Main contributions

**Theoretical:**
- Definition of a new class of valid inequalities, called 2-links, for the standard linearization polytope.
- The 2-links (added to the standard linearization) provide a complete description for functions with two monomials.

**Computational:**
- Great improvements of the continuous relaxation bounds.
- Decrease of computation times (in many cases by factor 10).
- The number of 2-links is only quadratic in the number of terms.

### Problem definition: multilinear 0-1 optimization

\[
\min \sum_{i=1}^{n} \alpha_i x_i + l(x)
\]

subject to:

- \(s.t. \ x_i \in \{0,1\}\)
- \(i = 1, \ldots, n\)
- \(S\): subsets of \(\{1, \ldots, n\}\) with \(as \neq 0\) and \(|S| \geq 2\).
- \(l(x)\): linear part.

### Standard linearization (SL) [2, 3]

\[
\min \sum_{i=1}^{n} \alpha_i y_i + l(x)
\]

subject to:

- \(s.t. \ y_i \leq x_i \wedge y_T = \prod_{i \in T} x_i\)
- \(x_j \geq y_i (\forall i \in S)\)
- \(\forall S \subseteq \{1, \ldots, n\}\)

- For variables \(x_i, y_S \in \{0,1\}\), the convex hull of feasible solutions is \(P_{\text{SL}}\).
- For continuous variables \(x_i, y_S \in [0,1]\), the set of feasible solutions is \(P_{\text{SL}}\).

### The 2-link inequalities [1]

**Definition:**

For \(S, T \in \mathcal{S}\) and \(y_T, y_S\) such that \(y_S = \prod_{i \in S} x_i\), \(y_T = \prod_{i \in T} x_i\),

- the 2-link associated with \((S, T)\) is the linear inequality

\[
y_S \leq y_T - \sum_{i \in T \setminus S} x_i + |T| y_S.
\]

- \(P_{\text{2L}}\) is the polytope defined by the 2-link inequalities and the 2-links.

### Theorem 1: A complete description for the case of two monomials

For the case of two nonlinear monomials, \(P_{\text{SL}} = P_{\text{2L}}\), i.e., the standard linearization and the 2-links provide a complete description of \(P_{\text{SL}}\).

### Theorem 2: Facet-defining inequalities for the case of two monomials

For the case of two nonlinear monomials defined by \(S, T\) with \(|S \cap T| \geq 2\), the 2-links are facet-defining for \(P_{\text{SL}}\).

### Computational experiments: are the 2-links helpful for the general case?

**Objectives:**
- compare the bounds obtained when optimizing over \(P_{\text{SL}}\) and \(P_{\text{2L}}\), i.e., the standard linearization and the 2-links.
- compare the computational performance of exact resolution methods.

Software used: CPLEX 12.06.

### Results random instances

**Opt. gap (%) fixed degree**

- SL
- SL&cplex

**Opt. gap (%) random degree**

- SL
- SL&cplex

**Run times (in sec.) fixed degree**

- SL
- SL&cplex

**Run times (in sec.) random degree**

- SL
- SL&cplex

### Results image restoration instances

**Opt. gap (%) 10x15 images**

- SL
- SL&cplex

**Opt. gap (%) 15x15 images**

- SL
- SL&cplex

**Run times (in sec.) 10x15 images**

- SL
- SL&cplex

**Run times (in sec.) 15x15 images**

- SL
- SL&cplex

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