked-in and thus paying large switching costs in the second period (see (iii) of Proposition 3). Third, profits are decreasing in network externalities, as network externalities not only have their own direct effect of intensifying price competition as in Armstrong (2006a) but also have an indirect effect of strengthening the indirect bargain (see (iv) of Proposition 3). Thus, the combination of switching costs and network externalities can decrease overall profits more dramatically than can each of these ingredients alone.

This result can be useful for thinking about managerial policies in platform markets such as that for smartphone and video games, where switching costs are present on both sides. In the smartphone market, switching from Apple's iOS to Google's Android system, application developers need to re-code their programs for different interfaces, as well as to create additional support and maintenance, whereas application users need to migrate and re-purchase their applications. In the video games market, switching from Sony's PlayStation to Windows' Xbox, gamers need to re-learn how to use the controller and lose the progress of their games, whereas developers have to buy a separate development kit to create games for different consoles. Since both switching costs and network externalities reduce platforms' overall profits, it would be desirable for platforms to lower either or both of them. Let us explore each of these strategies in turn.

In terms of reducing switching costs, platforms can provide guides on how to make a switch across platforms, and introduce apps and services that help data migration, thereby allowing users to access the same media content across platforms (e.g., Google Play Movies & TV on iTunes and cloud computing technology). Moreover, instead of interpreting switching costs simply as transaction costs and learning costs brought about by moving to another platform, one can think of them as measures of backward compatibility without changing the qualitative results. More specifically, when technologies are completely backward compatible, there is no additional cost of staying with the same platform. However, when technologies are only

[16] When e is small, profits are U-shaped in switching costs, for similar reasons described in part (i) of Proposition 3.

partially backward compatible, staying with the same platform creates an additional cost of backward incompatibility. One can incorporate these additional costs in this model by reinterpreting switching costs as the difference between the cost of migration across platforms and the cost of backward compatibility when staying with one platform. Thus, a reduction in switching costs may also mean offering new versions of products being less compatible with older versions.[17]

In terms of reducing network externalities, platforms can introduce more valuable services and functionalities that are not available on other platforms. This has two effects: first, the stand-alone value becomes more important relative to the network benefits; second, as the more valuable services are now integrated with the platform, this lowers the average value of the network benefits that consumers can derive from other services. These two effects taken together decrease the value of network effects. In reality, this fits with Microsoft's recent strategy of integrating its services such as Azure, Cortana, Office, and Xbox more deeply with its own operating system,[18] as well as Apple's strategy of integrating its OS X Yosemite on Mac and iOS on iPhones more closely, both of which aim at improving the value of their services relative to their rivals'.

4 Welfare and Policy Implications

In the previous sections, I examined the effect of switching costs on prices and profits in the presence of network externalities. Because of the indirect bargain, the overall effect of switching costs on prices is not simply the sum of effects of switching costs and network externalities individually. Rather, the overall effect exceeds this sum. One should, therefore, be cautious about how to evaluate the impact of these two ingredients on consumer and social welfare also, both of which I discuss below. It is useful to keep in mind that the whole analysis has been carried out under the assumption that the market is fully covered and consumers have inelastic demand. First consider social welfare:

Proposition 4. *Social welfare decreases with switching costs.*

Proof. The first-period social welfare is constant in switching costs, while the second-period welfare is decreasing in switching costs. More specifically, the second-period welfare loss is the sum of two deadweight losses:

$$2[\underbrace{(\frac{1-s_i}{2})s_i}_{DWL\ from\ switchers} + \underbrace{\frac{s_i^2}{4}}_{DWL\ from\ non-switchers}],$$

which is increasing in s_i. □

[17]Notice, however, that there may be other forms of backward compatibility that are not covered here. For example, when there are different generations of consumers, backward compatibility may mean later generations benefiting from earlier generations through network effects, but this form of compatibility is not included here, because there is no cross-generation network effect.

[18]In particular, the new Windows 10 allows users' data to integrate more smoothly from phones and tablets to PC's and Xbox game consoles.

The first-period social welfare is constant in switching costs because all consumers buy one unit of good, the size of the two groups is fixed, and the whole market is served. There are no demand-expansion and demand-reduction effects of switching costs as the total demand is fixed. However, the second-period welfare is decreasing in switching costs. The second-period welfare loss is the sum of two deadweight losses. Considering consumers who have previously bought from platform 0, those whose tastes change a lot will switch to platform 1 with probability $(1-s_i)/2$ and each pays s_i; those whose tastes change a little will continue to buy from platform 0 even though they prefer platform 1, which happens with probability $s_i/2$ and each suffers an average loss of mismatch with an inferior product $s_i/2$.[19] A similar distortion arises for consumers who have previously bought from platform 1, and for both groups of consumers.

Another welfare criterion concerns consumer surplus:

Proposition 5. *In the symmetric case of $e_i = e$, $s_i = s$ and $\delta_i = \delta_F = \delta$, $i \in \{A, B\}$, if network externalities are strong ($e \geq 3/7$), consumer surplus is higher with switching costs than without. However, if network externalities are weak ($e < 3/7$), there exists a threshold $\hat{s} \in (0, 1)$ such that consumer surplus is higher with switching costs than without for small switching costs ($s < \hat{s}$), and consumer surplus is lower for larger switching costs ($s \geq \hat{s}$).*

Proof. See Appendix E. □

Although switching costs lower social welfare, consumers may enjoy a net gain when the benefit from a lower first-period price is larger than the sum of the two deadweight losses. This is especially true when network externalities are strong: the indirect bargain weakens the standard consumer anticipation effect in Klemperer's result and may even reverse it, and it further decreases the first-period price through the interaction between switching costs and network externalities, which is in addition to the direct effect of network externalities as in Armstrong's result.

However, although total consumer surplus increases with switching costs under strong network externalities, higher switching costs may hurt some consumers. More specifically, consider consumers who have purchased from platform 0 in the first period, and suppose that there is an increase in s by Δs. Consumers can then be categorized into three types according to the impact of such an increase on them. First, consumers whose second-period preference falls into the interval of $[0, 1/2 + s/2)$ do not switch before and after the increase, and therefore benefit from a decrease in the first-period prices. Second, consumers whose second-period preference falls into the interval of $[1/2 + (s + \Delta s)/2, 1]$ indeed switch, and they have to balance the benefit from lower first-period prices against the cost of switching. Third, consumers whose second-period preference falls into the interval of $(1/2 + s/2, 1/2 + (s + \Delta s)/2)$ do not switch but would have switched if there were no increase in s. For these consumers, they enjoy the benefits of lower first-period prices, but because they are stuck with their first-period choice,

[19] The gain in better match is not relevant to the analysis of the effect of switching costs on social welfare because such gain is not affected by switching costs. To see why, suppose that there are no switching costs, consumers can switch if they prefer, and thus get a better match of product with their tastes. Suppose now that switching costs are positive, consumers who switch will still gain from a better match. On the contrary, the loss in mismatch is affected by switching costs, as switching costs may prevent some consumers from switching.

they also incur a cost of product mismatch, which is equal to the value of switching cost s. More formally, for the latter two categories of consumers, the marginal benefit of raising s is

$$\frac{\partial \Delta p}{\partial s} = \frac{2(1-e^2) + 2(3e^2 + e - 2)s}{3(1-e^2)},$$

which can be obtained by differentiating the benefit of price reduction associated with an increase in s with respect to s (the details of these calculations are given in Appendix E), whereas the marginal cost is $ds/ds = 1$. Thus, they are indeed worse off following an increase in s if

$$2s(3e^2 + e - 2) < 1 - e^2.$$

This model thus provides two general policy rules in two-sided markets with switching costs. First, policymakers need to consider demand interdependencies more carefully, especially because they will affect how the indirect bargain works. However, since most of the theoretical models that study the welfare effects of switching costs rely on the assumption that the market is one-sided (see Section 2.9 in Farrell and Klemperer (2007)), they are generally not applicable to studying two-sided markets. Many papers in the two-sided literature, for instance, Wright (2004) and Evans and Schmalensee (2014), have pointed out that various policies (without focusing on switching costs) that apply one-sided results to two-sided markets are prone to commit errors. The analysis here is complementary to their view. In particular, I show that in the presence of switching costs, demands can be interdependent in two ways: through the direct effect of network externalities, which is the traditional channel in the two-sided literature, and through the indirect effect of network externalities in changing the way prices respond to switching costs, which is the main novelty of this paper. Further, because of this indirect effect, the switching costs of one group of consumers can affect the prices of another group (as shown in Proposition 3), and in addition, switching costs can be less harmful to consumers in a market with network externalities than in a market without network externalities (as shown in Proposition 5). Hence, policymakers should be cautious about consumer protection policies that reduce switching costs in two-sided markets. For instance, because it is common to have bargains-then-ripoffs pricing in one-sided markets with switching costs, attractive introductory offers may call for consumer protection policies that lower switching costs in later periods. In two-sided markets, however, the lowering of switching costs of one group will unambiguously raise the first-period price of the other group, and such a change will benefit consumers on one side while making consumers on the other side worse off. Moreover, even within one group of consumers, when network externalities are strong, it is possible that lowering switching costs may hurt some consumers (especially the non-switchers) because of the increase in first-period prices. Accurate welfare analysis should account for these cross-group and within-group effects associated with switching costs and network effects.

Second, policymakers need to evaluate carefully the size of the indirect bargain. This paper derives new insights on the bargain effect of switching costs: I show in Proposition 3 that the interaction between switching costs and network externalities may lead to a yet bigger bargain effect of switching costs than the sum of effects in traditional models. Policies should avoid mechanical analysis of simply adding up the effects of switching costs and network externalities because this may overestimate the extent to which switching costs can reduce welfare.

From a broader perspective, these rules also apply to compatibility policy as we can easily interpret switching costs as the difference between the cost of migration across platforms and the cost of backward compatibility when staying with one platform, as mentioned in the previous section. Hence, a parallel policy conclusion would be that more compatibility across different versions of one platform's products could be more beneficial to consumers in the presence of network externalities, because an increase in switching costs may mean reducing the compatibility between the products of different platforms or increasing the compatibility between the products of one platform.

5 Extensions

In the model discussed so far, consumers are farsighted and their preferences are independent across periods: what will happen if some consumers care only about their utility in the current period (myopic) and some has unchanged preferences (loyal)? Moreover, the main analysis is based on a single-homing model but this is not the only market configuration in reality. One may consider the case where one group single-homes while the other group joins both (commonly termed as "competitive bottlenecks"). It might also be interesting to explore the consequences of asymmetric compatibility between platforms' products, and price discrimination between old consumers of one platform and new consumers from the other platform. In this section, I discuss these extensions in turn, and examine the conditions under which the indirect bargain remains valid.

5.1 Other Forms of Consumer Heterogeneity

Let us first extend the model to incorporate other forms of consumer heterogeneity. Specifically, a consumer can now be one of four types: he can be either farsighted or myopic, and be either loyal or disloyal. Farsighted consumers on side i behave as in the basic model: they make decisions based on their lifetime utility and discount the second-period utility at rate δ_i. Myopic consumers on side i make decisions based on their first-period utility, and therefore have a discount factor $\delta_i = 0$. Disloyal consumers, as in the basic model, have independent preferences across periods and can switch to another platform at a cost of s_i. Loyal consumers do not switch. Their preference, which is represented by their location on the Hotelling line, does not change across the two periods, and hence in the second period, they always stay with the same platform from which they have purchased in the first period.[20]

On each side, a proportion α_i of the consumers is myopic, while $1 - \alpha_i$ of them is farsighted.[21] Moreover, a consumer is loyal with probability μ_i, and has independent preferences

[20]Klemperer (1987b) makes a similar assumption, but he assumes that those consumers, who have unchanged preferences, respond to prices in both periods, so his consumers are not exactly "loyal" in the sense of this paper.

[21]This is different from Klemperer (1987b) because he does not consider the possibility of having a mixture of myopic and farsighted consumers. That is, his consumers are either all myopic or all farsighted.

with probability $1 - \mu_i$.[22] Both α_i and μ_i are common knowledge among platforms and consumers,[23] but in the first period, consumers do not learn their realized switching costs for the second period.[24] The timing of this modified game is as follows:

- At the beginning of the first period, consumers are unattached to any platforms. They learn their initial preferences. Platforms set the first-period prices. Consumers choose which platform to join.

- At the beginning of the second period, consumers learn their switching costs (and hence their loyalty). If indeed consumers are not loyal, they also learn their second-period preferences. Platforms set the second-period prices. Consumers decide to switch or not.

The equilibrium of this game is derived in Appendix F. Similar to the basic model, the existence of the symmetric equilibrium requires that platforms' profits are concave, there is no profitable deviation from the equilibrium prices both on and off the equilibrium path, and in addition, it requires that there is no profitable deviation for platforms to serve only their loyal customers.

First, consider the case where consumers are myopic.[25] The effect of switching costs on prices and welfare are as follows.

Proposition 6. *In a model with myopic consumers ($\delta_i = 0$ and $\alpha_i = 1$), independent preferences $\mu_i = 0$, and symmetric externalities $e_i = e$ and switching costs $s_i = s$, $i \in \{A, B\}$, the first-period price is given by*

$$p^i_{0,1} = 1 - e - \frac{2}{3}\delta_F s.$$

The first-period price $p^i_{0,1}$ is decreasing in s, social welfare is decreasing in s, and consumer surplus is increasing in s, regardless of the level of network externalities e.

Proof. See Appendix G □

Myopic consumers do not anticipate that a first-period price cut will lead to a second-period price rise, and will therefore react more responsively to price cut in the first period. This increases the incentives of platforms to compete for myopic consumers. This result is broadly consistent with that of von Weizsäcker (1984) and Borenstein, MacKie-Mason and

[22] Ruiz-Aliseda (2013) shows that the assumption of independent preferences may have unintended consequences of price competition in the second period ending up being too soft. The presence of loyal consumers here relaxes such an assumption. This could also support the fact that in practice not all consumers have changing preferences.

[23] Myopia is therefore defined as consumers ignoring the link between their utility in the two periods, even though they observe prices set by both platforms, and α_i and μ_i.

[24] This means that loyalty depends on switching costs and, in particular, s is drawn from a two-point distribution, where s is small with probability $1 - \mu$, and s is big with probability μ.

[25] A straightforward interpretation of myopic consumers is that these consumers only care about their utility in the current period. Or, alternatively, this could be interpreted as "new" consumers, who are different in every period. For example, a company buys some software for their workers in the first period. Some workers leave the company in the second period, and purchase their own software. These workers have a switching cost of learning the new software product that is different from that purchased by their former company, but the company will not take into consideration such a switching cost when making its purchase in the first period.

Netz (2000). They show, for example, that if consumers expect that a firm's price cut is more permanent than their tastes, which can be interpreted as consumers being myopic, then switching costs tend to lower prices.

In addition, this result illustrates that the indirect bargain affects prices only through consumer anticipation: when all consumers are myopic (and hence there is no consumer anticipation), the indirect bargain fully disappears, and the first-period price is indeed the sum of the effect in Armstrong (2006a) and that in Klemperer (1987b). Therefore, the indirect bargain effect remains valid as long as not all consumers are myopic.

Switching costs reduce social welfare because of the two types of deadweight costs borne by the switchers and the non-switchers, who suffer a mismatch between the product and their tastes, as mentioned before.[26] Compared to the case where consumers are farsighted (more precisely, Proposition 5), what is different here is that consumer surplus is always increasing in switching costs regardless of the level of network externalities. The reason is that myopia weakens the consumer anticipation effect, which increases the platforms' incentives for cutting prices, which in turn increases consumer surplus.

On the other hand, when all consumers are farsighted, but some of them are loyal, the indirect bargain is always present.[27] In particular,

Proposition 7. *In a model with loyal and farsighted consumers ($\mu_i > 0$ and $\alpha_i = 0$) and symmetric externalities $e_i = e$, $i \in \{A, B\}$, there exists a threshold $\tilde{e} > \bar{e}$ such that the first-period price $p^i_{0,1}$ is decreasing in switching costs s_i if network externalities are sufficiently strong ($e \geq \tilde{e}$).*

Proof. See Appendix H. □

Hence, the indirect bargain effect is unchanged and the first-period prices can still be decreasing in switching costs, provided that network externalities are strong enough.[28] Moreover, Propositions 3, 6, and 7 together highlight a new explanation for aggressive pricing behavior in two-sided markets. In particular, the strategy of lowering price is not simply due to network externalities, a view that is central to the work of Rochet and Tirole [2003] and Armstrong [2006a], but also depends on switching costs (as shown in Proposition 3) and the characteristics of these consumers (as shown in Propositions 6 and 7).

[26]Myopia itself does not affect social welfare, because when myopic consumer preferences do not change, they make the right product choice and do not switch. When myopic consumer preferences change, switchers have to bear switching costs; and some of the non-switchers are forced into buying an inferior product that does not match their tastes. Hence, the only thing that matters is whether consumer preferences change or not.

[27]Gabszewicz, Pepall and Thisse (1992) also explore the consequences of heterogeneity in consumer brand loyalty, but they consider the pricing strategy of a monopoly incumbent, who anticipates the entry of a rival in the subsequent period, and focus on the effect of loyalty on entry, which is a different issue from this model.

[28]When $e < \tilde{e}$, the first-period price is U-shaped in switching costs as long as μ is small enough (e.g., when $\mu < 1/2$); otherwise, the first-period price could increase with switching costs when there are too many loyal consumers. This is because loyal consumers are farsighted. They expect that a price cut today will be followed by a price rise tomorrow, and such a price rise is larger with loyal consumers than without. As shown in Appendix F, the second-period equilibrium price here is $p^i_{0,2} = \frac{1-e_j(1-\mu_i)}{1-\mu_i}$, which is larger than the price in a model without loyal consumers, $p^i = 1 - e_j$. This makes the demand of loyal consumers even more price inelastic, which in turn increases the incentives for raising prices in the first period.

Another interesting issue arises when we reinterpret $1 - \mu_i$ as the probability of being "ignorant" instead of being "disloyal". In this extension, all consumers know their initial preference before their first-period purchase. While loyal consumers always know their first- and second-period tastes, disloyal consumers, given that their tastes change randomly in the second period, only learn their second-period taste after their first-period purchase. We can then reinterpret μ_i as the fraction of consumers who know their preference, while $1 - \mu_i$ as the remaining consumers who do not know theirs. The latter are ignorant consumers, who receive a random signal about their first-period taste at the beginning of the period and only discover their true taste after consuming the product. In this case, all consumer locations on the line are fixed across periods, but the previous results remain valid provided that the signal received by the ignorant consumers is uniformly distributed. Since the cutoff threshold of e in Proposition 7 above which prices are no longer a U shape but instead a decreasing function of switching costs, $\tilde{e}$, is larger than that in Proposition 3, $\bar{e}$, an increase in μ_i makes it less likely that the first-period price is decreasing in switching costs. This, in turn, suggests that it can be profitable for platforms to provide more information about their products in order to enhance consumers' understanding of their own preferences.

5.2 Competitive Bottlenecks

This model can be extended to the case of competitive bottlenecks. Suppose that side A continues to single-home, while side B may multi-home. The competitive bottleneck framework is typical in markets such as computer operating systems, and online air ticket and hotel bookings. In the operating system market, users use a single OS, Windows OS, Apple's Mac OS X platform or Linux-based OS, while engineers write software for different OS. In the market for online travel booking, consumers rely on one comparison site such as skyscanner.com, lastminute.com or booking.com, but airlines and hotels join multiple platforms in order to gain access to each comparison site's customers. Since side B can join both platforms, switching costs and loyalty on this side are not relevant, so that $s_B, \mu_B = 0$.[29] Thus, a benchmark for comparison to the prices in the multi-homing model would be the case where $s_B, \mu_B = 0$ but both sides single-home. It follows from Proposition 2 that

Corollary 4. *In the single-homing model, with all consumers and both platforms equally patient ($\delta_i = \delta_F = \delta > 0$ and $\alpha_i = 0$), independent preferences $\mu_i = 0$, $i \in \{A, B\}$, and side B does not incur any switching costs ($s_B = 0$), switching costs on one side s_A do not affect the first-period*

[29] Note that the concept of multi-homing is not compatible with switching costs in the current framework. I use two examples to illustrate. First, think of the smartphone market. If the option to multi-home means consumers are able to use both iPhone and Android systems, then it is not reasonable to impose an additional learning cost on them if they switch platform. Another example is the media market. If multi-homing means that advertisers are free to put ads on either or both platforms, then it does not make sense to impose an additional switching cost on these advertisers. One may argue that we can distinguish between learning switching costs (incurred only at a switch to a new supplier) and transactional switching costs (incurred at every switch), as in Nilssen (1992), but switching costs are not relevant on the multi-homing side because learning costs and transaction costs are equivalent in a two-period model. This also explains why it is not useful to consider the case in which both sides multi-home.

price on the other side $p_{0,1}^{B,sh}$, where sh denotes prices in the single-homing model. That is, $p_{0,1}^{B,sh} = 1 - e_A$.

The intuition is that since preferences of consumers on side B in the two periods are unrelated and they do not have switching costs, every period's choice is independent. This means that the first-period price is not affected by that in the second period. Consequently, although side A's switching costs affect side B's second-period price in the subgame equilibrium, it does not affect side B's first-period price.

The main difference between the multi-homing and the single-homing model lies in the market share of consumers on side B, which can be described as follows. In period t, $t \in \{1, 2\}$, a consumer on side B located at $\theta_{0,t}^B$ is indifferent between buying and not buying from platform 0 if

$$v_B + e_B n_{0,t}^A - \theta_{0,t}^B - p_{0,t}^B = 0,$$

which can be simplified to

$$\theta_{0,t}^B = v_B + e_B n_{0,t}^A - p_{0,t}^B.$$

Similarly, a consumer on side B located at $\theta_{1,t}^B$ is indifferent between buying and not buying from platform 1 if

$$v_B + e_B(1 - n_{0,t}^A) - (1 - \theta_{1,t}^B) - p_{1,t}^B = 0,$$

which can be simplified to

$$\theta_{1,t}^B = v_B + e_B(1 - n_{0,t}^A) - p_{1,t}^B.$$

We solve the game by backward induction as before, and consider symmetric equilibrium (see Appendix I for its existence conditions). Then, we can derive the equilibrium prices.

Proposition 8. *In the multi-homing model, with all consumers and both platforms equally patient ($\delta_i = \delta_F = \delta > 0$ and $\alpha_i = 0$), independent preferences $\mu_i = 0$, and symmetric externalities $e_i = e > 0$, $i \in \{A, B\}$,*

- *i. For the single-homing consumers, if network externalities are weak ($e < \sqrt{1/2}$), the first-period price $p_{0,1}^{A,mh}$ is U-shaped in switching costs s_A, whereas if network externalities are strong ($e \geq \sqrt{1/2}$), $p_{0,1}^{A,mh}$ is decreasing in s_A.*
- *ii. The first period price of the multi-homing consumers $p_{0,1}^{B,mh} = v_B/2$ does not depend on switching costs s_A or network externalities e.*
- *iii. First-period prices tend to be higher on the multi-homing side and lower on the single-homing side with respect to the single-homing model in Corollary 4 if the market is fully covered (that is, $e + v_B/2 > 1$), where mh denotes prices in the multi-homing model.*

Proof. See Appendix I. □

Part (i) implies that for single-homing consumers stronger externalities make it more likely that first-period equilibrium prices decrease with switching costs, which is consistent with Proposition 3 in the single-homing model.[30]

As for multi-homing consumers, part (ii) shows that switching costs do not affect the price paid by them. Even though switching costs of the single-homing consumers can in principle affect the second-period price of the multi-homing consumers through network externalities, there is no connection between the second-period and first-period prices of the multi-homing consumers, because for them, each period's choice is independent due to their independent preferences and the lack of switching costs. Notice that although this intuition is similar to that of Corollary 4, $p_{0,1}^{B,mh} \geq p_{0,1}^{B,sh}$, as will be explained below. Moreover, the price of the multi-homing side is not sensitive to network externalities because with multi-homing, consumers on both sides can always interact with each other, and this eliminates the impact of network externalities on prices.

Part (iii) further compares the prices in the multi-homing case to that in the single-homing case. Since side B multi-homes, there is no competition between the two platforms to attract this group. Compared to the single-homing case with $s_B, \mu_B = 0$, the higher first-period price faced by the multi-homing side is a consequence of each platform having monopoly power over this side, and the large revenue is used in the form of lower first-period prices to convince the single-homing side to join the platform.

Regarding welfare, as the prices of the multi-homing consumers do not depend on switching costs, the welfare effect of switching costs is determined solely by their effect on single-homing consumers. Therefore, as in Propositions 4 and 5, social welfare is decreasing in switching costs, and consumer welfare may be increasing in switching costs, depending on the benefits of price reduction in the first period and the deadweight losses associated with switching costs.[31]

5.3 Asymmetric Compatibility

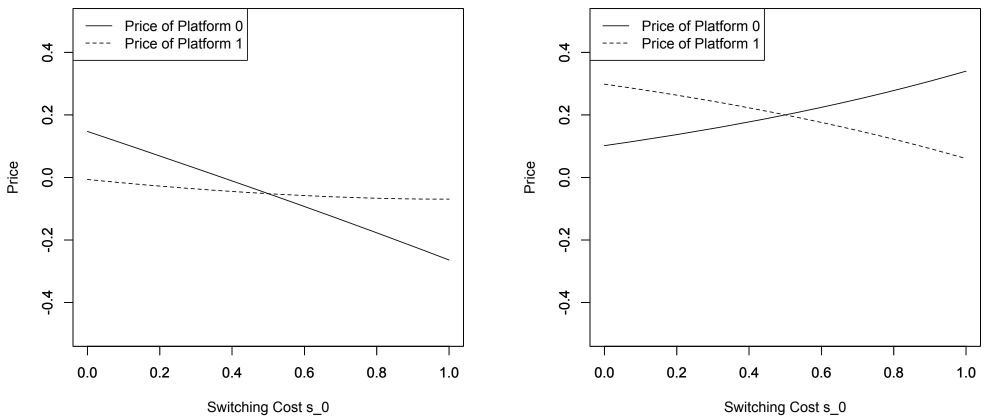
Let us now consider competition between platforms of products with asymmetric compatibility: the cost of switching from platform 0 to 1, denoted s_0, is different from the cost of switching from platform 1 to 0, denoted s_1. In the smartphone market, whereas most iPhones cater for the high-end market, some of the Samsung Android phones are more affordable and cover the lower end of the spectrum. Can we attribute the difference in the pricing of devices between Apple and Samsung to the fact that Apple has successfully built an ecosystem that makes it hard for users to switch?

In this case, the platform-symmetric equilibrium would no longer exist because the platforms become asymmetric. Therefore, in each period, instead of having two different prices in equilibrium, we have four, which makes the analysis less tractable. I consider the following numerical example to illustrate how asymmetric compatibility influences the equilibrium

[30] Notice that the threshold value of e above which the U shape disappears is larger in this case than that in Proposition 3 (that is, $\bar{e} \leq \sqrt{1/2}$). This is because here, there are no switching costs on the multi-homing side that intensify price competition on the single-homing side.

[31] If there is quality choice as in Anderson et al. (2015), then welfare effects are less clear-cut: a platform's investment in quality may change depending on whether multi-homing is allowed.

pricing: $\delta_A = \delta_B = \delta_F = 0.8$, $\mu_A = \mu_B = 0$, $e_A = e_B = 0.8$, $s_1 = 0.5$, and $s_0 \in [0,1]$.[32] In addition, assume that only consumers on side A bear switching costs, and that all consumers single-home. In Figure 1, I illustrate the equilibrium pricing of the platforms on side A. The analysis of the effects of switching costs on side B is irrelevant for similar reasons in Corollary 4.



(a) First-period Prices

(b) Second-period Prices

Figure 1: Equilibrium Pricing with Asymmetric Compatibility.

Panel (a) presents the first-period pricing on side A, and panel (b) shows the second-period pricing on side A as functions of switching costs s_0. Pricing of platform 0 is shown with a solid line, and that of platform 1 is drawn as a dotted line. It is shown that if $s_0 < s_1$, platform 1 charges a lower price than platform 0 in the first period, but a higher price in the second period. The intuitive reason is that since platform 1 is relatively more expensive to switch away from in the second period, it is willing to charge a lower price in the first period in order to acquire more customers whom it can exploit later. On the contrary, if $s_0 > s_1$, platform 1, knowing that consumers will easily switch away tomorrow, will raise its price today.

5.4 Behavior-Based Price Discrimination

Finally, I extend the basic model, where consumers are farsighted and have independent preferences across periods, by allowing platforms to price discriminate based on consumers' purchase history. The difference is that in the second period, instead of setting one price on each side, a platform can set two prices depending on whether the consumer has purchased from it or from the other platform in the first period. To be more specific, in the second period, on side i, platform 0 charges $p_{0,2}^{0,i}$ to its own past customers, and $p_{0,2}^{1,i}$ to new customers who have bought from platform 1 in the first period. As for the first period, platforms charge the same price to all consumers on each side because there is no purchase history available. The game

[32]Notice that $e = 0.8 > \sqrt{1/2}$, which is the threshold for e above which the U shape disappears in Proposition 3 (the platform-symmetric case with $s_j = 0$). And even in this platform-asymmetric case, the U shape does not reappear, which means that the indirect bargain effect is still at work. However, when e is very large (such as $e \to 1$), tipping occurs and an interior solution does not exist anymore.

proceeds as in the basic model, and we solve for the platform-symmetric equilibrium, which can be summarized by the following proposition:

Proposition 9. *Suppose that $e_i = e$, $s_i = s$, and $\delta_i = \delta_F = \delta$, $i \in \{A, B\}$. When third-degree price discrimination is feasible, in the platform-symmetric equilibrium, the first-period prices are given by*

$$p_{0,1}^i = p_{1,1}^i = 1 - e + \frac{2\delta}{3}(s^2 - s) - \frac{\delta}{3}s^2[2 + \frac{e}{3(1-e)}],$$

and the second period-prices are given by

$$p_{0,2}^{0,i} = p_{1,2}^{1,i} = 1 - e + \frac{s}{3}, \quad p_{0,2}^{1,i} = p_{1,2}^{0,i} = 1 - e - \frac{s}{3}.$$

Proof. See Appendix J. □

The indirect bargain is now given by

$$\frac{\delta}{3}s^2[2 + \frac{e}{3(1-e)}],$$

which is still increasing in network externalities. Thus, the key intuitions on the indirect bargain in the basic model are unchanged. The difference is that when price discrimination is feasible, the U shape in Proposition 3 disappears, and the first-period prices are always decreasing in switching costs regardless of the level of network externalities. This is because price discrimination intensifies competition in the first period, which is a standard result in one-sided markets without network externalities.[33] Moreover, in the second period, since the discriminatory prices do not depend on first-period market shares, higher switching costs always increase the price for a platform's own past customers and decrease the price for its rival's customers. However, although switching costs have distributional effects across old and new consumers under price discrimination, it has no effect on the total welfare in the second period because each platform serves half of the consumers on each side in the symmetric equilibrium. More importantly, these results in both the first and second periods show that even if we allow for price discrimination, the main conclusion that switching costs can be less harmful to consumers on average in the presence of network externalities carries through. Thus, policymakers should worry less about switching costs in two-sided markets that exhibit strong network externalities, and this advice remains valid whether or not we allow the prices within each group to vary.

6 Conclusion

This paper has characterized the equilibrium pricing strategy of platforms competing in two-sided markets with switching costs, which can be applied to a wide range of industries, ranging from traditional industries such as shopping malls and credit cards to high-tech industries such as smartphones and video games. The main contribution is that it has provided a useful

[33] See Armstrong (2006b) for a survey of the relevant literature.

model for generalizing, and extending beyond, the traditional results in the switching cost and the two-sided literature. In line with earlier research, there are some conditions under which the first-period price is U-shaped in switching costs (à la Klemperer); and prices tend to be lower on the side that exerts stronger externalities (à la Armstrong). However, this model also provides new insights by proving that in a dynamic two-sided market—as opposed to a merely static one—under strong network externalities, the standard U-shaped result does not emerge and the first-period price always decreases with switching costs. This is due to the existence of the indirect bargain effect of switching costs that does not emerge in either Armstrong's or Klemperer's models. Recognizing the importance of this additional effect is critical for ensuring that consumer protection policies do not cause unintended consequence of reducing consumer welfare by causing more harm to some consumers than good to the other.

The literature on the interaction between switching costs and network externalities is relatively thin and does not provide a solid basis for evaluating their effects. This paper is a first attempt at analyzing the impact of such interaction, but much work remains to be done: First, this paper has taken switching costs as an exogenous feature of the market. Future research could consider endogenous switching costs. Second, this paper has focused on a two-period model, and it would be useful to understand the extent to which the results carry over to a multi-period model. Finally, this paper has explored consumer heterogeneity such as loyalty and farsightedness, but one can think of other forms of heterogeneity. For example, within-group switching costs may be different between the technologically advanced customers and the less advanced ones. Within-group externalities may also be different: youngsters use applications more heavily, and therefore care more about network externalities than their older counterparts, many of whom only use their smartphones for phone calls and text messages. However, including these forms of heterogeneity will complicate the analysis considerably.[34] The current model captures a lot of ingredients in reality, yet is sufficiently tractable to allow for a complete characterization of the equilibrium. This seems to be a reasonable first step to contribute to a literature that has not fully explored the consequences of consumer heterogeneity.

Appendices

A Second Period Equilibrium

Solving for $n^A_{0,2}$ and $n^B_{0,2}$ in Equation (2) simultaneously, we obtain the second-period market shares as follows:

$$n^i_{0,2} = \frac{\gamma + \beta_i + (p^i_{1,2} - p^i_{0,2}) + e_i(p^j_{1,2} - p^j_{0,2})}{2\gamma},$$

[34]See Ambrus and Argenziano (2009) for a model with heterogeneous network effects, where platforms can also price discriminate, but with no switching costs.

where

$$
\begin{aligned}
\gamma =& 1 - e_A e_B, \\
\beta_i =& (2n_{0,1}^i - 1)s_i + (2n_{0,1}^j - 1)e_i s_j.
\end{aligned}
$$

Because $e_i < 1$, we have $\gamma > 0$.

Substituting the market shares into the profit function in Equation (1), and differentiating it with respect to the prices, we obtain the following equations:

$$
\begin{aligned}
\frac{\partial \pi_{0,2}}{\partial p_{0,2}^i} &= n_{0,2}^i - \frac{p_{0,2}^i}{2\gamma} - \frac{p_{0,2}^j}{2\gamma} e_j, \\
\frac{\partial \pi_{1,2}}{\partial p_{1,2}^i} &= 1 - n_{0,2}^i - \frac{p_{1,2}^i}{2\gamma} - \frac{p_{1,2}^j}{2\gamma} e_j.
\end{aligned}
$$

Solving the system of first-order conditions, one finds the following second-period equilibrium prices:

$$
\begin{aligned}
p_{0,2}^i &= 1 - e_j + \frac{\eta_i \lambda_i s_i + \epsilon_i \lambda_j s_j}{\Delta}, \\
p_{1,2}^i &= 1 - e_j - \frac{\eta_i \lambda_i s_i + \epsilon_i \lambda_j s_j}{\Delta},
\end{aligned}
\tag{A.1}
$$

where

$$
\begin{aligned}
\Delta &= 9 - (e_A + 2e_B)(e_B + 2e_A) > 0, \\
\lambda_i &= 2n_{0,1}^i - 1, \\
\eta_i &= 3 - e_j(e_j + 2e_i) > 0, \\
\epsilon_i &= e_i - e_j.
\end{aligned}
$$

A.1 Proof of Corollary 1

Differentiate Equation (A.1) with respect to s_i, we have

$$
\begin{aligned}
\operatorname{sign} \frac{\partial p_{0,2}^i}{\partial s_i} =& \operatorname{sign}(n_{0,1}^i - \frac{1}{2}), \\
\frac{\partial p_{0,2}^i}{\partial s_i} =& -\frac{\partial p_{1,2}^i}{\partial s_i}.
\end{aligned}
$$

A.2 Proof of Corollary 2

Differentiate Equation (A.1) with respect to s_j, we have

$$
\begin{aligned}
\operatorname{sign} \frac{\partial p_{0,2}^i}{\partial s_j} =& \operatorname{sign}(e_i - e_j)(n_{0,1}^j - \frac{1}{2}), \\
\frac{\partial p_{0,2}^i}{\partial s_j} =& -\frac{\partial p_{1,2}^i}{\partial s_j}.
\end{aligned}
$$

B First Period Equilibrium

The market share of platform 0 on side i in the first period (i.e., the indifferent consumer) is given by

$$n_{0,1}^i = \theta_R^i = \frac{1}{2} + \frac{e_i(2n_{0,1}^j - 1) + p_{1,1}^i - p_{0,1}^i + \delta_i s_i \frac{[(e_i+2e_j)\lambda_j s_j + (3-\Delta)\lambda_i s_i]}{\Delta}}{2}.$$

Solving simultaneously for $n_{0,1}^A$ and $n_{0,1}^B$, we obtain:

$$n_{0,1}^i = \frac{1}{2} + \frac{e_i(1-\kappa_j s_j^2)(p_{1,1}^i - p_{0,1}^i) + (e_i + \sigma_i s_i s_j)(p_{1,1}^j - p_{0,1}^j)}{2[(1-\kappa_i s_i^2)(1-\kappa_j s_j^2) - (e_i + \sigma_i s_i s_j)(e_j + \sigma_j s_j s_i)]},$$

where

$$\begin{aligned}\kappa_i =& \frac{\delta_i(3-\Delta)}{\Delta}, \\ \sigma_i =& \frac{\delta_i(e_i + 2e_j)}{\Delta}.\end{aligned}$$

The expected profit of platform 0 is

$$\pi_0 = p_{0,1}^A n_{0,1}^A + p_{0,1}^B n_{0,1}^B + \delta_F \pi_{0,2}.$$

The first-order conditions for maximizing π_0 with respect to $p_{0,1}^A$ and $p_{0,1}^B$ are given as follows:

$$\frac{\partial \pi_0}{\partial p_{0,1}^i} = n_{0,1}^i - p_{0,1}^i \frac{(1-\kappa_j s_j^2)}{2\varphi} - p_{0,1}^j \frac{(e_j + \sigma_j s_j s_i)}{2\varphi} + \delta_F \left[\frac{\partial \pi_{0,2}}{\partial n_{0,1}^i}\frac{\partial n_{0,1}^i}{\partial p_{0,1}^i} + \frac{\partial \pi_{0,2}}{\partial n_{0,1}^j}\frac{\partial n_{0,1}^j}{\partial p_{0,1}^i}\right]$$

where

$$\begin{aligned}\varphi =& (1-\kappa_i s_i^2)(1-\kappa_j s_j^2) - (e_i + \sigma_i s_i s_j)(e_j + \sigma_j s_j s_i), \\ \frac{\partial \pi_{0,2}}{\partial n_{0,1}^i} =& \left[\frac{6}{\Delta} + \frac{(e_i - e_j) - (e_i + e_j)(e_j + 2e_i)}{\Delta}\right] s_i \stackrel{\text{def}}{=} \xi_i s_i.\end{aligned}$$

Similarly, there are two first-order conditions for platform 1.

In the platform-symmetric equilibrium, where $p_{0,1}^A = p_{1,1}^A = p^A$ and $p_{0,1}^B = p_{1,1}^B = p^B$, the sufficient condition for platform k's profit being concave in its prices is as follows:[35]

$$1 - \kappa_A s_A^2 > e_A + \sigma_A s_A s_B > 0; \quad 1 - \kappa_B s_B^2 > e_B + \sigma_B s_B s_A > 0. \tag{B.1}$$

In the symmetric equilibrium, the first-period equilibrium prices for side A and side B are given respectively by

$$p_{0,1}^A = 1 - e_B - \kappa_A s_A^2 - \sigma_B s_B s_A - \delta_F \xi_A s_A; \quad p_{0,1}^B = 1 - e_A - \kappa_B s_B^2 - \sigma_A s_A s_B - \delta_F \xi_B s_B, \tag{B.2}$$

and the second-period equilibrium prices are given by

$$p_{0,2}^A = 1 - e_B; \quad p_{0,2}^B = 1 - e_A.$$

[35]Notice that when $s_i = 0$, we obtain the same existence condition for the symmetric equilibrium as in Armstrong (2006a). That is, Equation (B.1) here is analogous to Equation (8) in Armstrong (2006a). However, I show that the equilibrium exists for a wider range of parameters and, in particular, when $s_i > 0$.

To ensure that the above prices are indeed the equilibrium, we need v_i to be big enough such that the market is covered; there is no profitable deviation from the equilibrium prices both on and off the equilibrium path, that is,

$$v_i > 2(1 - e_j) + 1 - e_i. \tag{B.3}$$

Second, we need to show that both platforms are active in the second period out of the equilibrium path. In particular, this requires that the second-period profit is non-negative even for the platform without any customer in the first period, that is,

$$p^i_{0,2}(n^i_{0,1} = 0, n^j_{0,1} = 0) \geq 0. \tag{B.4}$$

Notice that all these conditions hold even when some parameters on the two sides are not completely symmetric. Moreover, we can show that in the symmetric case where $e_A = e_B = e$ and $s_A = s_B = s$, this condition simplifies to

$$\frac{s}{3} + e \leq 1,$$

which means that s and e cannot be too large simultaneously. Similarly, for overall profits to be non-negative (that is, $\pi_i = \pi_{i,1} + \delta\pi_{i,2} \geq 0$), s and e cannot be too large simultaneously.

Indeed, when Conditions (B.1), (B.3) and (B.4) hold, the prices in Equation (B.2) are the unique symmetric equilibrium prices. It follows from each platform's profit function being concave that there is a unique solution to the set of first-order conditions, which are linear.

C Proof of Proposition 3

If $\delta_A = \delta_B = \delta_F = \delta > 0$, and $e_A = e_B = e > 0$, Equation (B.2) becomes

$$p^i_{0,1} = 1 - e + \frac{2\delta}{3}(s_i^2 - s_i) - \frac{\delta}{3(1-e^2)}(e^2 s_i^2 + e s_i s_j).$$

Differentiating $p^i_{0,1}$ with respect to s_i, we obtain

$$\begin{aligned}
\frac{\partial p^i_{0,1}}{\partial s_i} &= \frac{\delta}{3(1-e^2)}\left[2(2-3e^2)s_i - 2(1-e^2) - e s_j\right], \\
\frac{\partial^2 p^i_{0,1}}{\partial s_i^2} &= \frac{2\delta(2-3e^2)}{3(1-e^2)}\begin{cases} > 0 & \text{if } e < \sqrt{2/3}, \\ < 0 & \text{if } e \geq \sqrt{2/3}. \end{cases}
\end{aligned}$$

From this, we have

$$\frac{\partial p^i_{0,1}}{\partial s_i}|_{s_i=0} = \frac{\delta}{3(1-e^2)}\left[-2(1-e^2) - e s_j\right],$$

which is always negative. Moreover, we have

$$\frac{\partial p^i_{0,1}}{\partial s_i}|_{s_i=1} = \frac{\delta}{3(1-e^2)}(2 - 4e^2 - e s_j),$$

which is positive if

$$e < \bar{e} = \frac{-s_j + \sqrt{s_j^2 + 32}}{8}.$$

Notice that $\bar{e}$ is decreasing in s_j, and $\bar{e} = \sqrt{1/2} < \sqrt{2/3}$ when $s_j = 0$. Therefore, $p^i_{0,1}$ is U-shaped in s_i if $e < \bar{e}$, and decreasing in s_i if $e \geq \bar{e}$.

Differentiating $p^i_{0,1}$ with respect to s_j, we get

$$\frac{\partial p^i_{0,1}}{\partial s_j} = -\frac{\delta e s_i}{3(1-e^2)} < 0.$$

Therefore, $p^i_{0,1}$ is decreasing in s_j.

D Proof of Corollary 3

The first-order condition of π_0 with respect to s_i is

$$\begin{aligned}\frac{\partial \pi_0}{\partial s_i} =& \frac{1}{2}\frac{\partial p^i_{0,1}}{\partial s_i} + \frac{1}{2}\frac{\partial p^j_{0,1}}{\partial s_i} \\ =& \frac{\delta}{6(1-e^2)}\left[2(2-3e^2)s_i - 2(1-e^2) - 2es_j\right].\end{aligned}$$

As in Proposition 3, we have $\partial^2\pi_0/\partial s_i^2 > 0$ if $e < \sqrt{2/3}$, and $\partial^2\pi_0/\partial s_i^2 < 0$ if $e \geq \sqrt{2/3}$. Moreover, we have

$$\frac{\partial \pi_0}{\partial s_i}|_{s_i=0} = \frac{\delta}{6(1-e^2)}\left[-2(1-e^2) - 2es_j\right],$$

which is always negative, and

$$\frac{\partial \pi_0}{\partial s_i}|_{s_i=1} = \frac{\delta}{6(1-e^2)}(2-4e^2-2es_j),$$

which is positive if

$$e < \hat{e} = \frac{-s_j + \sqrt{s_j^2+8}}{4}.$$

Notice that $\hat{e}$ is decreasing in s_j, and $\hat{e} = \sqrt{1/2} < \sqrt{2/3}$ when $s_j = 0$. Therefore, π_0 is U-shaped in s_i if $e < \hat{e}$, and decreasing in s_i if $e \geq \hat{e}$.

The first-order condition with respect to e is

$$\frac{\partial \pi_0}{\partial e} = \frac{1}{2}\frac{\partial p^i_{0,1}}{\partial e} + \frac{1}{2}\frac{\partial p^j_{0,1}}{\partial e} + \delta\left(\frac{1}{2}\frac{\partial p^i_{0,2}}{\partial e} + \frac{1}{2}\frac{\partial p^j_{0,2}}{\partial e}\right).$$

Since prices in both periods are decreasing in e, it is easy to see that π_0 is also decreasing in e.

E Proof of Proposition 5

Since second-period prices are not affected by switching costs, the impact of switching costs on consumer surplus is equal to the price reduction consumers enjoy with switching costs relative to without minus the discounted second-period deadweight losses. More specifically, the change in consumer surplus is

$$\begin{aligned}\Delta W =& W(s) - W(0)\\ =&\delta[\underbrace{\frac{(3e^2-2)s^2+2(1-e^2)s+es^2}{3(1-e^2)}}_{price\ \ reduction} \underbrace{-(\frac{s}{2}-\frac{s^2}{4})}_{deadweight\ \ loss}]\\ =&\delta(\frac{(9e-5)s^2}{12(1-e)}+\frac{s}{6}),\end{aligned}$$

which is a quadratic function.

Because we have

$$\frac{\partial \Delta W}{\partial s}|_{s=0} = \frac{\delta}{6},$$

which is always positive, and

$$\frac{\partial \Delta W}{\partial s}|_{s=1} = \frac{\delta}{6}(1-\frac{5-9e}{1-e}),$$

which is positive if $e > 1/2$, consumer surplus is increasing in s if $e > 1/2$, and it is inverted U-shaped in s if $e < 1/2$. Furthermore, consumer surplus increases with switching costs for all $s \in [0, 1)$ if $\Delta W|_{s=1} > 0$, which is satisfied for $e \geq 3/7$. If $e < 3/7$, then there exists a threshold $\hat{s} = \frac{2(1-e)}{5-9e} \in (0, 1)$ such that $\Delta W|_{s\in[0,\hat{s})} > 0$ and $\Delta W|_{s\in[\hat{s},1)} < 0$.

F Other Forms of Consumer Heterogeneity

Second Period Equilibrium

Given the first-period market shares $n^A_{0,1}$ and $n^B_{0,1}$, a consumer on side i, located at θ^i_0 on the unit interval, purchased from platform 0 in the first period is indifferent between continuing to buy from platform 0 and switching to platform 1 if

$$v_i + e_i n^j_{0,2} - \theta^i_0 - p^i_{0,2} = v_i + e_i(1 - n^j_{0,2}) - (1 - \theta^i_0) - p^i_{1,2} - s_i.$$

The indifferent consumer is given by

$$\theta^i_0 = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,2} - 1) + p^i_{1,2} - p^i_{0,2} + s_i].$$

Another consumer on side i, positioned at θ^i_1, previously purchased from platform 1 is indifferent between switching to platform 0 and continuing to purchase from platform 1 if

$$v_i + e_i n^j_{0,2} - \theta^i_1 - p^i_{0,2} - s_i = v_i + e_i(1 - n^j_{0,2}) - (1 - \theta^i_1) - p^i_{1,2}.$$

The indifferent consumer is given by

$$\theta^i_1 = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,2} - 1) + p^i_{1,2} - p^i_{0,2} - s_i].$$

The second-period demand for platform 0 on side i is then given by

$$n^i_{0,2} = \mu_i n^i_{0,1} + (1 - \mu_i)n^i_{0,1}\theta^i_0 + (1 - \mu_i)(1 - n^i_{0,1})\theta^i_1.$$

Consumers of platform 0 consist of three types, and similarly for platform 1. The first type of consumers has unchanged preferences and buys from platform 0 in both periods. The second type of consumers has independent preferences across periods, but stays with platform 0 despite having changing preferences. The third type of consumers also has changing preferences, and indeed switches away from platform 1 to platform 0.

Solving for $n^A_{0,2}$ and $n^B_{0,2}$ in the above equations simultaneously, we obtain the second-period market shares as follows:

$$n^i_{0,2} = \frac{\gamma + \beta_i + (1-\mu_i)(p^i_{1,2} - p^i_{0,2}) + e_i(1-\mu_i)(1-\mu_j)(p^j_{1,2} - p^j_{0,2})}{2\gamma},$$

where

$$\begin{aligned}
\gamma =& 1 - (1-\mu_A)(1-\mu_B)e_A e_B, \\
\beta_i =& (2n^i_{0,1} - 1)(\mu_i + (1-\mu_i)s_i) + (2n^j_{0,1} - 1)(1-\mu_i)e_i(\mu_j + (1-\mu_j)s_j).
\end{aligned}$$

Because $e_i < 1$, we have $\gamma > 0$.

Substituting the market shares into the profit function in Equation (1), and differentiating it with respect to the prices, we obtain the following equations:

$$\begin{aligned}
\frac{\partial \pi_{0,2}}{\partial p^i_{0,2}} &= n^i_{0,2} - \frac{p^i_{0,2}}{2\gamma}(1-\mu_i) - \frac{p^j_{0,2}}{2\gamma}e_j(1-\mu_i)(1-\mu_j), \\
\frac{\partial \pi_{1,2}}{\partial p^i_{1,2}} &= 1 - n^i_{0,2} - \frac{p^i_{1,2}}{2\gamma}(1-\mu_i) - \frac{p^j_{1,2}}{2\gamma}e_j(1-\mu_i)(1-\mu_j).
\end{aligned}$$

Solving the system of first-order conditions, one finds the following second-period equilibrium prices:

$$\begin{aligned}
p^i_{0,2} &= \frac{1 - e_j(1-\mu_i)}{1-\mu_i} + \frac{\eta_i\lambda_i\bar{s}_i + \epsilon_i\lambda_j\bar{s}_j}{(1-\mu_i)\Delta}, \\
p^i_{1,2} &= \frac{1 - e_j(1-\mu_i)}{1-\mu_i} - \frac{\eta_i\lambda_i\bar{s}_i + \epsilon_i\lambda_j\bar{s}_j}{(1-\mu_i)\Delta},
\end{aligned}$$

where

$$\begin{aligned}
\Delta &= 9 - (1-\mu_A)(1-\mu_B)(e_A + 2e_B)(e_B + 2e_A) > 0, \\
\lambda_i &= 2n^i_{0,1} - 1, \\
\eta_i &= 3 - e_j(e_j + 2e_i)(1-\mu_i)(1-\mu_j) > 0, \\
\epsilon_i &= (1-\mu_i)(e_i - e_j), \\
\bar{s}_i &= \mu_i + (1-\mu_i)s_i.
\end{aligned}$$

First Period Equilibrium

I now turn to the first-period equilibrium outcomes when consumers are unattached. On side i, a proportion α_i of consumers are myopic (N) with $\delta_i = 0$. They make decisions based on their first-period utility only. The remaining $1 - \alpha_i$ of consumers on side i is farsighted (R) with $\delta_i > 0$. They make decisions based on their lifetime utility.

A myopic consumer on side i located at θ_N^i is indifferent between buying from platform 0 and platform 1 if

$$v_i + e_i n_{0,1}^j - \theta_N^i - p_{0,1}^i = v_i + e_i(1 - n_{0,1}^j) - (1 - \theta_N^i) - p_{1,1}^i,$$

which can be simplified to

$$\theta_N^i = \frac{1}{2} + \frac{1}{2}[e_i(2n_{0,1}^j - 1) + p_{1,1}^i - p_{0,1}^i].$$

As for farsighted consumers, they also take into consideration their second-period utility. If a consumer on side i located at θ_R^i joins platform 0 in the first period, his expected second-period utility is given by

$$\begin{aligned} U_{0,2}^i = \mu_i(v_i + e_i n_{0,2}^j - \theta_R^i - p_{0,2}^i) + (1-\mu_i)\int_0^{\theta_0^i}(v_i + e_i n_{0,2}^j - x - p_{0,2}^i)dx \\ + (1-\mu_i)\int_{\theta_0^i}^1 (v_i + e_i(1 - n_{0,2}^j) - (1-x) - p_{1,2}^i - s_i)dx, \end{aligned}$$

which is the sum of three terms. With probability μ_i the consumer is loyal and chooses to join platform 0 in both periods; with probability $(1-\mu_i)\theta_0^i$ he has independent preferences but still chooses to stay with platform 0; and with probability $(1-\mu)(1-\theta_0^i)$ he has independent preferences and he switches to platform 1.

Similarly, if he joins platform 1 in the first period, his expected second-period utility is given by

$$\begin{aligned} U_{1,2}^i = \mu_i(v_i + e_i(1 - n_{0,2}^j) - (1 - \theta_R^i) - p_{1,2}^i) \\ + (1-\mu_i)\int_{\theta_1^i}^1 (v_i + e_i(1 - n_{0,2}^j) - (1-x) - p_{1,2}^i)dx \\ + (1-\mu_i)\int_0^{\theta_1^i}(v_i + e_i n_{0,2}^j - x - p_{0,2}^i - s_i)dx. \end{aligned}$$

A farsighted consumer on side i is indifferent between purchasing from platform 0 and platform 1 if

$$v_i + e_i n_{0,1}^j - \theta_R^i - p_{0,1}^i + \delta_i U_{0,2}^i = v_i + e_i(1 - n_{0,1}^j) - (1 - \theta_R^i) - p_{1,1}^i + \delta_i U_{1,2}^i,$$

and the indifferent consumer is given by

$$\theta_R^i = \frac{1}{2} + \frac{e_i(2n_{0,1}^j - 1) + p_{1,1}^i - p_{0,1}^i + \delta_i \bar{s}_i \frac{[(1-\mu_i)(e_i+2e_j)\lambda_j\bar{s}_j + (3-\Delta)\lambda_i\bar{s}_i]}{(1-\mu_i)\Delta}}{2(1+\delta_i\mu_i)}.$$

The first-period market share of platform 0 on side i is

$$n_{0,1}^i = \alpha_i\theta_N^i + (1-\alpha_i)\theta_R^i.$$

Substitute θ_N^i and θ_R^i into the above equation, and solve simultaneously for $n_{0,1}^A$ and $n_{0,1}^B$:

$$n_{0,1}^i = \frac{1}{2} + \frac{e_i(1-\kappa_j\bar{s}_j^2)(p_{1,1}^i - p_{0,1}^i) + \tau_j(e_i\tau_i + \sigma_i\bar{s}_i\bar{s}_j)(p_{1,1}^j - p_{0,1}^j)}{2[(1-\kappa_i\bar{s}_i^2)(1-\kappa_j\bar{s}_j^2) - (e_i\tau_i + \sigma_i\bar{s}_i\bar{s}_j)(e_j\tau_j + \sigma_j\bar{s}_j\bar{s}_i)]},$$

where

$$\tau_i = \alpha_i + \frac{1-\alpha_i}{1+\delta_i\mu_i},$$
$$\kappa_i = \frac{\delta_i(3-\Delta)(1-\alpha_i)}{(1-\mu_i)\Delta(1+\delta_i\mu_i)},$$
$$\sigma_i = \frac{\delta_i(e_i+2e_j)(1-\alpha_i)}{\Delta(1+\delta_i\mu_i)}.$$

The expected profit of platform 0 is

$$\pi_0 = p^A_{0,1}n^A_{0,1} + p^B_{0,1}n^B_{0,1} + \delta_F\pi_{0,2}.$$

The first-order conditions for maximizing π_0 with respect to $p^A_{0,1}$ and $p^B_{0,1}$ are given as follows:

$$\frac{\partial\pi_0}{\partial p^i_{0,1}} = n^i_{0,1} - p^i_{0,1}\frac{\tau_i(1-\kappa_j\bar{s}_j^2)}{2\varphi} - p^j_{0,1}\frac{\tau_i(e_j\tau_j+\sigma_j\bar{s}_j\bar{s}_i)}{2\varphi} + \delta_F\left[\frac{\partial\pi_{0,2}}{\partial n^i_{0,1}}\frac{\partial n^i_{0,1}}{\partial p^i_{0,1}} + \frac{\partial\pi_{0,2}}{\partial n^j_{0,1}}\frac{\partial n^j_{0,1}}{\partial p^i_{0,1}}\right]$$

where

$$\varphi = (1-\kappa_i\bar{s}_i^2)(1-\kappa_j\bar{s}_j^2) - (e_i\tau_i+\sigma_i\bar{s}_i\bar{s}_j)(e_j\tau_j+\sigma_j\bar{s}_j\bar{s}_i),$$
$$\frac{\partial\pi_{0,2}}{\partial n^i_{0,1}} = \left[\frac{6}{(1-\mu_i)\Delta} + \frac{(e_i-e_j)-(e_i+e_j)(e_j+2e_i)(1-\mu_j)}{\Delta}\right]\bar{s}_i \overset{\text{def}}{=} \xi_i\bar{s}_i.$$

Similarly, there are two first-order conditions for platform 1.

In the platform-symmetric equilibrium, where $p^A_{0,1} = p^A_{1,1} = p^A$ and $p^B_{0,1} = p^B_{1,1} = p^B$, the sufficient condition for platform k's profit being concave in its prices is as follows:

$$1-\kappa_A\bar{s}_A^2 > e_A\tau_A + \sigma_A\bar{s}_A\bar{s}_B > 0; \quad 1-\kappa_B\bar{s}_B^2 > e_B\tau_B + \sigma_B\bar{s}_B\bar{s}_A > 0.$$

In the symmetric equilibrium, the first-period equilibrium prices for side A and side B are given respectively by

$$p^A_{0,1} = \frac{1-\kappa_A\bar{s}_A^2}{\tau_A} - \frac{\sigma_B\bar{s}_B\bar{s}_A}{\tau_B} - e_B - \delta_F\xi_A\bar{s}_A; \quad p^B_{0,1} = \frac{1-\kappa_B\bar{s}_B^2}{\tau_B} - \frac{\sigma_A\bar{s}_A\bar{s}_B}{\tau_A} - e_A - \delta_F\xi_B\bar{s}_B, \tag{F.1}$$

and the second-period equilibrium prices are given by

$$p^A_{0,2} = \frac{1-e_B(1-\mu_A)}{1-\mu_A}; \quad p^B_{0,2} = \frac{1-e_A(1-\mu_B)}{1-\mu_B}.$$

To ensure that the above prices are indeed the equilibrium, we need the following conditions to ensure that there is no profitable deviation from the equilibrium prices both on and off the equilibrium path:

$$v_i > 2(\frac{1}{1-\mu_i} - e_j) + 1 - e_i,$$
$$p^i_{0,2}(n^i_{0,1}=0, n^j_{0,1}=0) \geq 0,$$
$$\pi_i = \pi_{i,1} + \delta\pi_{i,2} \geq 0,$$
$$\pi^i_{0,2}(n^i_{0,1}=1, n^j_{0,1}=1) \geq \mu(v_i+e_i-1).$$

The first inequality ensures that v_i is big enough such that the market is covered. The second inequality means that both platforms are active in the second period, given any market shares in the first period and, in particular, the platform without any customer makes nonnegative profit in the second period. In the symmetric case where $e_A = e_B = e$ and $s_A = s_B = s$, this condition simplifies to $s/3 + e \leq 1$, which is also equivalent to the third condition under symmetry assumptions, which in turn ensures that the overall equilibrium profit of each platform is nonnegative. The last condition ensures that the platform with a large customer base has no incentive to deviate to serve only its loyal customers in the second period (this condition is derived as if the platform sells to all consumers in the first period, as this is the most tempting scenario that provides the strongest incentive to sell only to the loyal consumers), and this condition is satisfied when μ is small enough and v_i is big, but not too big. For example, all these conditions are satisfied when μ, s and e are not too large.

G Proof of Proposition 6

If $\delta_A = \delta_B = 0$, $\alpha_A = \alpha_B = 1$, $\mu_A = \mu_B = 0$, $e_A = e_B = e > 0$, and $s_A = s_B = s > 0$, Equation (F.1) becomes

$$p^i_{0,1} = 1 - e - \frac{2}{3}\delta_F s.$$

Differentiating it with respect to s, we obtain

$$\frac{\partial p^i_{0,1}}{\partial s} < 0.$$

Moreover, the change in consumer surplus is

$$\begin{aligned}
\Delta W = & \; W(s) - W(0) \\
= & \; \delta[\underbrace{\frac{2s}{3}}_{price\ \ reduction} \underbrace{-(\frac{s}{2} - \frac{s^2}{4})}_{deadweight\ \ loss}] \\
= & \; \delta(\tfrac{s}{6} + \tfrac{s^2}{4}),
\end{aligned}$$

which is increasing in s.

H Proof of Proposition 7

Setting $\alpha_A = \alpha_B = 0$, $\mu_A = \mu_B = \mu > 0$, and $e_A = e_B = e > 0$, we have $\frac{\partial p^i_{0,1}}{\partial s_i}|_{s_i=0,\mu>0} < 0$ if

$$2e^2(1-\mu)^2 < 2\mu(3e^2(1-\mu)^2 - 2) + e(1-\mu)(\mu + (1-\mu)s_j) + 2,$$

which is always satisfied when $e \geq \frac{1}{1-\mu}\sqrt{2/3}$. Moreover, we have

$$\frac{\partial^2 p^i_{0,1}}{\partial s_i^2} = \frac{2\delta(1-\mu)}{3(1-e^2(1-\mu)^2)}(2 - 3e^2(1-\mu)^2).$$

Clearly, when $e \geq \frac{1}{1-\mu}\sqrt{2/3}$, we have $\frac{\partial^2 p^i_{0,1}}{\partial s_i^2} < 0$. Thus, if $e \geq \frac{1}{1-\mu}\sqrt{2/3}$ then $p^i_{0,1}$ is decreasing in s_i. However, if $e < \frac{1}{1-\mu}\sqrt{2/3}$, $p^i_{0,1}$ may also be decreasing in s_i as long as $\frac{\partial p^i_{0,1}}{\partial s_i}|_{s_i=0,\mu>0} < 0$, which is satisfied when μ is small enough (e.g., $\mu < 1/2$ is a sufficient condition), and $\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu>0} \leq 0$. From

$$\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu>0} = \frac{2\delta(\Delta-3)}{\Delta} - \frac{3\delta e}{\Delta}(1-\mu)(\mu+(1-\mu)s_j) - \frac{2\delta}{3}(1-\mu),$$

where $\Delta = 9(1-(1-\mu)^2e^2)$, we can see that the right hand side of this equation decreases with e and is positive at $e=0$. Taken together with the previous result that $p_{0,1}^i$ is decreasing in s_i if $e \geq \frac{1}{1-\mu}\sqrt{2/3}$, we can deduce that there exists a threshold $\tilde{e}$ such that $\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu>0} \leq 0$ if $e \geq \tilde{e}$, provided that μ is sufficiently small.

Recall that, in Proposition 3, with $\mu = 0$, we have

$$\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu=0} = \frac{\delta}{3(1-e^2)}(2-4e^2-es_j),$$

and $\bar{e}$ is such that $\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu=0} = 0$ at $e = \bar{e}$. Then, it is straightforward to show that

$$\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu=0} < \frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu>0},$$

and because $\frac{\partial p_{0,1}^i}{\partial s_i}|_{s_i=1,\mu\geq 0}$ is decreasing in e, we must have $\tilde{e} > \bar{e}$.

I Proof of Proposition 8

The first-order conditions of π_k, $k \in \{0,1\}$, with respect to $p_{0,1}^A$ and $p_{0,1}^B$ are, respectively,

$$n_{k,1}^A - \frac{1}{2\omega}p_{k,1}^A - \frac{e}{2\omega}p_{k,1}^B - \frac{\delta}{2\omega}\frac{\partial \pi_{k,2}}{\partial n_{0,1}^A} = 0,$$
$$n_{k,1}^B - (1+\frac{e^2}{2\omega})p_{k,1}^B - \frac{e}{2\omega}p_{k,1}^A - \frac{\delta e}{2\omega}\frac{\partial \pi_{k,2}}{\partial n_{0,1}^A} = 0,$$

where

$$\omega = 1 - e^2 - \frac{\delta s_A^2(e^2-2\gamma)}{3\gamma}.$$

To ensure that platform k's profit is concave in its prices, we need $\omega \geq 0$, which means that δ, s_A and e are not too big.

The first-period equilibrium prices are as follows.

$$p_{0,1}^A = 1 - e^2 - \frac{\delta(3e^2-2)s_A^2}{3(1-e^2)} - \frac{2\delta s_A}{3} - \frac{v_B e}{2},$$
$$p_{0,1}^B = \frac{v_B}{2}.$$

To ensure that the above prices are indeed the equilibrium, we need v_A and v_B to be relatively large so that the market is covered, and $v_A, v_B \geq 3$ is a sufficient condition. This condition also ensures that both platforms make nonnegative overall profits. Furthermore, we need both platforms to be active in the second period, and this requires $s_A/3 + e^2 \leq 1$.

For part (i), differentiate $p_{0,1}^A$ with respect to s_A.

$$\begin{aligned}\frac{\partial p_{0,1}^A}{\partial s_A} &= -\frac{2\delta}{3} - \frac{2\delta(3e^2-2)s_A}{3(1-e^2)},\\ \frac{\partial^2 p_{0,1}^A}{\partial s_A^2} &= -\frac{2\delta(3e^2-2)}{3(1-e^2)}\begin{cases} >0 & \text{if } e<\sqrt{2/3},\\ <0 & \text{if } e\geq\sqrt{2/3}.\end{cases}\end{aligned}$$

From this, we have

$$\frac{\partial p_{0,1}^A}{\partial s_A}|_{s_A=0} = -\frac{2\delta}{3},$$

which is always negative. Moreover, we have

$$\frac{\partial p_{0,1}^A}{\partial s_A}|_{s_A=1} = \frac{2\delta}{3}(\frac{2-3e^2}{1-e^2}-1),$$

which is positive if $e<\sqrt{1/2}$. Therefore, $p_{0,1}^A$ is U-shaped in s_A if $e<\sqrt{1/2}$, and decreasing in s_A if $e\geq\sqrt{1/2}$.

For part (iii), we compare the first-period prices paid by consumers who bear switching costs (side A) and those who do not (side B) in the multi-homing model (denoted mh) with that in the single-homing model (denoted sh) in Corollary 4.

For side A,

$$p_{mh}^A < p_{sh}^A \ \text{ if } e+\tfrac{v_B}{2}>1.$$

For side B,

$$p_{mh}^B > p_{sh}^B \ \text{ if } e+\tfrac{v_B}{2}>1.$$

J Proof of Proposition 9

We focus on the case where $e_A=e_B=e$, $s_A=s_B=s$, and $\delta_A=\delta_B=\delta$. Given first-period market shares $n_{0,1}^i$, let $p_{k,2}^{l,i}$ denote the second-period price charged by platform $k\in\{0,1\}$ to consumers on side $i\in\{A,B\}$ who have purchased from platform $l\in\{0,1\}$ in the first period. Then, a consumer on side i, who has purchased from platform 0 in the first period, is indifferent between staying with platform 0 and switching to platform 1 if he is located at

$$\theta_0^i = \frac{1}{2}+\frac{1}{2}[e(2n_{0,2}^j-1)+p_{1,2}^{0,i}-p_{0,2}^{0,i}+s].$$

Similarly, a consumer, who has purchased from platform 1 in the first period, is indifferent between staying with platform 1 and switching to platform 0 if he is located at

$$\theta_1^i = \frac{1}{2}+\frac{1}{2}[e(2n_{0,2}^j-1)+p_{1,2}^{1,i}-p_{0,2}^{1,i}+s].$$

The second-period market share of platform 0 on side i is then

$$n_{0,2}^i = n_{0,1}^i\theta_0^i+(1-n_{0,1}^i)\theta_1^i.$$

The second-period profit of platform 0 is given by

$$\pi_{0,2} = p_{0,2}^{0,A}n_{0,1}^A\theta_0^A + p_{0,2}^{1,A}(1-n_{0,1}^A)\theta_1^A + p_{0,2}^{0,B}n_{0,1}^B\theta_0^B + p_{0,2}^{1,B}(1-n_{0,1}^B)\theta_1^B.$$

The profit function of platform 1 in the second period can be written in a similar way. Then, we take first-order condition of each profit function with respect to the four prices (two on each side). Together, we have eight first-order conditions with eight unknowns. Solving the system of equations, we obtain the following second-period prices:

$$p_{0,2}^{0,i} = p_{1,2}^{1,i} = 1 - e + \frac{s}{3}, \quad p_{0,2}^{1,i} = p_{1,2}^{0,i} = 1 - e - \frac{s}{3}, \quad i \in \{A, B\}.$$

From these prices, we can calculate the difference in the expected second-period utility for a consumer on side i from joining platform 0 in the first period compared to that from joining platform 1, which is given by

$$U_{0,2}^i - U_{1,2}^i = (\frac{s}{3})^2 \frac{e}{1-e^2}[(2n_{0,1}^j - 1) + e(2n_{0,1}^i - 1)].$$

Then, in the first period, the indifferent consumer is located at

$$\theta_R^i = \frac{1}{2} + \frac{1}{2}[e(2n_{0,1}^j - 1) + p_{1,1}^i - p_{0,1}^i + \delta(U_{0,2}^i - U_{1,2}^i)],$$

which also gives us the market share of platform 0 on side i in the first period, that is, $n_{0,1}^i = \theta_R^i$. The expected profit of platform 0 is then given by

$$\pi_0 = p_{0,1}^A n_{0,1}^A + p_{0,1}^B n_{0,1}^B + \delta_F \pi_{0,2}.$$

Taking the first-order conditions for both platforms, we obtain

$$p_{0,1}^i = p_{1,1}^i = 1 - e + \frac{2\delta}{3}(s^2 - s) - \frac{\delta}{3}s^2[2 + \frac{e}{3(1-e)}], \quad i \in \{A, B\}.$$

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