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Outstanding Student Poster & PICO Contest

Mars Topography Investigated Through the Wavelet Leaders Method: a Multidimensional Study of its Fractal Structure

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joint work with **Thomas Kleyntssens and Samuel Nicolay.** Supported by F.R.S.-FNRS.

1. Introduction and Data

Previous works about the scaling properties of Mars topography revealed two distinct scaling regimes. The scale break and the scaling exponents H vary from one paper to another:

Method	**Small scales**	**Large scales**
Power spectral density [1]	$H \approx 1.2$ (< 10 km)	$H \approx 0.2 - 0.5$
Variance of a wavelet transform [3]	$H \approx 1.25$ (< 24 km)	$H \approx 0.5$
Statistical moments [4]	$H \approx 0.76$ (< 10 km)	$H \approx 0.52$

These studies are all based on along-track measurements, which implies that the 2D part of the topographic field has not been taken into account. We perform longitudinal and latitudinal 1D analysis and a complete 2D study of the surface roughness of Mars using the wavelet leaders method (WLM).

In this work, we use the MOLA data, using the 128 pix/deg map (`http://pds-geosciences.wustl.edu`).

2. The Wavelet Leaders Method

For a given function $f : \mathbf{R}^n \to \mathbf{R}$ and a wavelets basis $(\psi^{(i)})_{1\leq i<2^n}$ of $L^2(\mathbf{R}^n)$, we denote by c_λ the wavelet coefficient associated to the dyadic interval $\lambda := \frac{i}{2^{j+1}} + \frac{k}{2^j} + [0, \frac{1}{2^{j+1}})^n$:

$$c_\lambda = 2^{nj} \int_{\mathbf{R}^n} f(x)\psi^{(i)}(2^j x - k)\,dx.$$

The wavelet leader associated to the interval λ is the quantity

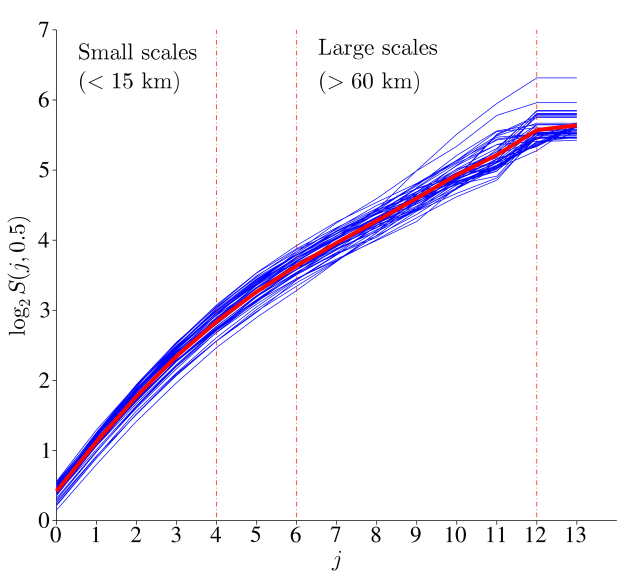
$$d_\lambda = \sup_{\lambda' \subset 3\lambda} |c_{\lambda'}|,$$

where 3λ is the set of intervals consisting of λ and the 2 intervals adjacent to λ. The method is based on the function η defined by

$$\eta(q) = \liminf_{j\to+\infty} \frac{\log S(j,q)}{\log 2^{-j}} \quad \text{where} \quad S(j,q) = 2^{-nj}\sum_\lambda d_\lambda^q.$$

If η is linear, i.e. in practice η has a linear correlation coefficient greater that some threshold $1-\alpha$ (here, $\alpha = 0.02$), then the signal is *monofractal* and the slope of η gives the Hurst exponent ([2]). Otherwise, the signal is *multifractal* and other notions are needed in addition to the slope to fully characterize the fractal nature of the signal (see e.g. [4, 5]).

3. Results for the 1D analysis



Function $j \mapsto \log_2 S(j, 0.5)$ for several longitudinal bands. The scale j corresponds to $0.463 * 2^{j+1}$ kilometers (1 pixel corresponds to 0.463 kilometers). The first vertical dashed line indicates that a scale break occurs at ≈ 15 kilometers.

Results obtained for the longitudes (l) and latitudes (L):

scales	**mono (l)**	**mean H (l)**	**mono (L)**	**mean H (L)**
small	99.7%	1.15 ± 0.06	92.1%	1.05 ± 0.13
large	8.3%	0.78 ± 0.087	36.8%	0.65 ± 0.11

At small scales, the signals are mostly monofractal. At large scale, they are mostly multifractal. A difference appears in the values of H. The influence of latitude can also be noted; it could be the crustal dichotomy of Mars. The difference between latitude and longitude may indicate a slight anisotropy of the surface roughness of Mars.

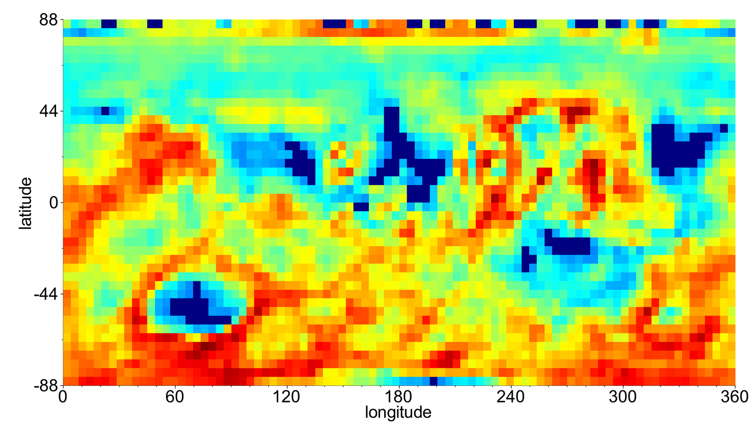
4. Results for the 2D analysis

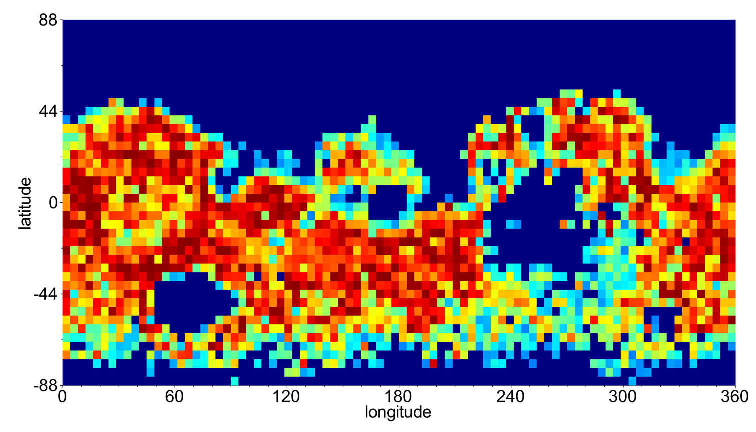
- The map is gridded into tiles of 1024×1024 pixels, corresponding to windows of $8° \times 8°$ on Mars. This choice of the tile size is a good compromise between a local analysis (size not too large) and provides enough results for statistical analyzes (size not too small).
- As in the 1D case, a scale break at ≈ 15 km can be noted.
- Results obtained for the 2D analysis:

scales	**proportion mono**	**mean H**
small (< 15 km)	96.1%	1.12 ± 0.13
large (> 60 km)	54.2%	0.67 ± 0.3

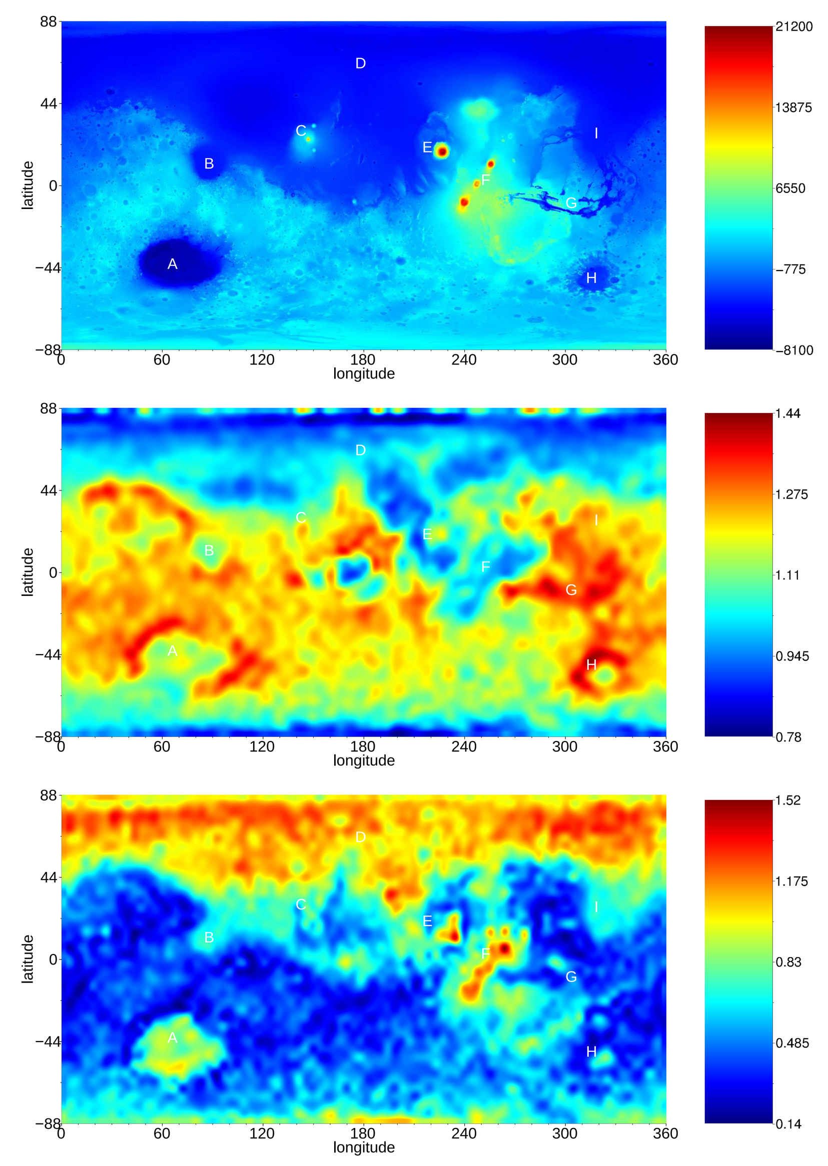
The switch to a multifractal behaviour is still noted for many areas but is less pronounced than in the 1D case. This could be influenced by the lower number of wavelet coefficients available at large scales for each tile which facilitates the detection of a monofractal behaviour.

- The multifractal tiles at small scales and the monofractal tiles at large scales are represented in dark blue in the following figure. It can be seen that they mostly correspond to particular features of the surface of Mars.





- The spatial distribution of the scaling exponents at small and large scales shows that the most distinctive features of Mars can be recovered.



From top to bottom: the topographic map of Mars in false colors, the map representing the spatial distribution of the exponents H at small scales, then at large scales. The regions of interest are Hellas Planitia (A), Isidis Planitia (B), Elysium Mons (C), Vestitas Borealis - Northern plains (D), Olympus Mons (E), Tharsis (F), Valles Marineris (G), Argyre Planitia (H) and Acidalia Planitia (I). It can be noted that most of these regions are clearly detected in at least one of the maps representing the exponents H.

5. Conclusion

This work shows that the WLM is well-suited for studying the irregularity of planetary bodies. Since the WLM is able to handle multidimensional signals, we have done the first complete study of Mars in 2D. It allows, for example, to exhibit a link between the scaling exponents and several famous features of the Martian topography.

References :

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[2] Jaffard, S.: Wavelet techniques in multifractal analysis. Proceedings of symposia in pure mathematics, 72:91–152, 2004.

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[4] Landais, F., Schmidt, F., Lovejoy, S.: Universal multifractal Martian topography. Nonlinear Processes in Geophysics Discussions **2** (2015) 1007–1031.

[5] Lovejoy, S., Schertzer, D.: The Weather and Climate: Emergent Laws and Multifractal Cascades. Cambridge University Press, 2013.