Identification of hardening behavior using inverse modeling and Image Correlation

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ABSTRACT

The basic principle of an inverse modeling procedure as it is used for parameter identification, is the generation of a complex and heterogeneous deformation field that contains as much information as possible about the parameters to be identified. One way of obtaining such a non-homogeneous deformation is by making the geometry of the specimen less regular. Another possibility is to make the loading conditions more complex. In this paper both options are actually combined by using a biaxial tensile test on a cruciform specimen in order to identify the parameters of a Swift isotropic hardening law. The yield criterion is modeled by the isotropic Von Mises criterion. The optimization technique used is a constrained gradient based Newton-type routine, which means that in every iteration step, a sensitivity calculation has to be performed in order to indicate the direction in which the parameters are to be optimized. The functional to be minimized is a least-squares expression of the discrepancy between the measured and the simulated strain fields at a certain load. The numerical routines as well as the identification results, based on simulated strain fields, are discussed.

INTRODUCTION

The accuracy of a Finite Element Simulation for plastic deformation strongly depends on the chosen constitutive laws and the value of the material parameters within these laws. The identification of those mechanical parameters can be done based on homogeneous stress and strain fields such as those obtained in uni-axial tensile tests and simple shear tests performed in different plane material directions. Another way to identify plastic material parameters is by inverse modeling of an experiment exhibiting a heterogeneous stress and strain field. Material parameter identification methods, which integrate optimization techniques and numerical methods such as the finite element method (FEM), indeed offer an alternative tool. The most common approach is to determine the optimal estimates of the model parameters by minimizing a selected measure-of-fit between the responses of the system and the model [1-3]. In the present study a method is proposed for the identification of the yield stress and the two parameters of a Swift isotropic hardening law, based on surface measurements of a cruciform specimen subjected to biaxial tensile loading. Experimental forces and strains are in this case compared to the simulated values and it is tried to reduce the discrepancy in a least-squares sense by optimizing the model parameters. A finite element model of the perforated specimen serves as numerical counterpart for the experimental set-up. The difference between the experimental and numerical strains (ε_x, ε_y and ε_{xy}) is minimized in a least squares sense by updating the values of the three parameters. The sensitivities used to obtain the parameter updates are determined by finite differences, using small parameter perturbations. The optimization routine used, is based on a constrained Newton-type algorithm.

Paragraph two gives a description of the experimental equipment that will be used for the actual tensile testing of the cruciform specimens. Paragraph three briefly documents on the Digital Image Correlation technique, which will be used as the optical full-field measurement technique for the determination of the experimental strain field. In paragraph four, some information is given about the choice of the numerical model. Paragraph five concerns the numerical aspects of the optimization routine and the sensitivity analysis. And finally, paragraph six shows some results obtained based on simulated strain fields.

BIAXIAL TESTING

Different experimental techniques and specimens have been used to produce biaxial stress states. These techniques may be mainly classified into two categories [4]: (i) tests using a single loading system and (ii) tests using two or more independent loading systems. In the first category the biaxial stress ratio depends on the specimen geometry —their main disadvantage—, whereas in the second category it is specified by the applied load magnitude. Examples of the first category are bending tests on cantilever beams, anticlastic bending of rhomboidal shaped plates and bulge tests. Examples of the second category are thin-walled tubes subjected to a combination of tension/compression with torsion or internal/external pressure [5-6], and cruciform specimens under biaxial loading [7]. The most realistic technique to create biaxial stress states consists of applying in-plane loads along two perpendicular arms of cruciform specimens. The use of hydraulic actuators represents a very versatile technique for the application of the loads. The main difference between the existing techniques is the use of one or two actuators per loading direction. One actuator per loading direction [8] will cause movements of the centre of the specimen causing a side bending of the specimen. This results in undesirable non-symmetric strains. Systems with four actuators [9-11] with a close-loop servo control using the measured loads as feedback system, allow the centre of the specimen standing still.
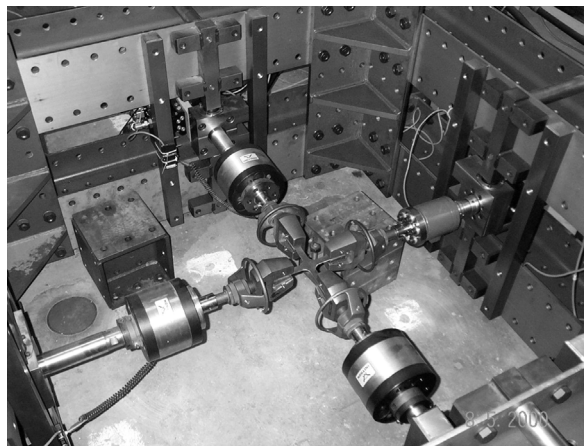

Figure 1: Plane biaxial test device for cruciform specimens.

The plane biaxial test device using cruciform specimens developed at the Free University of Brussels has four independent servo-hydraulic actuators with an appropriate control unit to keep the centre of the specimen explicitly still. The device (Figure 1) has a capacity of 100 kN in both perpendicular directions, but only in tension. As no cylinders with hydrostatic bearing were used, failure or slip in one arm of the specimen will result in sudden radial forces which could seriously damage the servo-hydraulic cylinders and the load cells. To prevent this, hinges were used to connect the specimen to the load cells and the servo-hydraulic cylinders to the test frame. Using four hinges for each loading direction results in an unstable situation in compression and consequently only tension loads can be performed.
In an ideal situation no displacement of the centre point of the specimen is observed. Even when using four actuators, a small displacement might occur in a real situation and an imbalance arises in the forces. However, as four load cells are used, it is possible to quantify this small load difference and to use this as a control signal.

DIGITAL IMAGE CORRELATION

The full field method that will be used for the displacement and strain determination on the cruciform specimens, is the digital image correlation method. This technique allows studying qualitatively as well as quantitatively the

mechanical behaviour of materials under certain loading conditions. It has been used in various technological domains and many of its applications have been reported [12-14]. Each picture taken with a CCD camera corresponds to a different load step. The cameras of the current set-up use a small rectangular piece of silicon, which has been segmented into a number of individual light-sensitive or pixels. Each pixel stores a certain grey value ranging from 0 to 255 in accordance with the intensity of the light reflected by the surface of the tested specimen. Two images of the specimen at different states of deformation are compared by using a set of pixels in the undeformed image and searching for it in the deformed image, in order to maximize a given similarity function. In most cases this function is based on a least squares formulation. Such a set of pixels is called a subset or correlation window. The displacement result, expressed in the centre point of the subset, is an average of the displacements of the pixels inside the subset.

Figure 2 depicts the sequence of taking a picture of an object before and after loading, storing the images onto a PC trough a frame grabber, performing the correlation of both images – i.e. locating the different undeformed subsets in the deformed image – and finally calculating the corresponding displacement of the centers of the subsets, which finally yields the desired displacement field. The strain field is then calculated by numerical differentiation of the smoothened displacement field.
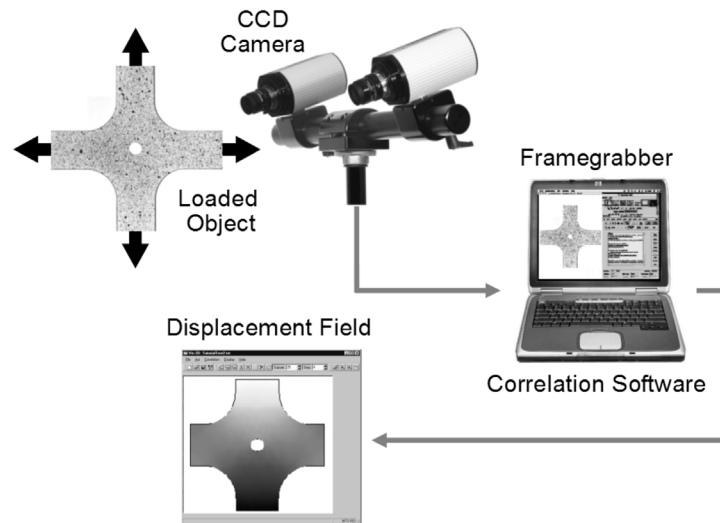

Figure 2: Working principle of the DIC-system

This technique has already been used successfully to identify the elastic parameters of composite materials [15] and it will be used in the future to obtain the displacement and strain fields corresponding to plastic deformation of biaxially loaded metal specimens. As the technique is well suited to measure large deformations it is indeed an adequate tool for the measurement of large plastic deformations occurring in metals.

NUMERICAL MODEL

The first assumption made, concerning the material used, is that it exhibits rate independent elastoplastic behaviour. The elastic part is assumed to be linear and isotropic. The elastoplastic law used in the FE-formulation is based on the decomposition of the total strain into a reversible elastic part ε^{el} and an irreversible plastic part ε^{pl}. In rate form this condition becomes:

$$\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{pl} \tag{1}$$

The stress rate is always related to the elastic strain rate by means of the elastic moduli E:

$$\dot{\sigma} = E\dot{\varepsilon}^{el} \tag{2}$$

For the determination of the plastic strain rate, an associative flow rule is used:

$$\dot{\varepsilon}^{pl} = \dot{\lambda} \frac{\partial \Phi}{\partial \sigma} \tag{3}$$

in which $\dot{\lambda}$ is the plastic multiplier and Φ is the yield function.

The yield function Φ which governs the onset and continuance of plastic deformation is chosen to be represented by the Von Mises yield criterion:

$$\sigma_{y0}^2 = \frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_x - \sigma_z\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + 6\left(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2\right)\right] \tag{4}$$

in which σ_{y0} represents the initial yield stress.

A hardening model is needed to represent the evolution of the yield surface during the process of plastic deformation. The type of hardening considered in the present study is a Swift type of isotropic hardening which describes the evolution of the yield surface size:

$$\sigma_y = \sigma_{y0} + K\left(\varepsilon_{eq}^{pl}\right)^n \tag{5}$$

in which σ_y represents the actual flow stress and ε_{eq}^{pl} the equivalent plastic strain. σ_{y0}, K and n are the three unknown parameters to be identified.

The specimen geometry that is chosen to perform the experimental tests in the near future is shown in figure 3 (left). It has a thickness of 0.8mm. The FE-simulation on the other hand is performed on a quarter of the piece, assuming symmetrical loading conditions. Figure 3 (right) shows an image of the FE-mesh.
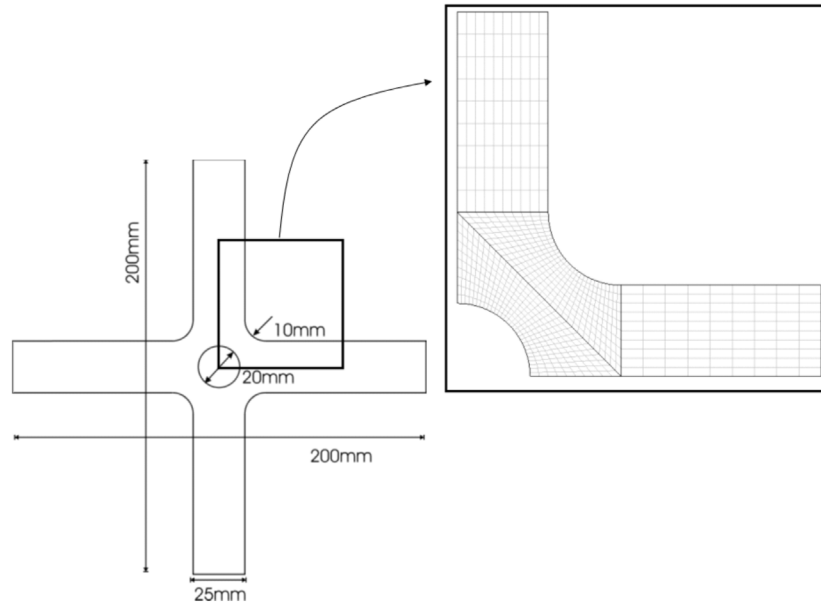

Figure 3: Geometry (left) and FE-mesh (right) of the cruciform specimen

The simulations are performed assuming plane stress conditions. This is an acceptable hypothesis given the thickness of the specimen. The FE-mesh is made up out of isoparametric quadrilateral Lagrange elements of the second order.

The reason for the hole in the specimen is the following: the aim is to localise the plastic deformation in the central part of the specimen where a biaxial stress and strain state exists. For the identification of the parameters of the hardening law, this may not be really necessary. However, for the identification of more complex anisotropic yield criteria – which is the ultimate goal – it is mandatory to obtain non-uniaxial plastic deformation.

INVERSE MODELLING AND SENSITIVITY CALCULATION

A direct problem is the classical problem where a given experiment is simulated in order to obtain the final geometry of the specimen, the stresses and the strains. Inverse problems on the other hand are concerned with the determination of the unknown state of a mechanical system considered as a black box, using information gathered from the response to stimuli on the system [16]. The inverse problem is a problem where certain input data of the direct problem is deduced from the comparison between the experimental results and the numerical FE-simulation of that same problem. Not only the boundary information is used, but relevant information coming from local or full-field surface measurements is also integrated in theevaluation of the behavior of a given material.

The values of the material parameters cannot be derived immediately from the experiment. A numerical analysis is necessary to simulate the actual experiment. However, this requires that the material parameters are known. The identification problem can then be formulated as an optimization problem where the function to be minimized is some error function that expresses the difference between numerical simulation results and experimental data. In the present case the strains are used as output data. Figure 4 represents the flow-chart of the present inverse modeling problem.
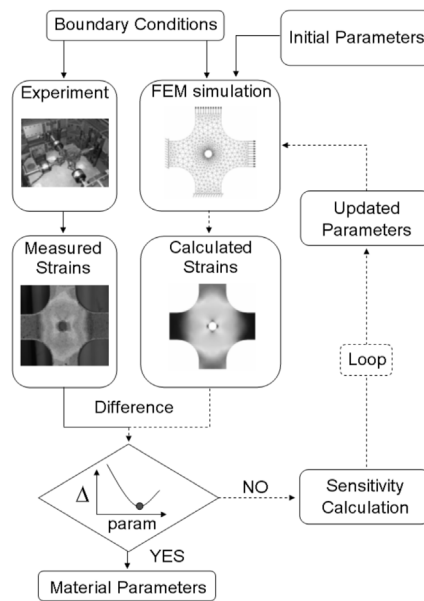

Figure 4- Inverse Modeling Flow-chart

Expression (6) shows the form of the least-squares cost function $C(\underline{p})$ that is minimized, with $\underline{p}$ the vector of material parameters to be identified. The residuals in the function are formed by the differences between the experimental and the numerical strains at every considered load step. The index "t" in expression (6) stands for the total number of elements and the index "s" stands for the total number of considered load steps.

$$C(\underline{p}) = \sqrt{\sum_{j=1}^{s}\left(\sum_{i=1}^{t}\left\langle\frac{\varepsilon_{x_i}^{num}(\underline{p})-\varepsilon_{x_i}^{exp}}{\varepsilon_{x_i}^{exp}}\right\rangle^2 + \left\langle\frac{\varepsilon_{y_i}^{num}(\underline{p})-\varepsilon_{y_i}^{exp}}{\varepsilon_{y_i}^{exp}}\right\rangle^2 + \left\langle\frac{\varepsilon_{xy_i}^{num}(\underline{p})-\varepsilon_{xy_i}^{exp}}{\varepsilon_{xy_i}^{exp}}\right\rangle^2\right)_j} \tag{6}$$

The necessary condition for a cost function to attain its minimum is expressed by (7). The partial derivative of the function with respect to the different material parameters has to be zero:

$$\frac{\partial C(\underline{p})}{\partial p_r} = \frac{1}{C(\underline{p})}\sum_{j=1}^{s}\sum_{i=1}^{t}\left(\frac{\varepsilon_{x_i}^{num}(\underline{p})-\varepsilon_{x_i}^{exp}}{\varepsilon_{x_i}^{exp}}\right)\frac{\partial\varepsilon_{x_i}^{num}}{\partial p_r} + \left(\frac{\varepsilon_{y_i}^{num}(\underline{p})-\varepsilon_{y_i}^{exp}}{\varepsilon_{y_i}^{exp}}\right)\frac{\partial\varepsilon_{y_i}^{num}}{\partial p_r} + \left(\frac{\varepsilon_{xy_i}^{num}(\underline{p})-\varepsilon_{xy_i}^{exp}}{\varepsilon_{xy_i}^{exp}}\right)\frac{\partial\varepsilon_{xy_i}^{num}}{\partial p_r} = 0 \tag{7}$$

By developing a Taylor expansion of the numerical (FEM) strains around a given parameter set, an expression is obtained in which the difference between the current parameters and their new estimates is given (8), m is the total number of parameters:

$$\varepsilon_i^{num}(\underline{p}) \cong \varepsilon_i^{num}(\underline{p}^k) + \sum_{j=1}^{m} \frac{\partial \varepsilon_i^{num}(\underline{p}^k)}{\partial p_j}\left(p_j - p_j^k\right) + \Theta^2 \tag{8}$$

When substituting this last expression into expression (7) and after rearranging some terms, the expression yielding the parameter updates is obtained (9):

$$\underline{\Delta p} = \left(\underline{\underline{S}}^t\, \underline{\underline{S}}\right)^{-1}\underline{\underline{S}}^t\left(\varepsilon^{exp} - \varepsilon^{num}\left(\underline{p}^k\right)\right) \tag{9}$$

in which the following elements are:

$\underline{\Delta p}$:   column vector of the parameter updates of σ_{y0}, K and n

$\underline{\varepsilon}^{exp}$:   column vector of the experimental strains

$\underline{\varepsilon}^{num}(\underline{p}^k)$:   column vector of the finite element strains as a function

     of the different parameters at iteration step k

$\underline{p}^k$:   the three parameters at iteration step k

$\underline{\underline{S}}$:   Sensitivity matrix

The sensitivity matrix groups the sensitivity coefficients of the strain components in every element of the FE mesh and for every considered load step with respect to the different material parameters.

$$\underline{\underline{S}} = \begin{bmatrix} \dfrac{\partial \varepsilon_{x_1}^1}{\partial \sigma_{y0}} & \dfrac{\partial \varepsilon_{x_1}^1}{\partial K} & \dfrac{\partial \varepsilon_{x_1}^1}{\partial n} \\[2ex] \dfrac{\partial \varepsilon_{y_1}^1}{\partial \sigma_{y0}} & \dfrac{\partial \varepsilon_{y_1}^1}{\partial K} & \dfrac{\partial \varepsilon_{y_1}^1}{\partial n} \\[2ex] \dfrac{\partial \varepsilon_{xy_1}^1}{\partial \sigma_{y0}} & \dfrac{\partial \varepsilon_{xy_1}^1}{\partial K} & \dfrac{\partial \varepsilon_{xy_1}^1}{\partial n} \\[1ex] \vdots & \vdots & \vdots \\[1ex] \dfrac{\partial \varepsilon_{xy_t}^1}{\partial \sigma_{y0}} & \dfrac{\partial \varepsilon_{xy_t}^1}{\partial K} & \dfrac{\partial \varepsilon_{xy_t}^1}{\partial n} \\[1ex] \vdots & \vdots & \vdots \\[1ex] \dfrac{\partial \varepsilon_{xy_t}^s}{\partial \sigma_{y0}} & \dfrac{\partial \varepsilon_{xy_t}^s}{\partial K} & \dfrac{\partial \varepsilon_{xy_t}^s}{\partial n} \end{bmatrix} \qquad \text{sensitivity matrix} \tag{10}$$

in which $\dfrac{\partial \varepsilon_{xy_t}^s}{\partial n}$ is the partial derivative of the shear strain component of element number "t" at load step "s" with respect to parameter "n".

IDENTIFICATION USING SIMULATED STRAIN FIELDS

In order to test the proposed routine, a virtual experiment is set up. The reference values of the three parameters used for the simulation of the direct problem and the starting values for the optimization routine are shown in table1.

Table 1: Reference values and starting values for the numerical validation

	σ_{y0} (MPa)	K	n
Reference values	100	1000	0.8
Starting values	90	800	0.95
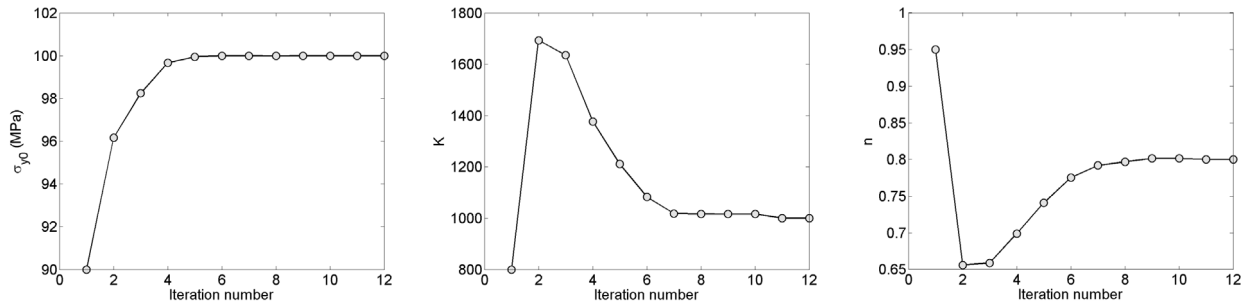
Figure 5: Results of the identification of σ_{y0} (left), K (middle) and n (right)

The results of the identification procedure are shown in figure 5. It has been found that the value of the starting parameters does not affect the final and optimal parameter values.

CONCLUSION

A method has been proposed to determine the parameters (σ_{y0}, K and n) of a Swift type of isotropic hardening law. The method is based on the inverse modeling of a perforated cruciform specimen under biaxial tension. A least-squares formulation of the difference between the experimental and the numerical strains is used along with a constrained Newton-type algorithm in the optimization process. A virtual experiment, in which the material parameters are known, is used to check the routine and to analyze the influence of the starting values on the obtained parameter estimates. The ultimate goal of using the cruciform geometry under biaxial tensile conditions is to be able to identify not only the parameters of a hardening law but those corresponding to a more complex yield criterion as well (e.g. Hill48).

ACKNOWLEDGEMENTS

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