Statistical physics of memory driven systems

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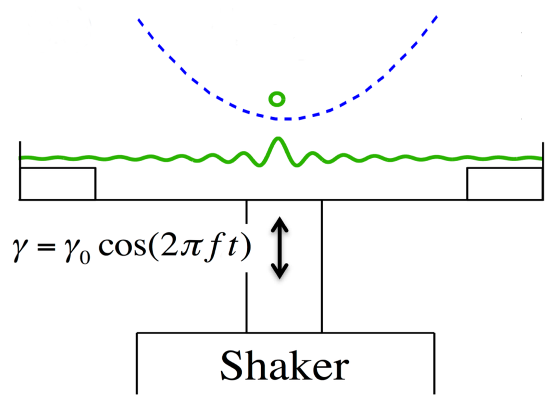
Walking droplets: a memory driven system

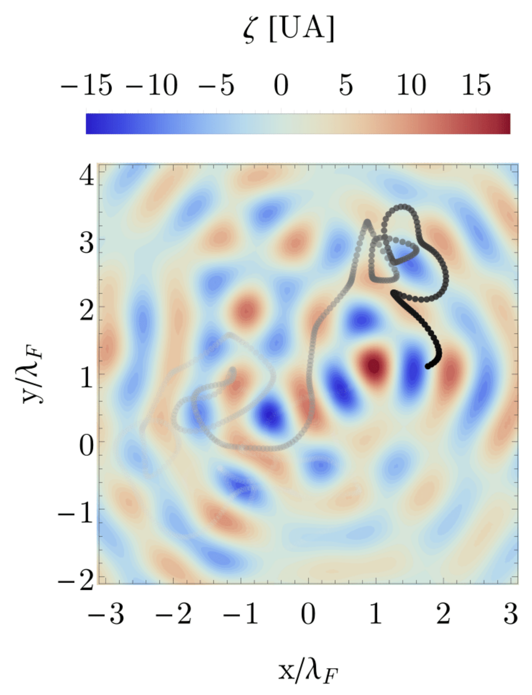
While bouncing on a liquid interface, a droplet may walk along the surface thanks to the Faraday waves it emits at each impact. By this mean, the droplet gets a horizontal "kick" due to the slope of the surface deformed by the waves

The subsequent dynamics results in a **macroscopic wave-particle duality**: the droplet (particle) creates the waves and the waves propel the particle



Thanks to non-linearity, the waves are damped over a characteristic time Me.τ_F where τ_F is the Faraday wave period and Me is the memory

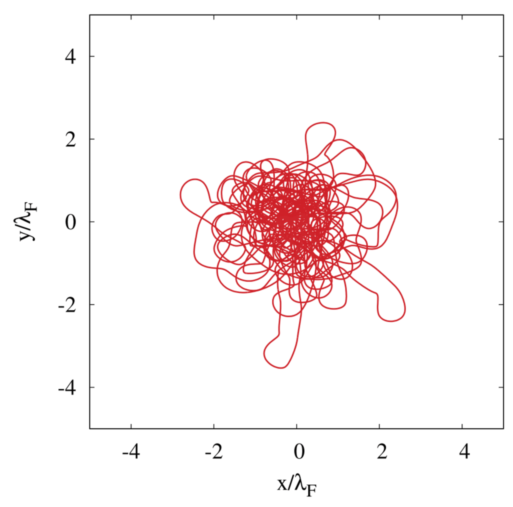
We consider this **walking droplet dynamics** in the **high memory regime** in which the particle is confined into an **harmonic potential.** The particle therefore interacts with its past through the waves it created on the surface, while the waves are damped over a large period of time.

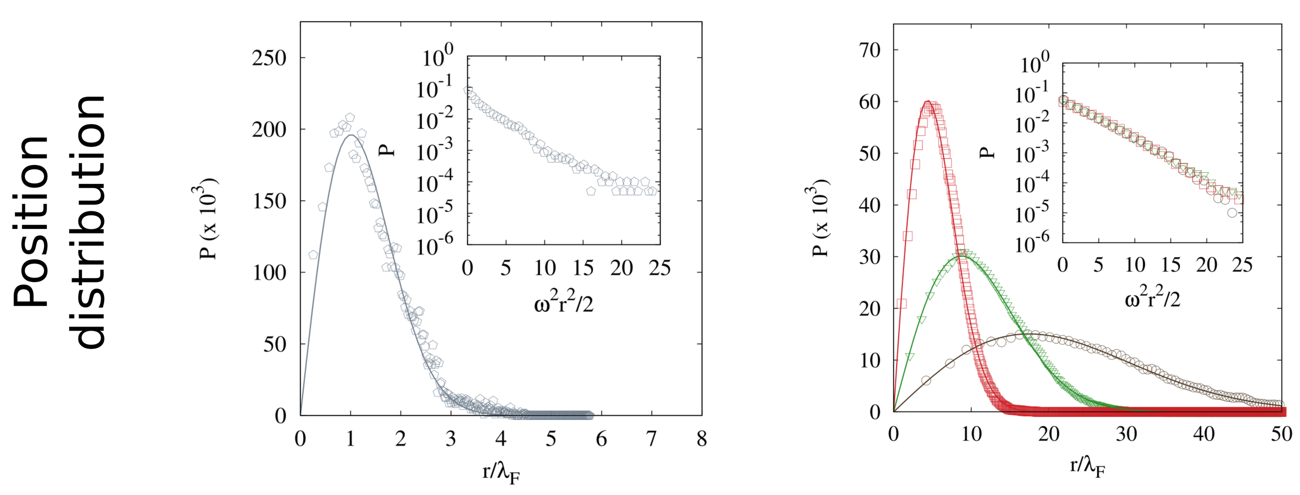


Focusing on the particle...

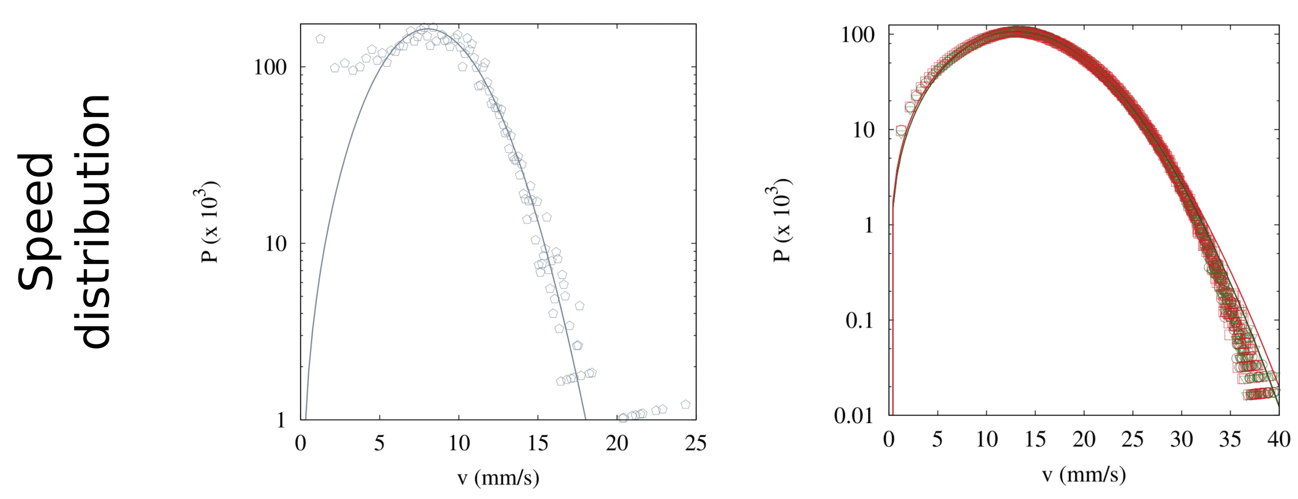
The force originating from the **wavefield** can be approximated by a **white noise** applied on the walker resulting in a **chaotic-like, self-propelled motion** within the harmonic potential

First order description via a **Langevin equation**: $\ddot{\mathbf{r}} + \gamma(\dot{\mathbf{r}})\dot{\mathbf{r}} + \omega^2\mathbf{r} + \eta(t) = 0$

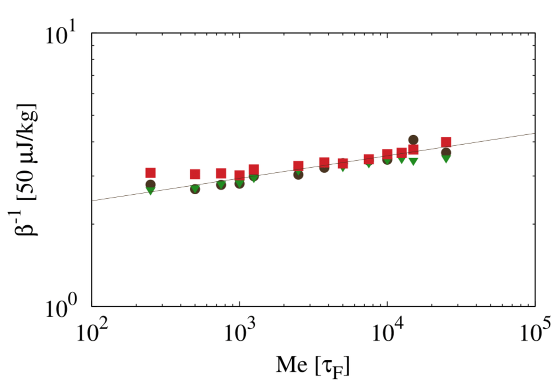




Boltzmann exponentials: $\mathcal{P}(r) \propto \exp(-\beta U(r))$



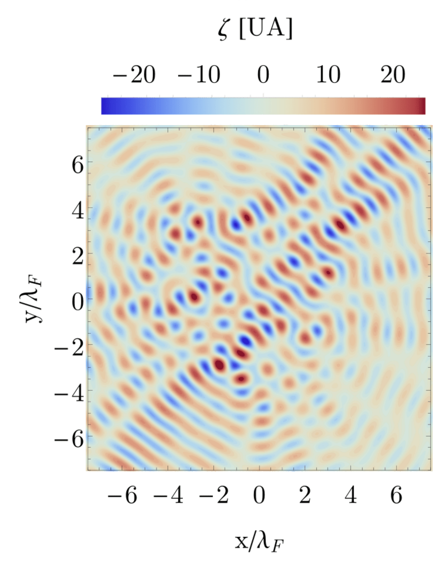
Self-propelled particles: $\mathcal{P}(v) \propto \exp(-\alpha(v^2 - v_0^2))$



The wavefield contains **hidden degrees of freedom** which define an effective **temperature** for the particle

What are the **properties** of this wave field and its degree of freedom?

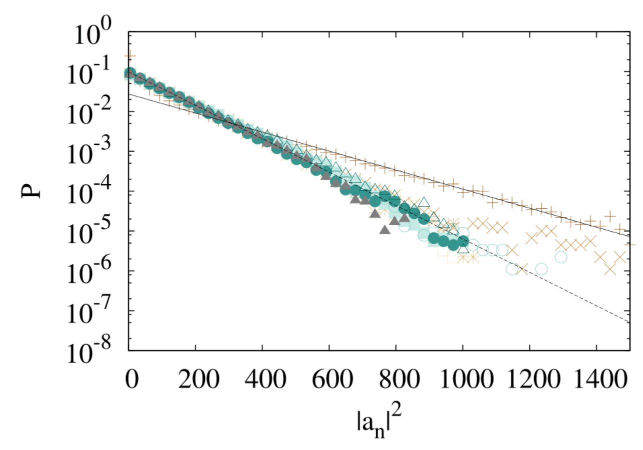
Focusing on the waves...



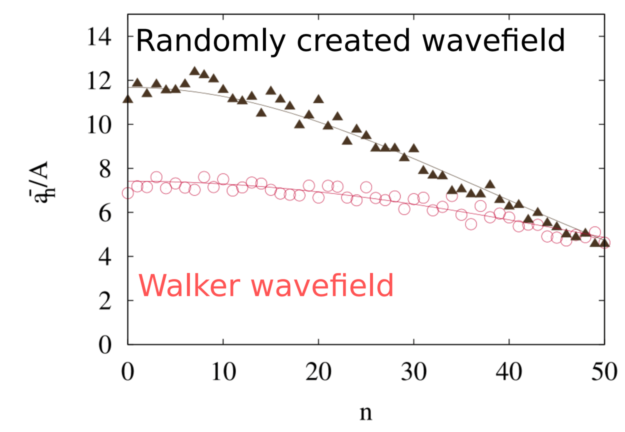
The wavefield can be decomposed in **cylindrical eigenmodes**

$$\zeta(\mathbf{x}) = \sum_{n=-\infty}^{\infty} a_n J_n\left(2\pi \frac{r}{\lambda_F}\right) \exp(-in\theta)$$

with $a_n(t) = \sum_{i \in \text{impacts}} J_n\left(2\pi \frac{r_i}{\lambda_f}\right) \exp(in\theta) \exp\left(-\frac{t - t_i}{\text{Me}}\right)$



Gaussian distribution of eigenmode values

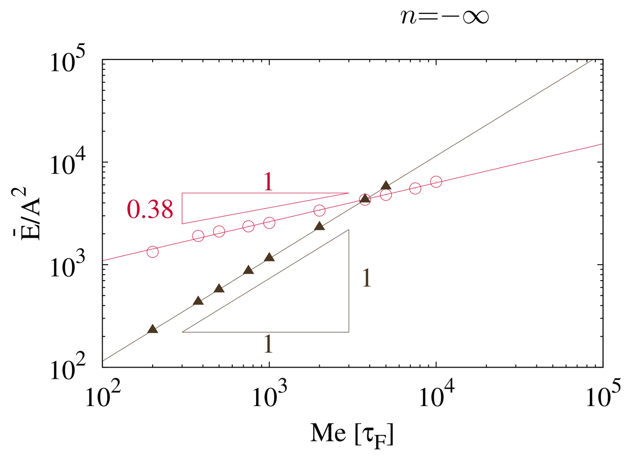


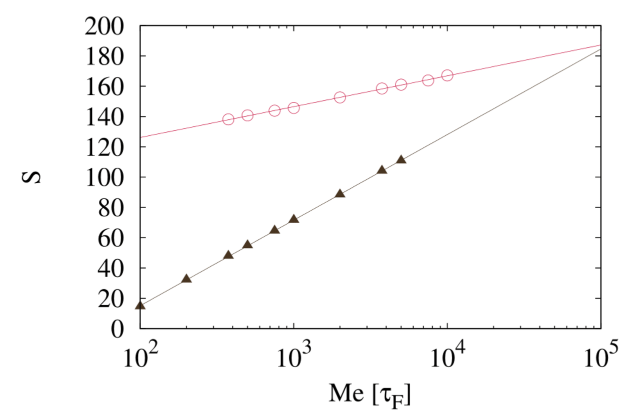
Equipartition of energy + large number of degrees of freedom

Even if the force applied by the waves on the particle can be described as random, the global stored energy is different from a random wave field. What about **energy** and **entropy**?

Energy $E = \sum_{n=-\infty}^{\infty} |a_n|^2$

Shannon entropy $S = \sum_{n=-\infty}^{\infty} \ln\left(\sigma_{a_n}\sqrt{2\pi e}\right)$





On average, the waves generated **interfere desctructively** because of the **correlations along the particle path**

Take home messages

- **For a walker, the thermodynamic limit can be reached at the single particle level**
- **The surface waves act as a thermal reservoir whose temperature is controlled through the memory**
- **The "wave thermal reservoir" stores energy equally in each of its degree of freedom**
- **The walker dynamics triggers a minimization process of the global wave energy**

Contact & Info

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