Finite element prediction of the Swift effect based on Taylor-type polycrystal plasticity models

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ABSTRACT: This paper describes the main concepts of the stress-strain interpolation model that has been implemented in the non-linear finite element code Lagamine. This model consists in a local description of the yield locus based on the texture of the material through the full constraints Taylor's model. The prediction of the Swift effect is investigated: the influence of the texture evolution is shown up. The LAMEL model is also investigated for the Swift effect prediction.

Key words: finite element method, LAMEL, Swift effect, Taylor's model, texture evolution, yield locus

1 INTRODUCTION

It is well-known that the plastic deformation of polycrystalline material induces reorientation of individual grains into preferred orientations. This phenomenon, i.e. the texture evolution, is responsible for the mechanical anisotropy of the material, which plays an important role in forming processes.

The implementation of texture evolution into finite element code is therefore of first importance. Unfortunately, micro-macro models generally require very large computation time and memory storage.

These considerations led us to the development of a local yield locus approach suitable for FE modelling of industrial forming processes. With this model, only a small zone of the yield locus is computed. This zone is updated when it is no more in the interesting part of the yield locus or when the yield locus changes due to texture evolution.

This local yield locus approach has already been successfully used to model deep drawing [12]. It is used here for the simulation of the Swift effect during torsion of a thin tube [1]. The results are compared to predictions of simple shear by a micro-macro modelling of crystal plasticity [13].

2 LOCAL YIELD LOCUS APPROACH

2.1 Stress-strain interpolation model

This model is particular in the sense that it does not use a yield locus formulation neither for the interpolation nor in the stress integration scheme. We use a linear stress-strain interpolation described by equation (1):

$$\underline{\sigma} = \tau \underline{\underline{C}} \bullet \underline{u} \qquad (1)$$

In this equation, $\underline{\sigma}$ is a 5-D vector containing the deviatoric part of the stress; the hydrostatic part being elastically computed according to Hooke's law. The 5-D vector $\underline{u}$ is the deviatoric plastic strain rate direction; it is a unit vector. τ is a scalar describing the work hardening according to the exponential relationship of equation (2) where the strength coefficient K, the offset Γ^0 and the hardening exponent n are material parameters fitted on experimental data and Γ is the polycrystal induced slip.

$$\tau = K \cdot \left(\Gamma^0 + \Gamma\right)^n \qquad (2)$$

The interpolation is included in the matrix $\underline{\underline{C}}$ of equation (1).

We assume 5 directions: $\underline{u}_i$ (i=1…5) advisedly chosen in the deviatoric strain rate space and the associated deviatoric stresses: $\underline{\sigma}_i$ (i=1…5) computed by the full constraints Taylor's model. These stress vectors lie on the yield surface according to Taylor's model. These points define the interpolation domain; they are located at the vertices of the domain and are called 'stress nodes'. The $\underline{\underline{C}}$ matrix is constructed on the basis of the 5 stress nodes.

With this method, only a small part of the yield locus is known. As long as the interpolation is achieved in the domain delimited by the 5 stress nodes, the interpolation matrix $\underline{\underline{C}}$ is valid. When the stress direction explored during the finite element computation falls out of the domain, updating of the stress nodes must take place; a new interpolation matrix is computed. The classical updating method consists in finding 5 new stress nodes defining a new domain containing the current stress direction. Our updating method makes use of the adjacent domain. Therefore, only 1 new stress node is computed with Taylor's model and 4 of the 5 old stress nodes are kept for the interpolation. The main advantages of this method are that updating requires only 1 (instead of 5) call to Taylor's model and that it improves the continuity of the resulting yield locus and the continuity of its normal.

The above considerations are sufficient to understand the basic concepts of the local yield locus implemented in Lagamine code. Further details and properties of such parameterisation of a N dimensional space were investigated by [9] and [12]. They studied different interpolation methods inside one domain: linear interpolation in Cartesian coordinates or hyperplane model, linear interpolation in spherical coordinates, approach enriched by bubble mode...

2.2 Texture evolution

Each point of the local yield locus presented above is calculated based on Taylor's model. Furthermore, Taylor's model is also used for the computation of texture evolution during finite element simulations. Texture evolution is computed on the basis of the strain history, which is available for each integration point of the FE mesh.

Texture is represented at each point of the mesh by a set of crystallographic orientations.

3 SWIFT EFFECT PREDICTION

The Swift effect has first been published in [1]. It refers to the lengthening (or shortening, according to the temperature range) of a cylinder when it is submitted to torsion.

The geometry of the sample can be a cylinder or a tube. During the torsion of the tube, its height tends to increase. This deformation mode, i.e. the Swift effect, only occurs for large torsion strains in the plastic regime.

The shear strain $\overline{\omega}$ (which is a measure of the torsion of the tube) is generally defined as:

$$\overline{\omega} = \frac{1}{2}\psi\overline{R} \qquad (3)$$

where ψ is the twist angle per unit length of the tube and $\overline{R}$ is the initial mean radius of the tube. Elongation strains have been measured by several authors and different numerical models have been developed for the Swift effect prediction (see table 1).

Table 1. Elongation strains reported in the literature

$\overline{\omega}$ (%)	Elongation strain (%)	Method used	Material investigated	ref.
up to 250	from 1 to 11	experimental	7 metals	[1]
130	7	experimental	304L stainless steel	[6], [7]
>120	6.8	experimental		[10]
20 to 30	0.4 to 0.8	experimental	iron and steel	[5]
140	10	numerical		[10]
25	1.5	numerical model based on [2]	aluminium	[11]
75	≈10	different numerical models	70-30 brass	[8]

3.1 Finite element model

Our stress-strain interpolation model has been used to simulate torsion of a tube. The geometry corresponds to the experimental measurements presented in [1]. The external diameter of the tube is 23.8mm; the internal diameter is 16.7mm. The length is 50.8mm. The tube is meshed with 48 elements in the circumferential direction, 20 elements along the axial direction and 1 element along the thickness of the tube.

The tube is meshed with BLZ3D finite elements (see [4]). BLZ3D is a solid finite element with 8 nodes using a mixed formulation based on the Hu-Washizu principle (see [3]). These elements are adapted for large strains and large displacements. One integration point per element is defined.

The lower end of the tube is fixed while the upper end is twisted.

The material's texture used for the Swift effect prediction corresponds to an isotropic material, i.e. 2000 crystallographic orientations uniformly distributed. The plastic deformation of each crystal is achieved through the 24 $\{1\ 1\ 0\}\langle 1\ 1\ 1\rangle$ and $\{1\ 1\ 2\}\langle 1\ 1\ 1\rangle$ slip systems classically used for bcc metals.

3.2 Micro-macro modelling of simple shear

We have also used the crystal plasticity code proposed in [13] to simulate simple shear. This constitutes a close approximation of torsion when the tube is thin. Texture was represented by the same set of 2000 crystal orientations as before.

Different assumptions about the interaction of adjacent grains have been tested. The classical Taylor full constraint model, that is already used in the Stress-strain interpolation technique, considers uniform deformation of all grains. The LAMEL model, which is detailed in [15], allows local strain heterogeneities. Two interaction modes are tested here:

- LAMEL 1: heterogeneous shearing in the plane normal to the tube axis;
- LAMEL 2: heterogeneous shearing in the plane normal to the radial direction.

LAMEL is expected to predict a different texture evolution and a different mechanical response. The LAMEL model leads to improved predictions of rolling textures in steel [14] and to a successful prediction of the yield locus [15]. Therefore, we wish to implement it in our finite element code in the future.

3.3 Numerical results

The Swift effect is represented in figure 1 through the significant positive value of the extension strain as a function of the shear strain.

The LAMEL 1 and 2 dots correspond to the LAMEL model with the 2 hypotheses described in section 3.2; the Taylor FC dots refer to the Taylor's model applied outside the FE code. The black curve corresponds to the Stress-strain interpolation model (S-s I) with the texture evolution computed during the FEM torsion simulation. The grey curve is also the Stress-strain interpolation model but the initial isotropic texture is used throughout the simulation.

A first remark is the good agreement between Taylor's model used outside the FE code with simple shear assumption and the FE simulation of torsion with the Stress-strain interpolation model.

If the initial isotropic texture is kept for the whole simulation, the Swift effect is almost non-existent.

The LAMEL model (whatever hypothesis 1 or 2 is used) predicts a larger Swift effect than Taylor's model.

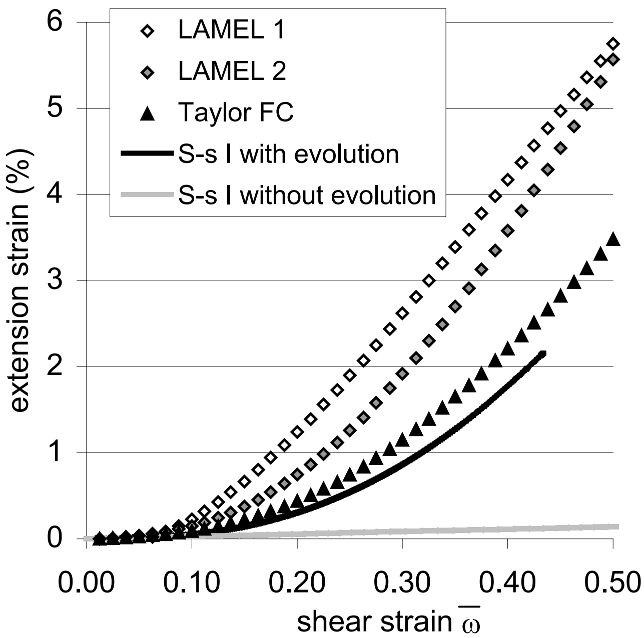

Fig. 1. Extension strain versus shear strain

As the tube length tends to increase, the FE simulation predicts a significant reduction of 1.47% of the tube mean radius (for a shear strain of 0.42).

3.4 Predicted texture

The 111 pole figures of the final texture (for a shear strain of 0.5) are presented in figure 2 for Taylor's model and figure 3 for LAMEL model with hypothesis 1. The corresponding figure with hypothesis 2 is very similar. The pole figures projection direction is the radial direction of the tube. One observes a significant texture development (the initial texture was isotropic). This texture results in anisotropic mechanical response, lengthening the tube.

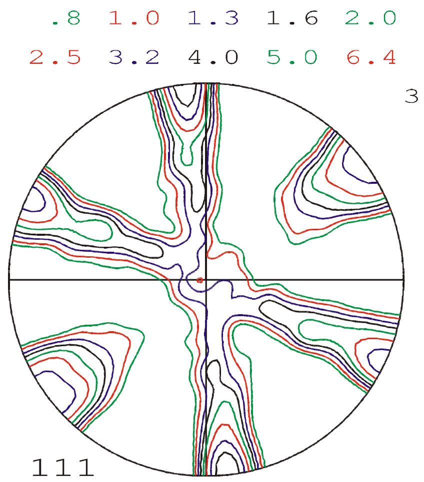

Fig. 2. Pole figure of the final texture predicted by Taylor's model

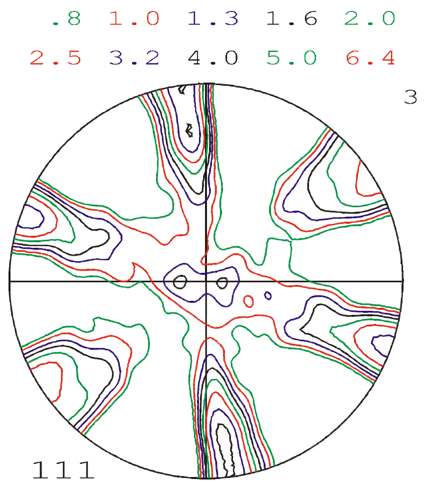

Fig. 3. Pole figure of the final texture predicted by the LAMEL model with hypothesis 1

4 CONCLUSIONS

The Stress-strain interpolation model described in section 2.1 and implemented in the finite element code Lagamine is able to predict the Swift effect. It appears that latent hardening is not necessary for the prediction of the Swift effect. Furthermore, the computation of the texture evolution throughout the simulation cannot be neglected. It appears that the texture developed during shearing is at the origin of the Swift effect.

The implementation of the LAMEL model in the finite element code Lagamine is under development.

The influence of the hardening model on the Swift effect should also be investigated.

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