Applications of the Multiple Timescale Spectral Analysis in wind engineering

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ABSTRACT: The random response of civil engineering structures to the buffeting action of wind loads is typically composed of several components, usually referred to as the background component, in the low frequency zone and the resonant component(s) in the neighborhood of modal natural frequencies. It has become customary to study separately and add the contributions of these components to the total response, at least as far as the second order response (variance of structural responses) is concerned. Such a decomposition exists but is less usual for the computation of covariances of modal coordinates or of structural displacements, which are in turn necessary for the determination of internal stresses. The question of such a decomposition also holds for nonlinear systems, or even for the higher statistical moments of a linear structural system, should the response be non Gaussian. With very wide ranges of applicability, the Multiple Timescale Spectral Analysis summarizes under a unified framework recent works aiming at the development of such decompositions. This paper briefly pictures this particular theory based on perturbation methods, and provides illustrations of its applicability to the problems cited above.

1 INTRODUCTION

The steady state statistics of the response of structures with a linear structural behavior subject to the buffeting action of turbulent winds are usually determined in the frequency domain and modal basis, by means of the well-known background/resonant decomposition. For single degree-of-freedom systems, this decomposition is sometimes presented as the replacement of the actual power spectral density of the aerodynamic forces by an equivalent constant value (a white noise intensity), the magnitude of which being chosen as the value of the power spectral density of the loading at the natural frequency of the oscillator. This simple and straightforward view also holds for the estimation of correlation coefficients between modal responses. However, this simple approach is not necessarily the appropriate reasoning in non Gaussian, nonlinear or non stationary situations. Actually, the root motivation that is valid in all these cases is the existence of several timescales in the response.

This observation has triggered the development of a unified approach, the Multiple Timescale Spectral Analysis (Denoël 2015), which is based on the sole assumption of separation of timescales. In the particular case of second order statistics (variances and covariances) in linear elasto-dynamics, it degenerates into the well-known background/resonant decomposition, and in other cases, it allows the derivation of closed form asymptotic approximations of the statistical properties of the structural responses. Some of these more general cases are reviewed in this paper.

To possess closed form expressions for the statistics of the structural responses has an evident advantage over estimates obtained with Monte Carlo simulations, since it offers a simple analytical tool. This usually translates into a clear understanding of the problem as offered for instance by the background/resonant decomposition, while time domain simulations would not reveal the structural behavior as clearly.

As hinted by its name, The Multiple Timescale Spectral Analysis requires the problem to be formulated in a spectral approach, i.e. a frequency domain. Contrary to what is usually thought, a frequency domain approach might still be applicable in non Gaussian, nonlinear and non stationary contexts. For instance, the Volterra and Wiener theories of nonlinear systems provide a backdrop to the analysis of weakly nonlinear systems while the evolutionary spectral analysis provides a theoretical framework for the extension to a wide class of non stationary processes. Applications of the Multiple Timescale Spectral Analysis therefore requires the considered problem to be formulated within the scope of one of these theories that are more general than standard spectral analysis of linear oscillators.

2 THE MULTIPLE TIMESCALE SPECTRAL ANALYSIS

In a spectral analysis, the loading is defined by a set of spectra defined on multidimensional frequency spaces. Among them, the well-known power spectral density, defined for $\omega \in \mathbb{R}$, represents the distribution of the second cumulant (the variance) over the frequency domain. More generally, the j^{th} order spectrum represents the distribution of the j^{th} stationary cumulant over a multidimensional frequency domain $\mathbb{R}^{j-1}$ ($j = 2$: power spectral density, $j = 3$: bispectrum, $j = 4$: trispectrum, etc.) The objective of a spectral analysis is to determine the spectra of the response in terms of those of the input and eventually integrate them in the corresponding frequency spaces in order to determine the cumulants of the response.

Spectral analysis develops in various versions, but in all of them, a canonical form for the j^{th} order spectrum of the response (or contribution thereof in the nonlinear case) is

$$\mathbf{S_x}\left(\boldsymbol{\omega}_j\right) = \mathbf{K}\left(\boldsymbol{\omega}_j\right)\mathbf{S_p}\left(\boldsymbol{\omega}_j\right) \quad (1)$$

where $\boldsymbol{\omega}_j = \{\omega_1, \cdots, \omega_{j-1}\}$, with $j \geq 2$, gathers the independent variables of the frequency space, $\mathbf{K}\left(\boldsymbol{\omega}_j\right)$ is a frequency kernel function and $\mathbf{S_p}\left(\boldsymbol{\omega}_j\right)$ and $\mathbf{S_x}\left(\boldsymbol{\omega}_j\right)$ respectively stand for the j^{th} order spectra of the loading and of the response. A modified version, adding time t as an additional independent variable is required in case of evolutionary spectrum.

The spectral analysis then consists in the determination of

$$\mathbf{k}_j = \int_{\mathbb{R}^{j-1}} \!\!\cdots\!\! \int \mathbf{S_x}\left(\boldsymbol{\omega}_j\right) d\boldsymbol{\omega}_j \quad (2)$$

which represents the j^{th} cumulant of the response (or a contribution thereof in the nonlinear case).

In the stochastic analysis of large structures, the computational cost associated with the estimation of the spectrum of the loading for a particular frequency $\boldsymbol{\omega}_j$ might be relatively high whilst application of the background/resonant decomposition promptly solves the problem as it only requires one evaluation of $\mathbf{S_p}\left(\boldsymbol{\omega}_j\right)$ for a single frequency. When seeking higher statistical moments of the response, which requires integration of high order spectra, numerical techniques are only applicable to systems with few degrees-of-freedom. As seen in the sequel, application of the the Multiple Timescale Spectral Analysis actually drops by one (at least) the dimensionality of the integral, which makes the estimation of higher moments affordable, even for large structures.

The Multiple Timescale Spectral Analysis consists in a sequential consideration of the different contributions to the cumulant of the response. The methodology precisely follows the example presented by (Hinch 1991); it is composed of the following steps: (i) identify a contribution to the integral (for instance, background or resonant), (ii) rescale the problem in the corresponding zone of frequencies, (iii) find a simple and accurate local approximation that decreases fast enough in the far field, (iv) subtract the approximation from the initial function, in order to obtain a residue, (v) iterate by identifying and estimating the different contributions to this residue; finally stop when all leading order contributions are treated.

Four different examples are developed in the full-length paper to illustrate the accuracy and quality of the proposed method.

3 CONCLUSIONS

This paper summarizes the basis of the Multiple Timescale Spectral Analysis. Without giving any detail on how to proceed with the sequential approximation of the leading terms in the total integrals that arise in spectral analysis, several examples are used for illustrations. First, the response of a single degree-of-freedom model, in order to validate the approach; then the non-Gaussian response of a linear oscillator and then the evaluation of the covariances between the modal coordinates; finally some information about its application to a nonlinear single degree-of-freedom model were given. In all cases, the method achieves a remarkable accuracy in estimating the statistics of the response. This is because the only assumptions are the clear separation of different timescales. What is the most important is that the method provides very simple analytical solutions, that capture the most important features of the response (see for instance the estimation of the covariance). Furthermore, it is noteworthy to underline that the method is potentially very wide as to the domains and problems it targets. In particular, it is not limited to markovian processes as it is the case for the alternative methods based on the Fokker-Planck-Kolmogorov or the so-called moment equations.

REFERENCES

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