# ON-LINE DAMAGE ASSESSMENT USING OPERATING DEFLECTION SHAPES

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## ABSTRACT

Structural health monitoring using measured vibration data may (or not) be based on a numerical model. If a structural model is not available, the measurements of the nominally healthy structure have to be used as the baseline for comparison. In this way, alterations of the behavior may be tracked. This approach can be considered as a reactive way to handle the problem since the engineer will detect that the characteristics of the structure changed, but he will not know if the modification(s) render(s) the structure unreliable. The exploitation of a numerical model allows the application of a more rich, proactive strategy. In some conditions, the analyst may even be able to diagnose the remaining lifetime of the structure.

A common approach to health monitoring is to use identified modal data. In this paper, the direct use of operating deflection shapes (ODS) is considered. The main advantage of this approach is that ODS are more sensitive to structure modifications than mode shapes which have to be identified; moreover, the modal analysis efforts (and errors) are avoided. In this paper, a two level approach for damage assessment is presented. In the reactive level, current experimental ODS are compared to the healthy measured ODS. The Frequency Domain Assurance Criterion -FDAC- is used to track a global evolution and the shifted residual ODS technique is used to obtain a first damage localization. If changes in the ODS are significant, the proactive level of damage assessment is activated. It uses a FE model and is based on the theory of Minimization of Errors on Constitutive Equations -MECE- to locate and quantify damage in terms of physical parameters.

The proposed methods are applied on the example of an actual civil engineering structure on which the time evolution of damage is known.

## NOMENCLATURE

| | |
|---|---|
| $\mathbf{K}, \mathbf{M}$ | FE model stiffness and mass matrices |
| $\mathbf{K}^*, \mathbf{M}^*$ | perfectly identified matrices |
| $\triangle\mathbf{K}$, $\triangle\mathbf{M}$ | errors of the model matrices |
| $\tilde{\mathbf{K}}$ | condensed stiffness matrix |
| $\Phi$ | model modal matrix |
| $\bar{\mathbf{v}}$ | measured vector |
| $\mathbf{v}^*$ | perfectly expanded shape |
| $\mathbf{v}$ | expanded experimental shape |
| $\triangle\mathbf{v}$ | error in the expanded vector |
| $\mathbf{u}$ | instrument shape vector |
| $\mathbf{g}^*$ | perfectly known force vector |
| $\mathbf{g}$ | identified force vector |
| $\mathbf{f}_r$ | residual force vector |
| $\mathbf{q}^*$ | perfectly estimated modal coordinates |
| $\hat{\mathbf{q}}$ | estimated modal coordinates |
| $\alpha$ | control parameter |
| $\triangle\omega$ | error in the identification of the experimental frequency |
| $_1$ | measured partition of dofs |

## 1 INTRODUCTION

Vibration data may come in three different formats : time series, FRF vectors and mode shapes. The direct use of time data requires to measure the whole set of coordinates at the same time (implying an expensive acquisition hardware). FRF vectors and mode shapes are intrinsic properties of the structure (provided it shows a linear behavior) and may be averaged. In this case, acquisition may be much more flexible. A common reason to use modes is that they are able to reconstruct FRFs with some level of quality. The difference between the measured and the modereconstructed FRFs is mainly due to those modes that were not identified, to errors of damping identification and to the effect of noise. Given a known force configuration, FRFs present the advantage of being estimated directly from time data, avoiding the extraction of modal parameters and thus identification errors. Damping effects also appear more clearly than with mode shapes. However, a drawback to the direct use of FRF data may be the difficulty of choice. As a great number of FRFs is usually available, the question becomes to determine which is the more representative subset of data (i.e. containing more information about damage). Note that computation cost may become high when a structural model is exploited in the damage detection procedure. If only experimental data is considered, this quantity of information may be handled easily by current computers. As FRF vectors are linear combinations of all the eigenmodes that were excited (identified or not), they are more sensitive to changes in the parameters. In general, only global modes are accurately identified; and it has been observed that these modes are relatively insensitive to changes in localized parameters [1], [3]. Local modes which are more difficult to identify show bigger changes in their shapes and frequency.

The aim of this paper is to present a two level approach for damage assessment by using operating deflection shapes. In the reactive level, current experimental ODS are compared to the healthy measured ODS. The Frequency Domain Assurance Criterion -FDAC [2]- is used to track a global evolution and the Shifted Residual ODS technique is used to obtain a first idea on damage localization. If changes in the ODS are *significant*, the proactive level of damage assessment is activated. It uses a FE model and is based on the theory of Minimization of Errors on Constitutive Equations -MECE [5]- to locate and quantify damage in terms of physical parameters.

The proposed methods are applied on the example of an actual civil engineering structure on which the time evolution of damage is known.

## 2 NON-MODEL BASED HEALTH MONITORING

Non-model based techniques assume that local model errors produce changes in the local responses. They are limited by the number of sensors and may be used by comparing modal indicator variations with regard to a previous healthy situation. Evolution of the damage influence on the response can be tracked in this way, but it cannot assess if the structure is still reliable or safe. However, these methods present the advantage of not requiring the expensive process of building a model. An extensive survey of this type of methods is presented in reference [1].

### 2.1 Frequency Domain Assurance Criterion (FDAC)

An alternative solution to the use of identified mode shapes is the direct exploitation of measured operating deflection shapes. It was proposed in reference [2] to measure the degree of correlation between two sets of ODS $(\mathbf{x}_{\omega_1}, \mathbf{y}_{\omega_2})$ by the Frequency Domain Assurance Criterion defined as :

$$FDAC(\mathbf{x}_{\omega_1}, \mathbf{y}_{\omega_2}) = \frac{\mathbf{x}_{\omega_1}^T \, \mathbf{y}_{\omega_2}}{|\mathbf{x}_{\omega_1}| \, |\mathbf{y}_{\omega_2}|} \tag{1}$$

Note that FDAC allows to take into account the frequency shift that is produced by the displacement of the modal base on the frequency axis when model parameters are perturbed.

The idea proposed here is to use FDAC in a cascade analysis to observe global changes in the ODS when the undamaged situation is taken as reference state.

### 2.2 Shifted residual ODS

By exploiting the frequency shift, it is possible to compare ODS in a more rational way. Rather than comparing the shapes at the same frequency, it is proposed to compare the reference shape $\mathbf{x}_{\omega_1}$ taken at the reference frequency with the corresponding FRF shape $\mathbf{y}_{\omega_2^*}$ paired using FDAC. In order to do this, a residual ODS is defined by :

$$\Delta \mathbf{v} = \mathbf{x}_{\omega_1} - \mathbf{y}_{\omega_2^*} \tag{2}$$

This will provide a graphical representation about where the errors might be.

## 3 MODEL BASED HEALTH MONITORING

As stated in reference [4], a validated mathematical model updated to the current state of the system constitutes a rich knowledge base, which can be exploited by comparing it with the correlated models of previous states. This serves the purposes of fault detection and structural assessment. Model based damage identification methods are superior since they allow to detect not only the location of changes in the structure, but also its nature and its amplitude level. The engineer may have a better understanding of the physics behind the damage and is able to exploit the rich knowledge base used to build the model.

The model based localization approach considered here makes use of the model equilibrium conditions. In this category of methods, one finds the force residual technique and the method based on the minimization of errors on constitutive equations (MECE). A drawback of these techniques however is that they require a matching process in order to estimate the non measured degrees of freedom of the model (commonly referred as *expansion*).

### 3.1 Model based error localization

Let us assume that *modeling* errors are negligible (i.e. equations are correct, discretization is adequate) and that only model *parameter* errors (e.g. material or geometric properties) are actually present. Under these conditions, the dynamic equilibrium equation of the experimental structure corresponding to a particular forced response vector (ODS) may be written in the frequency domain as

$$\mathbf{K}^* \, \mathbf{v}^* = \omega_{v^*}^2 \, \mathbf{M}^* \, \mathbf{v}^* + \mathbf{g}^* \tag{3}$$

The validity of this equation is crucial for the rest of the developments. The assumption of the existence of such a structural model for the experimental structure tells implicitly that both the "undamaged" and "damaged" experimental structures have a linear behavior and that dissipation effects do not influence the operating deflection shapes being considered.

The correspondence between the damaged and the initial FE model can be established through the following relations

$$\mathbf{K}^* = \mathbf{K} + \Delta\mathbf{K} \tag{4}$$
$$\mathbf{M}^* = \mathbf{M} + \Delta\mathbf{M} \tag{5}$$
$$\mathbf{v}^* = \mathbf{v} + \Delta\mathbf{v} \tag{6}$$
$$\omega_{v^*} = \omega_v + \Delta\omega \tag{7}$$

The approximation $\mathbf{v}$ to the true experimental response vector $\mathbf{v}^*$ may be found by assuming that the numerical model is close to the true structure. Using a standard formulation, an expanded vector $\mathbf{v}$ is sought by minimizing the residue of the equilibrium equation in some adequate metric :

$$\min \; \mathbf{f}_r^T \, \Theta \, \mathbf{f}_r \tag{8}$$

where

$$\mathbf{f}_r = (\mathbf{K} - \omega^2 \, \mathbf{M}) \; \mathbf{v} - \mathbf{g} \tag{9}$$

In order to solve problem (8), the experimental data is exploited by requiring that the expanded vectors should be similar to the reference measured shapes. It can be done by minimizing the difference between measured and calculated displacements in some adequate metric i.e.

$$\min \, (\mathbf{v}_I - \bar{\mathbf{v}})^T \; \Xi \; (\mathbf{v}_I - \bar{\mathbf{v}}) \tag{10}$$

A great number of expansion techniques are based on the exact verification of equation (10) (in this case, $\mathbf{v}_I = \bar{\mathbf{v}}$) while solving equation (8). In the MECE expansion method [5], [6] considered here, the two conditions (8 and 10) are performed simultaneously.

If metric $\Theta = \mathbf{K}^{-1}$ is chosen to solve problem (8) and $\Xi = \tilde{\mathbf{K}}$ in equation (10), an optimization problem may be written as

$$\min \; \mathbf{f}_r^T \, \Theta \, \mathbf{f}_r + \alpha \, (\mathbf{v}_I - \bar{\mathbf{v}})^T \; \Xi \; (\mathbf{v}_I - \bar{\mathbf{v}}) \tag{11}$$

It can be shown [5] that the expansion problem (11) may also be expressed in the form

$$\min \; (\mathbf{v} - \mathbf{u})^T \; \mathbf{K} \; (\mathbf{v} - \mathbf{u}) + \alpha \; (\mathbf{v}_I - \bar{\mathbf{v}})^T \; \tilde{\mathbf{K}} \; (\mathbf{v}_I - \bar{\mathbf{v}}) \tag{12}$$

subject to $\quad \mathbf{K} \, \mathbf{u} = \omega_v^2 \, \mathbf{M} \, \mathbf{v} + \mathbf{g}$

where $\mathbf{u}$ is an instrument shape vector. In order to solve this optimization problem, the expanded vector is expressed as a combination of the FE modal base :

$$\mathbf{v} = \Phi \, \hat{\mathbf{q}} \tag{13}$$

In equation (13) $\tilde{\mathbf{K}}$ assures a good balance between the two terms, but other weight matrices may be used. The instrument vector $\mathbf{u}$ introduced in the constraint equation of the optimization problem (13) allows the definition of an local error indicator that quantifies a residual strain energy (element-by-element or substructure-by-substructure):

$$e_s = (\mathbf{u} - \mathbf{v})^T \; \mathbf{K}_s \; (\mathbf{u} - \mathbf{v}) \tag{14}$$

where $\mathbf{K}_s$ is the stiffness matrix of substructure $s$.

## 4 APPLICATION EXAMPLE

The procedure for health monitoring based on the exploitation of operating deflection shapes is illustrated on the example of the I-40 bridge over the Rio Grande river in New Mexico. This civil engineering structure has been presented in details in the literature [1], [7]. The first mode shape of the bridge model is shown in figure 1.

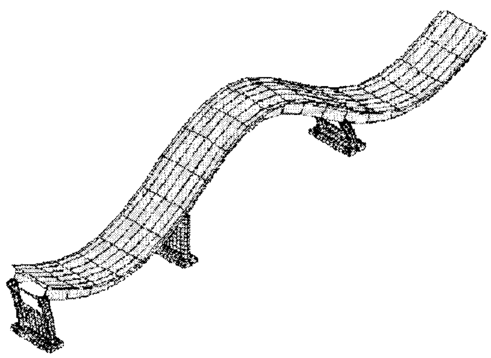

**Figure 1: FE model of the bridge-1st mode**

The true structure was tested in its undamaged state using ambient and forced excitations. Damage was induced on the bridge by cutting through one of the main girders in four increasing states (figures 2 and 3). Modal analysis was performed at each state using a set of 26 accelerometers placed along the two sides of the bridge deck.

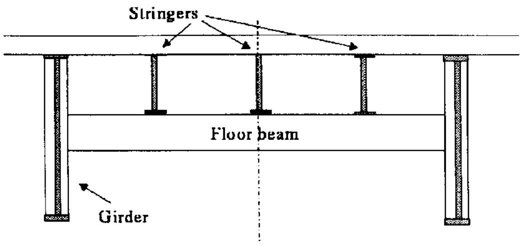


**Figure 2: Bridge cross section**

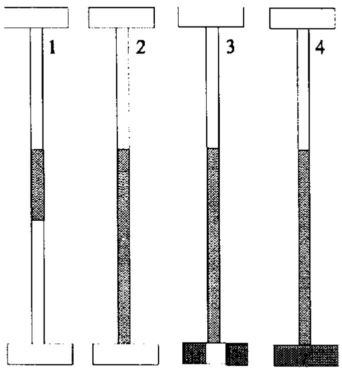


**Figure 3: Section of the damaged girder vs state**

In the following, the different techniques presented here are applied to the measured vibration data.

### 4.1 Frequency Domain Assurance Criterion

The reference FDAC is depicted in figure 4 where the undamaged ODS are compared with themselves. This obviously generates a FDAC without frequency shift (note that the maximal correlation for a given ODS($\omega$) is found at $\omega$ and FDAC values on the axis $\omega = \omega$ are unitary. The cross points on the diagonal correspond to the resonance frequencies. Without performing a classical modal analysis, five eigenfrequencies are identified in the interval [2 - 6] Hz.

Figure 5 shows the FDAC obtained for the last state of damage. It can be observed that the first two eigenfrequencies moved away from the diagonal. The maximum correlation is now less than unity and happens on a discontinuous line. In the interval [3 - 4.5] Hz however, the correlation between ODS remains very close to one and no systematic frequency shift is observed.

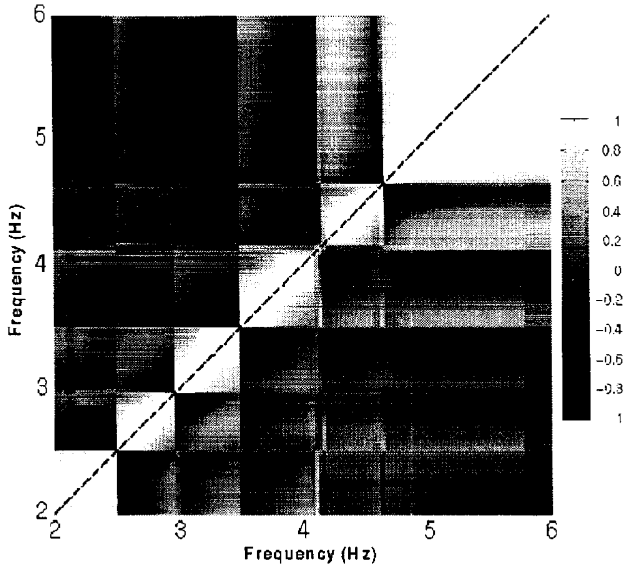


**Figure 4: Reference FDAC (undamaged structure)**

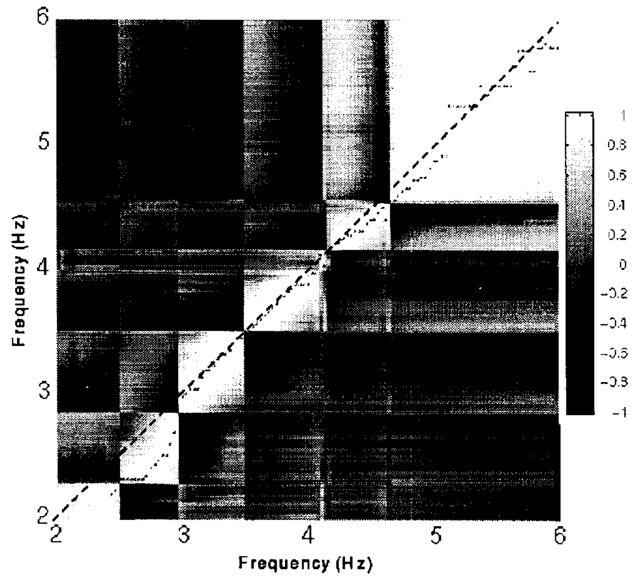


**Figure 5: FDAC (last damage vs reference**

### 4.2 Shifted Residual ODS

The use of FDAC allows the comparison of ODS taking into account the frequency shift. Let us consider for instance the ODS measured at the frequency of 2.72 Hz which is located between the two first eigenfrequencies (2.48 and 2.94 Hz respectively) of the undamaged structure. The reference ODS of the undamaged structure at 2.72 Hz is shown in figure 6.

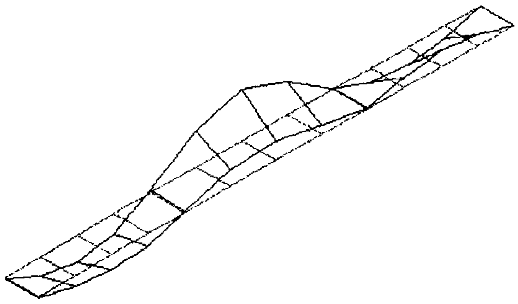

**Figure 6: Reference ODS at 2.72 Hz**

According to FDAC (figure 5), the ODS of the damaged structure which better correlates to it, is situated at the shifted frequency of 2.41 Hz. The situation is also shown in figure 7 where the Bode plot at the damage position for the reference and last damage situation are plotted. The decrease of the resonances is indicated by the resonance peaks shift. The shift of the ODS given by FDAC is also depicted.

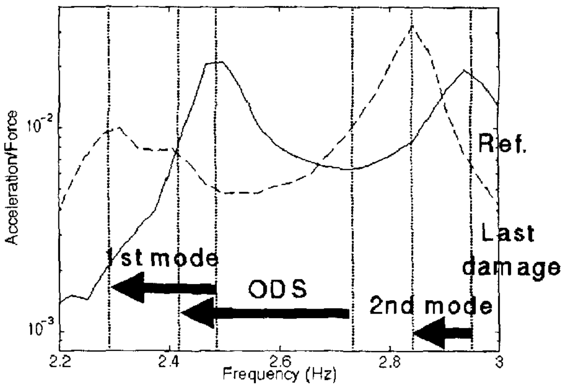


**Figure 7: Bode plot at damage position**

The ODS of the damaged structure at 2.41 Hz and at 2.72 Hz are given respectively in figures 8 and 9. When these ODS are compared with the one given in figure 6, it can be seen that the ODS of the damaged structure at the shifted frequency (2.41 Hz) shows a higher level of similarity with the reference ODS (undamaged structure at 2.72 Hz); the discrepancy between these two ODS appears mainly in the zone where damage was induced. At the opposite, the ODS of the damaged structure at 2.72 Hz looks very different from the reference ODS at the same frequency (2.72 Hz).

The residual ODS which are defined as the difference between these ODS and the reference ODS are shown in figures 10 and 11. It can be observed that the use of the shifted ODS allows a correct localization of the damaged zone which is not the case when comparison frequencies are the same.

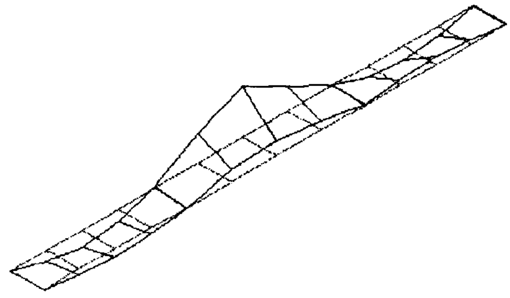

**Figure 8: ODS of the damaged structure at 2.41 Hz**

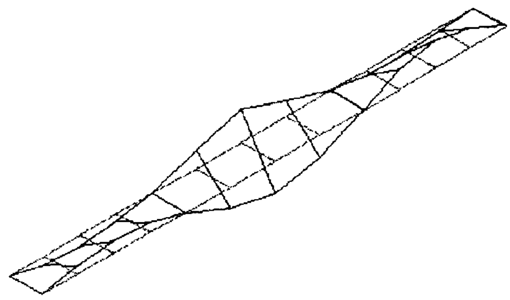

**Figure 9: ODS of the damaged structure at 2.72 Hz**

### 4.3 Model based error localization using MECE

The expanded ODS of the damaged structure at 2.41 Hz is shown in figure 12. It can be compared to figure 8. Results of the MECE error localization using this damaged ODS are shown in figure 13. Results are similar to those obtained with the mode shapes in [7].

## 5 CONCLUSION

The feasibility of a strategy to assess damage evaluation by using measured operating deflection shapes has been studied. The considered procedure is based on a two-level analysis. The first level considers the comparison of ODS by use of non-model based techniques such as FDAC and the residual ODS using the concept of frequency shift. The second level is based on the existence of a validated analytical model of the structure. In this case, the ODS may be expanded and a residual energy based indicator at the local level (element-by-element) is exploited. If necessary, this second level may be enhanced by using a model updating procedure.

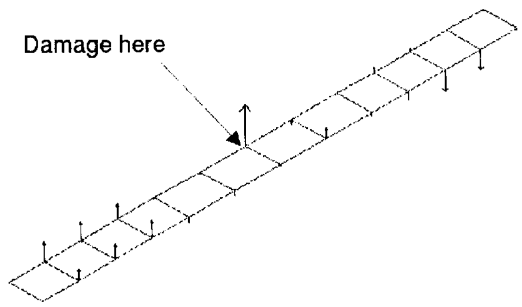


**Figure 10: Residual ODS using the shifted frequency**

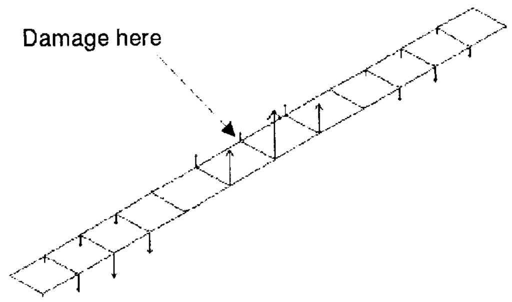


**Figure 11: Residual ODS at 2.72 Hz**

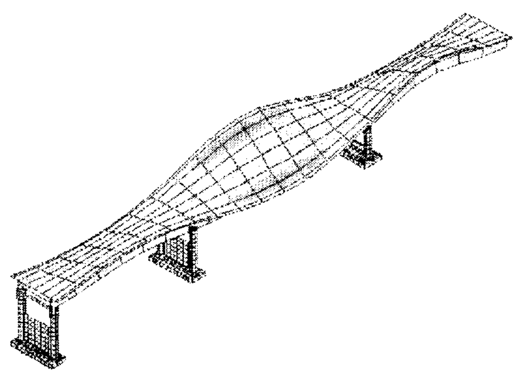

**Figure 12: Expanded ODS at 2.72 Hz**

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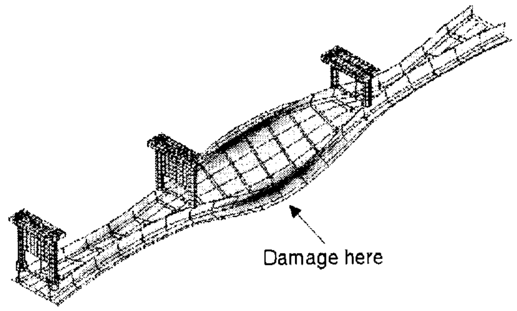


**Figure 13: MECE error localization at 2.72 Hz**

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